Chapter 13

Mathematical Reasoning

1. **Statement-1 :** $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

   Statement-2 : $\sim (p \leftrightarrow \sim q)$ is a tautology.

   [AIEEE-2009]

   (1) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

   (2) Statement-1 is true, Statement-2 is false

   (3) Statement-1 is false, Statement-2 is true

   (4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

2. Let S be a non-empty subset of R Consider the following statement :

   P : There is a rational number $x \in S$ such that $x > 0$.

   Which of the following statements is the negation of the statement P?

   [AIEEE-2010]

   (1) There is a rational number $x \in S$ such that $x \leq 0$

   (2) There is no rational number $x \in S$ such that $x \leq 0$

   (3) Every rational number $x \in S$ satisfies $x \leq 0$

   (4) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational

3. The only statement among the followings that is a tautology is

   [AIEEE-2011]

   (1) $[A \wedge (A \rightarrow B)] \rightarrow B$

   (2) $B \rightarrow [A \wedge (A \rightarrow B)]$

   (3) $A \wedge (A \vee B)$

   (4) $A \vee (A \wedge B)$

4. The negation of the statement

   "If I become a teacher, then I will open a school", is **[AIEEE-2012]**

   (1) Either I will not become a teacher or I will not open a school

   (2) Neither I will become a teacher nor I will open a school

   (3) I will not become a teacher or I will open a school

   (4) I will become a teacher and I will not open a school

5. Consider :

   Statement - I : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

   Statement - II : $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology. **[JEE (Main)-2013]**

   (1) Statement - I is true; Statement-II is true; Statement - II is a correct explanation for Statement - I.

   (2) Statement - I is true; Statement - II is true; Statement-II is **not** a correct explanation for Statement-I.

   (3) Statement-I is true; Statement - II is false.

   (4) Statement - I is false; Statement - II is true.

6. The statement $\sim(p \leftrightarrow \sim q)$ is **[JEE (Main)-2014]**

   (1) A tautology

   (2) A fallacy

   (3) Equivalent to $p \leftrightarrow q$

   (4) Equivalent to $\sim p \leftrightarrow q$

7. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to **[JEE (Main)-2015]**

   (1) $s \wedge \sim r$ (2) $s \wedge (r \wedge \sim s)$

   (3) $s \vee (r \vee \sim s)$ (4) $s \wedge r$

8. The Boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to : **[JEE (Main)-2016]**

   (1) $p \wedge q$ (2) $p \vee q$

   (3) $p \vee \sim q$ (4) $\sim p \wedge q$

9. The following statement $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ is

   [JEE (Main)-2017]

   (1) Equivalent to $\sim p \rightarrow q$

   (2) Equivalent to $p \rightarrow \sim q$

   (3) A fallacy

   (4) A tautology

10. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to

[JEE (Main)-2018]

(1) ~p (2) p

(3) q (4) $\sim q$

11. If the Boolean expression

$(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to

$p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is

[JEE (Main)-2019]

(1) $(\vee, \wedge)$ (2) $(\vee, \vee)$

(3) $(\wedge, \wedge)$ (4) $(\wedge, \vee)$

12. The logical statement

$[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$ is equivalent to

[JEE (Main)-2019]

(1) $(p \wedge r) \wedge \sim q$ (2) $(p \wedge \sim q) \vee r$

(3) $(\sim p \wedge \sim q) \wedge r$ (4) $\sim p \vee r$

13. Consider the following three statements

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true?

JEE (Main)-2019]

(1) $(\sim P) \wedge (\sim Q \wedge R)$

(2) $(\sim P) \vee (Q \wedge R)$

(3) $P \vee (\sim Q \wedge R)$

(4) $(P \wedge Q) \vee (\sim R)$

14. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?

[JEE (Main)-2019]

(1) $p \vee r$ (2) $(p \wedge r) \to (p \vee r)$

(3) $(p \vee r) \to (p \wedge r)$ (4) $p \wedge r$

15. Contrapositive of the statement

"If two numbers are not equal, then their squares are not equal." is **[JEE (Main)-2019]**

(1) If the squares of two numbers are equal, then the numbers are equal

(2) If the squares of two numbers are not equal, then the numbers are equal

(3) If the squares of two numbers are equal, then the numbers are not equal

(4) If the squares of two numbers are not equal, then the numbers are not equal

16. The Boolean expression $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to **[JEE (Main)-2019]**

(1) $p \wedge q$ (2) $(\sim p) \wedge (\sim q)$

(3) $p \wedge (\sim q)$ (4) $p \vee (\sim q)$

17. The expression $\sim(\sim p \to q)$ is logically equivalent to **[JEE (Main)-2019]**

(1) $p \wedge q$ (2) $p \wedge \sim q$

(3) $\sim p \wedge \sim q$ (4) $\sim p \wedge q$

18. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is:

[JEE (Main)-2019]

(1) If you are born in India, then you are not a citizen of India.

(2) If you are not born in India, then you are not a citizen of India.

(3) If you are a citizen of India, then you are born in India.

(4) If you are not a citizen of India, then you are not born in India.

19. Which one of the following statements is not a tautology? **[JEE (Main)-2019]**

(1) $(p \wedge q) \to (\sim p) \vee q$

(2) $(p \wedge q) \to p$

(3) $(p \vee q) \to (p \vee (\sim q))$

(4) $p \to (p \vee q)$

20. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is **[JEE (Main)-2019]**

(1) $\sim p \wedge \sim q$ (2) $\sim p \vee \sim q$

(3) $p \wedge q$ (4) $p \leftrightarrow q$

21. If $p \Rightarrow (q \vee r)$ is false, then truth values of p, q, r are respectively **[JEE (Main)-2019]**

(1) T, F, F (2) F, F, F

(3) T, T, F (4) F, T, T

22. Which one of the following Boolean expressions is a tautology? **[JEE (Main)-2019]**

(1) $(p \vee q) \vee (p \vee \sim q)$ (2) $(p \wedge q) \vee (p \wedge \sim q)$

(3) $(p \vee q) \wedge (p \vee \sim q)$ (4) $(p \vee q) \wedge (\sim p \vee \sim q)$

23. The negation of the Boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to **[JEE (Main)-2019]**

(1) $s \wedge r$ (2) r

(3) $\sim s \wedge \sim r$ (4) $s \vee r$

24. If the truth value of the statement $p \to (\sim q \vee r)$ is false (F), then the truth values of the statements p, q, r are respectively **[JEE (Main)-2019]**
(1) F, T, T (2) T, T, F
(3) T, F, F (4) T, F, T

25. The Boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to **[JEE (Main)-2019]**
(1) $(\sim p) \Rightarrow q$ (2) $p \vee q$
(3) $p \wedge q$ (4) $q \Rightarrow \sim p$

26. The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to **[JEE (Main)-2020]**
(1) $\sim q$ (2) p
(3) q (4) $\sim p$

27. Which one of the following is a tautology?
(1) $Q \to (P \wedge (P \to Q))$ **[JEE (Main)-2020]**
(2) $P \vee (P \wedge Q)$
(3) $P \wedge (P \vee Q)$
(4) $(P \wedge (P \to Q)) \to Q$

28. Which of the following statements is a tautology? **[JEE (Main)-2020]**
(1) $\sim(p \wedge \sim q) \to p \vee q$
(2) $p \vee (\sim q) \to p \wedge q$
(3) $\sim(p \vee \sim q) \to p \vee q$
(4) $\sim(p \vee \sim q) \to p \wedge q$

29. Negation of the statement :
'$\sqrt{5}$ is an integer or 5 is irrational' is **[JEE (Main)-2020]**
(1) $\sqrt{5}$ is not an integer and 5 is not irrational
(2) $\sqrt{5}$ is an integer and 5 is irrational
(3) $\sqrt{5}$ is not an integer or 5 is not irrational
(4) $\sqrt{5}$ is irrational or 5 is an integer

30. If $p \to (p \wedge \sim q)$ is false, then the truth values of p and q are respectively **[JEE (Main)-2020]**
(1) T, T (2) F, F
(3) T, F (4) F, T

31. The contrapositive of the statement "If *I* reach the station in time, then *I* will catch the train" is **[JEE (Main)-2020]**
(1) If *I* will catch the train, then *I* reach the station in time
(2) If *I* do not reach the station in time, then *I* will catch the train
(3) If *I* do not reach the station in time, then *I* will not catch the train
(4) If *I* will not catch the train, then *I* do not reach the station in time

32. Which of the following is a tautology? **[JEE (Main)-2020]**
(1) $(\sim p) \wedge (p \vee q) \to q$
(2) $(\sim q) \vee (p \wedge q) \to q$
(3) $(p \to q) \wedge (q \to p)$
(4) $(q \to p) \vee \sim(p \to q)$

33. The proposition $p \to \sim(p \wedge \sim q)$ is equivalent to **[JEE (Main)-2020]**
(1) q (2) $(\sim p) \wedge q$
(3) $(\sim p) \vee (\sim q)$ (4) $(\sim p) \vee q$

34. Let p, q, r be three statements such that the truth value of $(p \wedge q) \to (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively **[JEE (Main)-2020]**
(1) T, F, T (2) F, T, F
(3) T, T, T (4) T, T, F

35. Given the following two statements **[JEE (Main)-2020]**
(S_1) : $(q \vee p) \to (p \leftrightarrow \sim q)$ is a tautology :
(S_2) : $\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then :
(1) only (S_1) is correct.
(2) both (S_1) and (S_2) are correct.
(3) only (S_2) is correct.
(4) both (S_1) and (S_2) are not correct.

36. Contrapositive of the statement **[JEE (Main)-2020]**
'If a function *f* is differentiable at *a*, then it is also continuous at *a*', is
(1) If a function *f* is continuous at *a*, then it is differentiable at *a*.
(2) If a function *f* is not continuous at *a*, then it is not differentiable at *a*.
(3) If a function *f* is not continuous at *a*, then it is differentiable at *a*.
(4) If a function *f* is continuous at *a*, then it is not differentiable at *a*.

37. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to **[JEE (Main)-2020]**
(1) $(x \wedge \sim y) \vee (\sim x \wedge y)$
(2) $(x \wedge y) \vee (\sim x \wedge \sim y)$
(3) $(x \wedge y) \wedge (\sim x \vee \sim y)$
(4) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

38. The statement $(p \to (q \to p)) \to (p \to (p \vee q))$ is **[JEE (Main)-2020]**

(1) A tautology

(2) Equivalent to $(p \vee q) \wedge (\sim p)$

(3) A contradiction

(4) Equivalent to $(p \wedge q) \vee (\sim q)$

39. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to **[JEE (Main)-2020]**

(1) $\sim p \vee q$ (2) $p \wedge \sim q$

(3) $\sim p \vee \sim q$ (4) $\sim p \wedge \sim q$

40. Consider the statement : "For an integer n, if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is **[JEE (Main)-2020]**

(1) For an integer n, if n is odd, then $n^3 - 1$ is even

(2) For an integer n, if n is even, then $n^3 - 1$ is even.

(3) For an integer n, if n is even, then $n^3 - 1$ is odd.

(4) For an integer n, if $n^3 - 1$ is not even, then n is not odd.

41. The statement among the following that is a tautology is : **[JEE (Main)-2021]**

(1) $A \vee (A \wedge B)$ (2) $B \to [A \wedge (A \to B)]$

(3) $[A \wedge (A \to B)] \to B$ (4) $A \wedge (A \vee B)$

42. For the statements p and q, consider the following compound statements: **[JEE (Main)-2021]**

(a) $(\sim q \wedge (p \to q)) \to \sim p$

(b) $((p \vee q) \wedge \sim p) \to q$

Then which of the following statements is correct?

(1) (a) and (b) both are tautologies.

(2) (a) is a tautology but not (b).

(3) (b) is a tautology but not (a).

(4) (a) and (b) both are not tautologies.

43. The negation of the statement

$\sim p \wedge (p \vee q)$ is : **[JEE (Main)-2021]**

(1) $p \wedge \sim q$ (2) $\sim p \vee q$

(3) $\sim p \wedge q$ (4) $p \vee \sim q$

44. The statement $A \to (B \to A)$ is equivalent to : **[JEE (Main)-2021]**

(1) $A \to (A \leftrightarrow B)$ (2) $A \to (A \wedge B)$

(3) $A \to (A \to B)$ (4) $A \to (A \vee B)$

45. The contrapositive of the statement "If you will work, you will earn money" is : **[JEE (Main)-2021]**

(1) If you will earn money, you will work

(2) You will earn money, if you will not work

(3) If you will not earn money, you will not work

(4) To earn money, you need to work

46. Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \to \sim A)$ be two logical expressions. Then : **[JEE (Main)-2021]**

(1) F_1 and F_2 both are tautologies

(2) Both F_1 and F_2 are not tautologies

(3) F_1 is a tautology but F_2 is not a tautology

(4) F_1 is not a tautology but F_2 is a tautology

[JEE (Main)-2021]

47. Which of the following Boolean expression is a tautology?

(1) $(p \wedge q) \to (p \to q)$ (2) $(p \wedge q) \wedge (p \to q)$

(3) $(p \wedge q) \vee (p \vee q)$ (4) $(p \wedge q) \vee (p \to q)$

[JEE (Main)-2021]

48. If the Boolean expression $(P \Rightarrow q) \Leftrightarrow (q * (\sim p))$ is a tautology, then the Boolean expression $p * (\sim q)$ is equivalent to : **[JEE (Main)-2021]**

(1) $p \Rightarrow q$ (2) $q \Rightarrow p$

(3) $\sim q \Rightarrow p$ (4) $p \Rightarrow \sim q$

49. If the Boolean expression $(p \wedge q) \circledast (p \otimes q)$ is a tautology, then $\circledast$ and $\otimes$ are respectively given by : **[JEE (Main)-2021]**

(1) $\to, \to$ (2) $\wedge, \to$

(3) $\wedge, \vee$ (4) $\vee, \to$

50. If P and Q are two statements, then which of the following compound statement is a tautology? **[JEE (Main)-2021]**

(1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$

(2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$

(3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$

(4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$

51. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to **[JEE (Main)-2021]**

(1) $q \Rightarrow p$ (2) $p \Rightarrow q$

(3) $p \Rightarrow \sim q$ (4) $\sim q \Rightarrow p$

52. Consider the following three statements

(A) If 3 + 3 = 7 then 4 + 3 = 8.

(B) If 5 + 3 = 8 then earth is flat.

(C) If both (A) and (B) are true then 5 + 6 = 17.

Then, which of the following statements is correct? **[JEE (Main)-2021]**

(1) (A) and (B) are false while (C) is true

(2) (A) is false, but (B) and (C) are true

(3) (A) and (C) are true while (B) is false

(4) (A) is true while (B) and (C) are false

53. Which of the following Boolean expressions is **not** a tautology? **[JEE (Main)-2021]**

(1) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

(2) $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$

(3) $(q \Rightarrow P) \vee (\sim q \Rightarrow p)$

(4) $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$

54. The Boolean expression

$(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to **[JEE (Main)-2021]**

(1) $\sim p$ (2) $\sim q$

(3) p (4) q

55. Consider the statement "The match will be played only if the weather is good and ground is not wet". Select the correct negation from the following **[JEE (Main)-2021]**

(1) The match will not be played or weather is good and ground is not wet

(2) The match will be played and weather is not good or ground is wet

(3) The match will not be played and weather is not good and ground is wet

(4) If the match will not be played, then either weather is not good or ground is wet

56. The compound statement $(P \vee Q) \wedge (\sim P) \Rightarrow Q$ is equivalent to : **[JEE (Main)-2021]**

(1) $P \vee Q$

(2) $\sim(P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$

(3) $P \wedge \sim Q$

(4) $\sim(P \Rightarrow Q)$

57. Which of the following is the negation of the statement "for all M > 0, there exists $x \in S$ such that $x \geq M$"? **[JEE (Main)-2021]**

(1) There exists M > 0, there exists $x \in S$ such that $x < M$

(2) There exists M > 0, there exists $x \in S$ such that $x \geq M$

(3) There exists M > 0, such that $x < M$ for all $x \in S$

(4) There exists M > 0, such that $x \geq M$ for all $x \in S$

58. If the truth value of the Boolean expression $((p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)) \rightarrow (p \wedge q)$ is false, then the truth values of the statements p, q, r respectively can be **[JEE (Main)-2021]**

(1) F F T (2) F T F

(3) T F F (4) T F T

59. Consider the two statements :

(S1): $(p \rightarrow q) \vee (\sim q \rightarrow p)$ is a tautology.

(S2): $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy.

[JEE (Main)-2021]

(1) Only (S2) is true

(2) Only (S1) is true

(3) Both (S1) and (S2) are true

(4) Both (S1) and (S2) are false

60. The statement $\left(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)\right) \rightarrow r$ is **[JEE (Main)-2021]**

(1) a fallacy

(2) equivalent to $q \rightarrow \sim r$

(3) equivalent to $p \rightarrow \sim r$

(4) a tautology

61. The Boolean expression $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$ is equivalent to **[JEE (Main)-2021]**

(1) $(p \wedge q) \Rightarrow (r \wedge q)$ (2) $(q \wedge r) \Rightarrow (p \wedge q)$

(3) $(p \wedge r) \Rightarrow (p \wedge q)$ (4) $(p \wedge q) \Rightarrow (r \vee q)$

62. Let $*, \square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p * \sim q) \Rightarrow (p \square q)$ is a tautology. Then: **[JEE (Main)-2021]**

(1) $* = \wedge$, $\square = \wedge$ (2) $* = \vee$, $\square = \wedge$

(3) $* = \vee$, $\square = \vee$ (4) $* = \wedge$, $\square = \vee$

63. Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is **[JEE (Main)-2021]**

(1) $\sim p \wedge q \wedge \sim r$ (2) $p \wedge \sim q \wedge \sim r$

(3) $\sim p \wedge q \wedge r$ (4) $p \wedge q \wedge r$

64. Which of the following is equivalent to the Boolean expression $p \wedge \sim q$? **[JEE (Main)-2021]**

(1) $\sim(q \rightarrow p)$ (2) $\sim(p \rightarrow \sim q)$

(3) $\sim(p \rightarrow q)$ (4) $\sim p \rightarrow \sim q$

65. The number of choices for $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, such that $(p \,\Delta\, q) \Rightarrow ((p \,\Delta \sim q) \vee ((\sim p) \,\Delta\, q))$ is a tautology, is [JEE (Main)-2022]

(1) 1 (2) 2

(3) 3 (4) 4

66. Consider the following statements:

A : Rishi is a judge.

B : Rishi is honest.

C : Rishi is not arrogant.

The negation of the statement “if Rishi is a judge and he is not arrogant, then he is honest” is

[JEE (Main)-2022]

(1) $B \to (A \vee C)$ (2) $(\sim B) \wedge (A \wedge C)$

(3) $B \to ((\sim A) \vee (\sim C))$ (4) $B \to (A \wedge C)$

67. Consider the following two propositions :

$P1 : \sim (p \to \sim q)$

$P2 : (p \wedge \sim q) \wedge ((\sim p) \vee q)$

If the proposition $p \to ((\sim p) \vee q)$ is evaluated as FALSE, then : [JEE (Main)-2022]

(1) $P1$ is TRUE and $P2$ is FALSE

(2) $P1$ is FALSE and $P2$ is TRUE

(3) Both $P1$ and $P2$ are FALSE

(4) Both $P1$ and $P2$ are TRUE

68. The negation of the Boolean expression $((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$ is logically equivalent to :

[JEE (Main)-2022]

(1) $p \Rightarrow q$ (2) $q \Rightarrow p$

(3) $\sim (p \Rightarrow q)$ (4) $\sim (q \Rightarrow p)$

69. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $p \nabla q \Rightarrow ((p \Delta q) \nabla r)$ is a tautology. Then $(p \nabla q) \Delta r$ is logically equivalent to : [JEE (Main)-2022]

(1) $(p \Delta r) \vee q$ (2) $(p \Delta r) \wedge q$

(3) $(p \wedge r) \Delta q$ (4) $(p \nabla r) \wedge q$

70. Let $r \in \{p, q, \sim p, \sim q\}$ be such that the logical statement $r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$ is a tautology. Then r is equal to : [JEE (Main)-2022]

(1) p (2) q

(3) $\sim p$ (4) $\sim q$

71. The boolean expression $(\sim(p \wedge q)) \vee q$ is equivalent to: [JEE (Main)-2022]

(1) $q \to (p \wedge q)$ (2) $p \to q$

(3) $p \to (p \to q)$ (4) $p \to (p \vee q)$

72. Which of the following statement is a tautology?

[JEE (Main)-2022]

(1) $((\sim q) \wedge p) \wedge q$

(2) $((\sim q) \wedge p) \wedge (p \wedge (\sim p))$

(3) $((\sim q) \wedge p) \vee (p \vee (\sim p))$

(4) $(p \wedge q) \wedge (\sim (p \wedge q))$

73. Let p, q, r be three logical statements. Consider the compound statements

$S_1 : ((\sim p) \vee q) \vee ((\sim p) \vee r)$ and

$S_2 : p \to (q \vee r)$

Then, which of the following is **NOT** true?

[JEE (Main)-2022]

(1) If S_2 is True, then S_1 is True

(2) If S_2 is False, then S_1 is False

(3) If S_2 is False, then S_1 is True

(4) If S_1 is False, then S_2 is False

74. The maximum number of compound propositions, out of $p \vee r \vee s$, $p \vee r \vee \sim s$, $p \vee \sim q \vee s$, $\sim p \vee \sim r \vee s$, $\sim p \vee \sim r \vee \sim s$, $\sim p \vee q \vee \sim s$, $q \vee r \vee \sim s$, $q \vee \sim r \vee \sim s$, $\sim p \vee \sim q \vee \sim s$ that can be made simultaneously true by an assignment of the truth values to p, q, r and s, is equal to ____________ . [JEE (Main)-2022]

75. Let $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ be such that $(p \wedge q) \Delta ((p \vee q) \Rightarrow q)$ is a tautology. Then Δ is equal to : [JEE (Main)-2022]

(1) $\wedge$ (2) $\vee$

(3) $\Rightarrow$ (4) $\Leftrightarrow$

76. Negation of the Boolean statement $(p \vee q) \Rightarrow ((\sim r) \vee p)$ is equivalent to [JEE (Main)-2022]

(1) $p \wedge (\sim q) \wedge r$ (2) $(\sim p) \wedge (\sim q) \wedge r$

(3) $(\sim p) \wedge q \wedge r$ (4) $p \wedge q \wedge (\sim r)$

77. Which of the following statements is a tautology?

[JEE (Main)-2022]

(1) $((\sim p) \vee q) \Rightarrow p$ (2) $p \Rightarrow ((\sim p) \vee q)$

(3) $((\sim p) \vee q) \Rightarrow q$ (4) $q \Rightarrow ((\sim p) \vee q)$

78. The statement $(\sim(p \Leftrightarrow \sim q)) \wedge q$ is : [JEE (Main)-2022]

(1) a tautology

(2) a contradiction

(3) equivalent to $(p \Rightarrow q) \wedge q$

(4) equivalent to $(p \Rightarrow q) \wedge p$

79. $(p \wedge r) \Leftrightarrow (p \wedge (\sim q))$ is equivalent to $(\sim p)$ when r is [JEE (Main)-2022]

(1) p (2) $\sim p$

(3) q (4) $\sim q$

80. If the truth value of the statement $(P \wedge (\sim R)) \rightarrow ((\sim R) \wedge Q)$ is F, then the truth value of which of the following is F? [JEE (Main)-2022]

(1) $P \vee Q \rightarrow \sim R$ (2) $R \vee Q \rightarrow \sim P$

(3) $\sim (P \vee Q) \rightarrow \sim R$ (4) $\sim (R \vee Q) \rightarrow \sim P$

81. Let the operations $*, \odot \in \{\wedge, \vee\}$. If $(p * q) \odot (p \odot \sim q)$ is a tautology, then the ordered pair $(*, \odot)$ is [JEE (Main)-2022]

(1) $(\vee, \wedge)$ (2) $(\vee, \vee)$

(3) $(\wedge, \wedge)$ (4) $(\wedge, \vee)$

82. Let

p : Ramesh listens to music.

q : Ramesh is out of his village.

r : It is Sunday.

s : It is Saturday.

Then the statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday" can be expressed as [JEE (Main)-2022]

(1) $((\sim q) \wedge (r \vee s)) \Rightarrow p$ (2) $(q \wedge (r \vee s)) \Rightarrow p$

(3) $p \Rightarrow (q \wedge (r \vee s))$ (4) $p \Rightarrow ((\sim q) \wedge (r \vee s)$

83. The statement $(p \Rightarrow q) \vee (p \Rightarrow r)$ is **NOT** equivalent to [JEE (Main)-2022]

(1) $(p \wedge (\sim r)) \Rightarrow q$ (2) $(\sim q) \Rightarrow ((\sim r) \vee p)$

(3) $p \Rightarrow (q \vee r)$ (4) $(p \wedge (\sim q)) \Rightarrow r$

84. Consider the following statements :

P : Ramu is intelligent.

Q : Ramu is rich.

R : Ramu is not honest.

The negation of the statement "Ramu is intelligent and honest if and only if Ramu is not rich" can be expressed as :

(1) $((P \wedge (\sim R)) \wedge Q) \wedge ((\sim Q) \wedge ((\sim P) \vee R))$

(2) $((P \wedge R) \wedge Q) \vee ((\sim Q) \wedge ((\sim P) \vee (\sim R)))$

(3) $((P \wedge R) \wedge Q) \wedge ((\sim Q) \wedge ((\sim P) \vee (\sim R)))$

(4) $((P \wedge (\sim R)) \wedge Q) \vee ((\sim Q) \wedge ((\sim P) \wedge R))$

[JEE (Main)-2022]

85. The statement $(p \wedge q) \Rightarrow (p \wedge r)$ is equivalent to :

(1) $q \Rightarrow (p \wedge r)$

(2) $p \Rightarrow (p \wedge r)$

(3) $(p \wedge r) \Rightarrow (p \wedge q)$

(4) $(p \wedge q) \Rightarrow r$

[JEE (Main)-2022]

86. Negation of the Boolean expression $p \Leftrightarrow (q \Rightarrow p)$ is **[JEE (Main)-2022]**

(1) $(\sim p) \wedge q$

(2) $p \wedge (\sim q)$

(3) $(\sim p) \vee (\sim q)$

(4) $(\sim p) \wedge (\sim q)$

❑ ❑ ❑

Chapter 13

Mathematical Reasoning

1. Answer (2)

p	q	~q	$p \leftrightarrow (\sim q)$	$\sim[p \leftrightarrow (\sim q)]$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

$\therefore$ Statement (1) is true and statement (2) is false.

2. Answer (3)

3. Answer (1)

A	B	$A \vee B$	$A \wedge (A \vee B)$	$(A \wedge B)$	$A \vee (A \wedge B)$
T	T	T	T	T	T
T	F	T	T	F	T
F	T	T	F	F	F
F	F	F	F	F	F

(3) and (4) are not tautology according to above table.

A	B	$A \to B$	$A \wedge (A \to B)$	$(A \wedge (A \to B)) \to B$	$B \to (A \wedge (A \to B))$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	T	F	T	F
F	F	T	F	T	T

$(A \wedge (A \to B)) \to B$ is tautology according to above table

4. Answer (4)

5. Answer (2)

p	q	$\sim p$	$\sim q$	$a = (p \to q)$	$b = (\sim q \to \sim p)$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$a \leftrightarrow b$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$
T	F
T	F
T	F
T	F

6. Answer (3)

$\sim(p \leftrightarrow \sim q)$

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim (p \leftrightarrow \sim q)$
F	F	T	F	T
F	T	F	T	F
T	F	T	T	F
T	T	F	F	T

Clearly equivalent to $p \leftrightarrow q$

7. Answer (4)

$\sim (\sim s \vee (\sim r \wedge s))$

$= s \wedge (r \vee \sim s)$

$= (s \wedge r) \vee (s \wedge \sim s)$

$= s \wedge r$

8. Answer (2)

$(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$= ((p \vee q) \wedge (\sim q \vee q)) \vee (\sim p \wedge q)$

$= ((p \vee q) \wedge t) \vee (\sim p \wedge q)$

$= (p \vee q) \vee (-p \wedge q)$

$= (p \vee q \vee -p) \wedge (p \vee q \vee q)$

$= t \wedge (p \vee q)$

$= p \vee q$

9. Answer (4)

p	q	$p \to q$	$(\sim p \to q)$	$(\sim p \to q) \to q$	$(p \to q) \to [(\sim p \to q) \to q]$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	T	T

(a tautology)

10. Answer (1)

$\sim (p \vee q) \vee (\sim p \wedge q)$

By property, $(\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$= \sim p$

11. Answer (4)

Check by options

(1) $(p \vee q) \wedge (\sim p \wedge q) = (\sim p \wedge q)$

(2) $(p \vee q) \wedge (\sim p \vee q) = q$

(3) $(p \wedge q) \wedge (\sim p \wedge q) = F$

(4) $(p \wedge q) \wedge (\sim p \vee q) = p \wedge q$

12. Answer (1)

$[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$

$= [(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$

$= [(p \wedge \sim q) \wedge (\sim q \wedge r)] \vee [(p \wedge r) \wedge (\sim q \wedge r)]$

$= [p \wedge \sim q \wedge r] \vee [p \wedge r \wedge \sim q]$

$= (p \wedge \sim q) \wedge r$

$= (p \wedge r) \wedge \sim q$

13. Answer (3)

P is True

Q is False

R is True

(1) $(\sim P) \wedge (\sim Q \wedge R) \equiv F \wedge (T \wedge T) \equiv F \wedge T = F$

(2) $(\sim P) \vee (Q \wedge R) \equiv F \vee (F \wedge T) \equiv F \vee F = F$

(3) $P \vee (\sim Q \wedge R) \equiv T \vee (T \wedge T) \equiv T \vee T = T$

(4) $(P \wedge Q) \vee (\sim R) \equiv (T \wedge F) \vee (F) \equiv F \vee F = F$

14. Answer (2)

q is false

$[(p \wedge q) \leftrightarrow r]$ is true

As $(p \wedge q)$ is false

[False $\leftrightarrow r$] is true

Hence r is false

Option (1): says $p \vee r$, As r is false

Hence $(p \vee r)$ can either be true or false

Option (2): says $(p \wedge r) \to (p \vee r)$

$(p \wedge r)$ is false

As $F \to T$ is true and

$F \to F$ is also true

Hence it is a tautology

Option (3): $(p \vee r) \to (p \wedge r)$

i.e. $(p \vee r) \to F$

It can either be true or false

Option (4): $(p \wedge r)$, As r is false

Hence $(p \wedge r)$ is false

15. Answer (1)

Contrapositive of "If A then B" is "If $\sim B$ then $\sim A$" Hence contrapositive of "If two numbers are not equal, then their squares are not equal" is "If squares of two numbers are equal, then the two numbers are equal".

16. Answer (2)

$((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$

$\equiv (p \vee \sim q) \wedge (\sim p \wedge \sim q)$

$\equiv ((p \vee \sim q) \wedge \sim p) \wedge ((p \vee \sim q) \wedge \sim q)$

$\equiv ((p \wedge \sim p) \vee (\sim q \wedge \sim p)) \wedge \sim q$

$\equiv (\sim p \wedge \sim q) \wedge \sim q \equiv (\sim p \wedge \sim q)$

17. Answer (3)

$\sim(\sim p \to q) \equiv \sim(p \vee q) \equiv \sim p \wedge \sim q$

18. Answer (4)

S: "If you are born in India, then you are a citizen of India."

Contrapositive of $p \to q$ is $\sim q \to \sim p$

So contrapositive of statement S will be :

"If you are not a citizen of India, then you are not born in India."

19. Answer (3)

By help of truth table :

p	q	$\sim q$	$p \vee \sim q$	$\sim p$	$p \wedge \sim q$	$p \vee q$	$p \to p \vee q$	$p \wedge q$	$(p \wedge q) \to p$	$\sim p \vee q$
T	T	F	T	F	F	T	T	T	T	T
T	F	T	T	F	T	T	T	F	T	F
F	T	F	F	T	F	T	T	F	T	T
F	F	T	T	T	F	F	T	F	T	T

$(p \wedge q) \to (\sim p) \vee q$	$(p \vee q) \to (p \vee (\sim q))$
T	T
T	T
T	F
T	T

20. Answer (1)

$$\begin{aligned} \sim(p \vee (\sim p \wedge q)) &= \sim(\sim p \wedge q) \wedge \sim p \\ &= (\sim q \vee p) \wedge \sim p \\ &= \sim p \wedge (p \vee \sim q) \\ &= (\sim q \wedge \sim p) \vee (p \wedge \sim p) \\ &= (\sim p \wedge \sim q) \end{aligned}$$

21. Answer (1)

For $p \to q \vee r$ to be F

r must be F & $p \to q$ must be F

For $p \to q$ to be F $p \to T$ & $q \to F$

$p, q, r \equiv$ T, F, F

22. Answer (1)

$(p \vee q) \vee (p \vee \sim q)$

$= p \vee (q \vee p) \vee \sim q$

$= (p \vee p) \vee (q \vee \sim q)$

$= p \vee T$

$= T$ so first statement is tautology

23. Answer (1)

$\sim s \vee (\sim r \wedge s)$

$\equiv (\sim s \vee \sim r) \wedge (\sim s \vee s)$

$\equiv (\sim s \vee \sim r)$ $(\because (\sim s \vee s)$ is tautology)

$\equiv \sim(s \wedge r)$

Hence its negation is $s \wedge r$

24. Answer (2)

$P \to (\sim q \vee r)$ is a fallacy

$\Rightarrow$ P is True and $\sim q \vee r$ is False

$\Rightarrow$ P is True and $\sim q$ is False and r is False

$\Rightarrow$ Truth values of p, q, r are

T, T, F respectively.

25. Answer (3)

$\sim(p \Rightarrow (\sim q))$ $\{\because p \Rightarrow q$ is same as $\sim p \vee q\}$

$\equiv \sim((\sim p) \vee (\sim q))$

$\equiv p \wedge q$

26. Answer (4)

$(\sim p \vee q) \wedge (\sim q \vee \sim p)$

$\equiv \sim p \vee (q \wedge \sim q)$

As $q \wedge \sim q$ is a fallacy

$\equiv \sim p$

27. Answer (4)

P	Q	P∨Q	P∧Q	P∨(P∨Q)	P∧(P∨Q)	P→Q	P∧(P→Q)	P∧(P→Q)→Q	Q→(P∨(P→Q))
T	T	T	T	T	T	T	T	T	T
T	F	T	F	T	T	F	F	T	T
F	T	T	F	T	F	T	F	T	F
F	F	F	F	F	F	T	F	T	T

$\therefore$ Hence $P \wedge (P \to Q) \to Q$ is a tautology.

28. Answer (3)

$\because$ $p \to q \equiv q \vee \sim P$

$\therefore$ Checking option 3 $\sim (p \vee \sim q) \to (p \vee q)$ is equivalent to

$\Rightarrow$ $(p \vee q) \vee (p \vee \sim q)$

$\Rightarrow$ $p \vee T \equiv T$

29. Answer (1)

Let $p : \sqrt{5}$ is an integer

q : 5 is irrational

Given statement is $(p \vee q)$

Negation of given statement will be $\sim(p \vee q)$

$= \sim p \wedge \sim q$

$= \sqrt{5}$ is not an integer and 5 is not irrational

30. Answer (1)

$p \to (p \wedge \sim q)$ is false

Than $p \to T$

$(p \wedge \sim q)$ is false

So when P is true and $p \wedge \sim q$ is false so there is only one possibility when q is also true.

So truth value of p and q will be TT.

31. Answer (4)

Contrapositive of if P then Q is

if not Q then not P

$\therefore$ If I will not catch the train then I do not reach the station in time

32. Answer (1)

Truth table

P	q	~p	p∨q	(~p)∧(p∨q)	(~p)∧(p∨q)→q
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

$\therefore$ $\sim p \wedge (p \vee q) \to q$ be a tautology

33. Answer (4)

p	q	p→ ~(p∧~q)	~p∨q
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

$\therefore$ $p \to \sim(p \wedge \sim q)$ is equivalent to $\sim p \vee q$

34. Answer (4)

$(p \wedge q) \rightarrow (\sim q \vee r)$

$= \sim(p \wedge q) \vee (\sim q \vee r)$

$= (\sim p \vee \sim q) \vee (\sim q \vee r)$

$= (\sim p \vee \sim q \vee r)$

$\because$ $\sim p \vee \sim q \vee r$ is false, then

$\sim p$, $\sim q$ and r all three must be false.

$\Rightarrow$ p is true, q is true and r is false.

35. Answer (4)

p	q	~p	~q	q∨p	p↔~q	(S_1)	~p↔q	(S_2)
T	T	F	F	T	F	F	F	F
T	F	F	T	T	T	T	T	T
F	T	T	F	T	T	T	T	F
F	F	T	T	F	F	T	F	F
						↓ not tautology		↓ not fallacy

$\therefore$ Both (S_1) and (S_2) are incorrect.

36. Answer (2)

Contrapositive statement will be

" If a function is not continuous at 'a', then it is not differentiable at 'a'.

37. Answer (2)

$p : x \leftrightarrow \sim y = (x \rightarrow \sim y) \wedge (\sim y \rightarrow x)$

$= (\sim x \ \vee \sim y) \wedge (y \vee x)$

$= \sim(x \wedge y) \wedge (x \vee y)$

Negation of p is $\sim p = (x \wedge y) \vee \sim (x \vee y)$

$= (x \wedge y) \vee (\sim x \wedge \sim y)$

38. Answer (1)

$(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$ is

By truth table

p	q	p∨q	p→(p∨q)	q→p	p→(q→p)	(p→(q→p)) → (p→(p∨q))
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	T	T	F	T	T
F	F	F	T	T	T	T

So it is tautology.

39. Answer (4)

$\sim(p \vee (\sim p \wedge q))$

$= \sim p \wedge (p \vee \sim q)$

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$= \sim p \wedge \sim q$

40. Answer (3)

Contrapositive statement will be

"For an integer n, if n is not odd then $n^3 - 1$ is not even".

OR

"For an integer n, if n is even then $n^3 - 1$ is odd.

41. Answer (3)

(1) $A \vee (A \wedge B) = A$

(2) $\because$ $A \wedge (A \rightarrow B) = A \wedge (\sim A \vee B) = A \wedge B$

So, $B \rightarrow (A \wedge B) = \sim B \vee (A \wedge B) = \sim B \vee A$

(3) $(A \wedge (A \rightarrow B)) \rightarrow B = (A \wedge B) \rightarrow B = \sim (A \wedge B) \vee B = \sim A \vee \sim B \vee B$ (Tautology)

(4) $A \wedge (A \vee B) = A$

42. Answer (1)

Truth table for required statements

p	q	~p	~q	p→q	(~q) ∧(p→q)	(~q∧ (p→q)) →~p	p∨q	(p∨q) ∧~p	((p∨q) ∧~p) →q
T	T	F	F	T	F	T	T	F	T
T	F	F	T	F	F	T	T	F	T
F	T	T	F	T	F	T	T	T	T
F	F	T	T	T	T	T (a) is tautology	F	F	T (b) is tautology

43. Answer (4)

$\sim(\sim p \wedge (p \vee q)) = p \vee \sim (p \vee q)$

$= p \vee (\sim p \wedge \sim q)$

$= (p \vee \sim p) \wedge (p \vee \sim q)$

$= p \vee \sim q$

44. Answer (4)

$B \rightarrow A = \sim B \vee A$

Also $A \rightarrow (B \rightarrow A) = A \rightarrow (\sim B \vee A) = \sim A \vee (\sim B \vee A)$

$= \sim A \vee \sim B \vee A = \sim A \vee A \vee \sim B = t \vee \sim B = t$

$A \rightarrow (A \vee B)$

$= \sim A \vee (A \vee B)$

$= (\sim A \vee A) \vee B$

$= t \vee B = t$

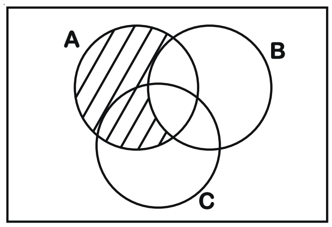
45. Answer (3)

Contrapositive of $A \rightarrow B$ is $\sim B \rightarrow \sim A$

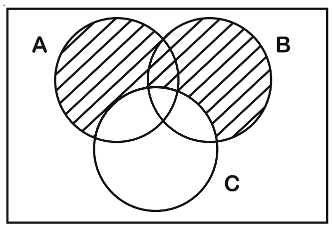
$\therefore$ Contrapositive of the given statement will be ~(you will earn money) → ~(you will work)

i.e., if you will not earn money, you will not work

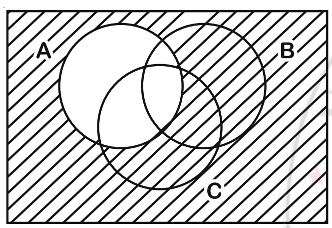
46. Answer (4)



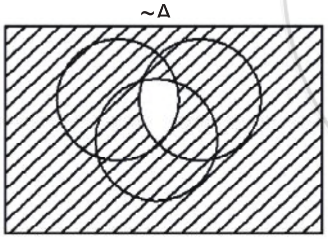
$A \wedge \sim B$



$\sim C \wedge (A \vee B)$



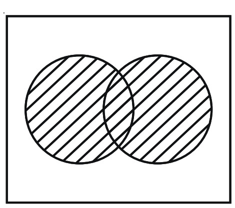
$\sim A$



$\Rightarrow$ **F_1 is not a tautology**

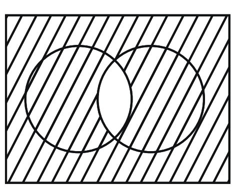
F_1

$B \to \sim A = \sim B \vee \sim A$



$A \vee B$

$\vee$



$\sim B \vee \sim A = \sim(A \cap B)$

$= F_2$

$\Rightarrow$ F_2 is a tautology

47. Answer (1)

$\because$ $p \to q = \sim p \vee q$

so $(p \wedge q) \to (p \to q) = \sim (p \wedge q) \vee (p \to q)$

$= (\sim p \vee \sim q) \vee (\sim p \vee q)$

$= \sim p \vee q \vee \sim q$

is a tautology

48. Answer (2)

$p \Rightarrow q \Leftrightarrow q^*(\sim p)$ is a tautology

$\therefore$ $p \Rightarrow q$ and $q^*(\sim p)$ have same truth value for all logical possibility

$\therefore$ $q*(\sim p) \equiv p \Rightarrow q$

And therefore, $p* \sim q \equiv q \Rightarrow p$

49. Answer (1)

$\because$ $p \to q = \sim p \vee q$

$(p \wedge q) \to (p \to q) = \sim(p \wedge q) \vee (p \to q)$

$= (\sim p \vee \sim q) \vee (\sim p \vee q)$

$= \sim p \vee q \vee \sim q$ is a tautology.

50. Answer (3)

$((P \to Q) \wedge \sim Q) \to \sim P$

$= ((\sim P \vee Q) \wedge \sim Q) \to \sim P$

$= ((\sim P \wedge \sim Q) \vee C) \to \sim P$

$= (\sim P \wedge \sim Q) \to \sim P$

$= \sim(\sim P \wedge \sim Q) \vee \sim P$

$= (P \vee Q) \vee \sim P$

$= (P \vee \sim P) \vee Q$

$= t \vee Q$

$= t$ (tautology)

51. Answer (2)

$\because$ $p \Rightarrow q$ is $\sim p \vee q$

$\therefore$ $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$

$= \sim (p \wedge \sim q) \vee (q \vee \sim p)$

$= (\sim p \vee q) \vee (\sim p \vee q)$

$= \sim p \vee q$

$= p \Rightarrow q$

52. Answer (3)

$\because$ 3 + 3 = 7 is false and 4 + 3 = 8 is false

then statement (A) is true

For (B) 5 + 3 = 8 is true and earth is flat is false.

Then statement (B) is false

For (C) if A and B are true then 5 + 6 = 17 is false, then (C) is true.

$\therefore$ (A) and (C) are true and (B) is false.

53. Answer (1)

1. $(\sim p \to q) \vee (\sim q \to p) \equiv (p \vee q) \vee (q \vee p)$

$\equiv p \vee q \not\equiv T$

2. $(\sim p \vee q) \vee (q \vee p) \equiv \sim(p \wedge \sim q) \vee (p \vee q) \equiv T$

3. $(\sim q \vee p) \vee (q \vee p) \equiv \sim(\sim p \wedge q) \vee (p \vee q) \equiv T$

4. $(\sim p \vee \sim q) \vee (q \vee p) \equiv \sim(p \wedge q) \vee (p \vee q) \equiv T$

54. Answer (1)

Making truth table, we get

p	q	$p \Rightarrow q$	$\sim p$	$q \Rightarrow \sim p$	$(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

$\therefore$ $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to $\sim p$

55. Answer (2)

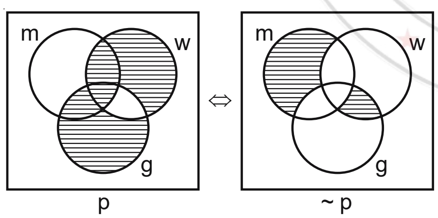
Consider the statements,

m : match will be played

w : weather is good

g : ground is not wet

$\because$ $p : m \leftrightarrow (w \wedge g) \Rightarrow$ In Venn diagram it will be represented by $[m \,\Delta\, (w \cap g)]^c$



So $\sim p : m \wedge (\sim w \vee \sim g)$ also $\sim p : (w \wedge g) \wedge (\sim m)$

$\Rightarrow$ $\sim p$: match will be played and weather is not good or ground is wet.

56. Answer (2)

$(P \vee Q) \wedge (\sim P) \to Q$

$= \sim(P \vee Q) \vee P \vee Q$

$= \sim(P \vee Q) \vee (P \vee Q) \Rightarrow$ It is a tautology.

Only option (2) is a tautology because

$\sim(P \to Q) = \sim(\sim P \vee Q) = P \wedge \sim Q$

57. Answer (3)

Statement : For all $M > 0$, there exists $x \in S$ such that $x \geq M$.

Negation : There exist $M > 0$, such that $x \not\geq M$ for all $x \in S$.

58. Answer (3)

$\because$ $((p \vee q) \wedge (q \to r) \wedge (\sim r)) \to (p \wedge q)$ is false

$\Rightarrow$ $(p \vee q) \wedge (q \to r) \wedge (\sim r)$ is true and $(p \wedge q)$ is false

$\Rightarrow$ $(p \vee q)$ is true, $(q \to r)$ is true, $(\sim r)$ is true and $(p \wedge q)$ is false

$\Rightarrow$ r is false and exactly one out of p and q is false

$\because$ $q \to r$ is true and r is false, so q is also false hence p must be true.

59. Answer (3)

For S1 and S2

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to p$	$(p \to q) \vee (\sim q \to p)$	$p \wedge \sim q$	$(\sim p \vee q)$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	T	T	T	F	T	F
T	F	F	T	F	T	T	T	F	F
F	T	T	F	T	T	T	F	T	F
F	F	T	T	T	F	T	F	T	F
						tautology S1			fallacy S2

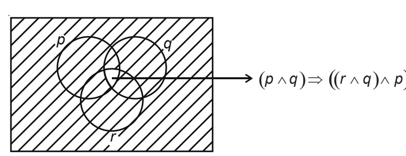
$\therefore$ Hence both S1 and S2 are true

60. Answer (4)

p	q	r	$p \to q$	$q \to r$	$p \wedge (p \to q)$	$p \wedge (p \to q) \wedge (q \to r)$	$p \wedge (p \to q) \wedge (q \to r) \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	T
T	F	T	F	T	F	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	F	F	T
F	T	F	T	F	F	F	T
F	F	T	T	T	F	F	T
F	F	F	T	T	F	F	T

Clearly it is a tautology

61. Answer (1)



This is equivalent to $(p \wedge q) \Rightarrow (r \wedge q)$

62. Answer (4)

(p * ~q) ⇒ (p □ q)

= ~ (p * ~q) ∨ (p □ q)

= (~ p □ q) ∨ (p □ q)

= T

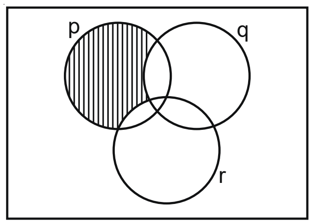
⇒ □ = ∨

and * = ~ □ = ∧

63. Answer (2)

~(p ∨ r) ⇒ (q ∨ r)

= (p ∨ r) ∧ ~(q ∨ r)



∴ p ∧ ~q ∧ ~r

64. Answer (3)

$p \wedge \sim q = \sim(\sim p \vee q)$

$= \sim(p \rightarrow q)$

65. Answer (2)

Let $x : (p \Delta q) \Rightarrow (p \Delta \sim q) \vee (\sim p \Delta q)$

Case-I

When Δ is same as $\vee$

Then $(p \Delta \sim q) \vee (\sim p \Delta q)$ becomes

$(p \vee \sim q) \vee (\sim p \vee q)$ which is always true, so x becomes a tautology.

Case-II

When Δ is same as $\wedge$

Then $(p \wedge q) \Rightarrow (p \wedge \sim q) \vee (\sim p \wedge q)$

If $p \wedge q$ is T, then $(p \wedge \sim q) \vee (\sim p \wedge q)$ is F

so x cannot be a tautology.

Case-III

When Δ is same as $\Rightarrow$

Then $(p \Rightarrow \sim q) \vee (\sim p \Rightarrow q)$ is same at $(\sim p \vee \sim q) \vee (p \vee q)$, which is always true, so x becomes a tautology.

Case-IV

When Δ is same as $\Leftrightarrow$

Then $(p \Leftrightarrow q) \Rightarrow (p \Leftrightarrow \sim q) \vee (\sim p \Leftrightarrow q)$

$p \Leftrightarrow q$ is true when p and q have same truth values, then $p \Leftrightarrow \sim q$ and $\sim p \Leftrightarrow q$ both are false. Hence x cannot be a tautology.

So finally x can be $\vee$ or $\Rightarrow$.

66. Answer (2)

∵ given statement is

$(A \wedge C) \rightarrow B$

Then its negation is

$\sim \{(A \wedge C) \rightarrow B\}$

or $\sim \{\sim (A \wedge C) \vee B\}$

∴ $(A \wedge C) \wedge (\sim B)$

or $(\sim B) \wedge (A \wedge C)$

67. Answer (3)

Given $p \rightarrow (\sim p \vee q)$ is false

$\Rightarrow$ $\sim p \vee q$ is false and p is true

Now p = True.

$\sim T \vee q = F$

$F \vee q = F \Rightarrow q$ is false

$P1 : \sim (T \rightarrow \sim F) \equiv \sim (T \rightarrow T) \equiv$ False.

$P2 : (T \wedge \sim F) \wedge (\sim T \vee F) \equiv (T \wedge T) \equiv (F \vee F)$

$\equiv T \wedge F \equiv$ False

68. Answer (3)

Let $S : ((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$

$\Rightarrow$ $S : \sim ((\sim q) \wedge p) \vee ((\sim p) \vee q)$

$\Rightarrow$ $S : (q \vee (\sim p)) \vee ((\sim p) \vee q)$

$\Rightarrow$ $S : (\sim p) \vee q$

$\Rightarrow$ $S : p \Rightarrow q$

So, negation of S will be $\sim (p \Rightarrow q)$

69. Answer (1)

Case-I

If ∇ is same as $\wedge$

Then $(p \wedge q) \Rightarrow ((p \Delta q) \wedge r)$ is equivalent to $\sim (p \wedge q) \vee ((p \Delta q) \wedge r)$ is equivalent to $(\sim (p \wedge q) \vee (p \Delta q)) \wedge (\sim (p \wedge q) \vee r)$

Which cannot be a tautology

For both Δ (i.e. $\vee$ or $\wedge$)

Case-II

If ∇ is same as $\vee$

Then $(p \vee q) \Rightarrow ((p \,\Delta\, q) \vee r)$ is equivalent to

$\sim(p \vee q) \vee (p \,\Delta\, q) \vee r$ which can be a tautology if Δ is also same as $\vee$.

Hence both Δ and ∇ are same as $\vee$.

Now $(p \,\nabla\, q) \,\Delta\, r$ is equivalent to $(p \vee q \vee r)$.

70. Answer (3)

Clearly r must be equal to $\sim p$

$\because \quad \sim p \vee \sim p = \sim p$

and $(p \wedge q) \vee \sim p = p$

$\therefore \quad \sim p \Rightarrow p$ = tautology.

71. Answer (4)

Making truth table

p	q	$p \wedge q$	$\sim p \wedge q$	$(\sim(p \wedge q)) \vee q$	$p \vee q$	$p \to q$	$p \to (p \vee q)$
T	T	T	F	T	T	T	T
T	F	F	T	T	T	F	T
F	T	F	T	T	T	T	T
F	F	F	T	T	F	T	T
				Tautology			Tautology

$\therefore \quad (\sim(p \wedge q)) \vee q \equiv p \to (p \vee q)$

72. Answer (3)

$\because \quad ((\sim q) \wedge p) \vee (p \vee (\sim p))$

$= (\sim q \wedge p) \vee t$ (t is tautology)

$\equiv t$

$\therefore$ option (3) is correct.

73. Answer (3)

S_1: $(\sim p \vee q) \vee (\sim p \vee r)$

$\cong (\sim p \vee q \vee r)$

S_2: $\sim p \vee (q \vee r)$

Both are same

So, option (3) is incorrect.

74. Answer (9)

There are total 9 compound propositions, out of which 6 contain $\sim s$. So if we assign s as false, these 6 propositions will be true.

In remaining 3 compound propositions, two contain p and the third contains $\sim r$. So if we assign p and r as true and false respectively, these 3 propositions will also be true.

Hence maximum number of propositions that can be true are 9.

75. Answer (3)

$(p \vee q) \Rightarrow q$

$\sim(p \vee q) \vee q$

$= (\sim p \wedge \sim q) \vee q$

$= (\sim p \vee q) \wedge (\sim q \vee q)$

$= (\sim p \vee q) \wedge T$

$= \sim p \vee q$

Now $(p \wedge q) \Delta (\sim p \vee q)$

p	q	$\sim p$	$p \wedge q$	$\sim p \vee q$	$(p \wedge q) \Delta (\sim p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	F	T	T
F	F	T	F	T	T

$\therefore \quad \Delta = \Rightarrow$

76. Answer (3)

$p \vee q \Rightarrow (\sim r \vee p)$

$\equiv \sim(p \vee q) \vee (\sim r \vee p)$

$\equiv (\sim p \wedge \sim q) \vee (p \vee \sim r)$

$\equiv [(\sim p \vee p) \wedge (\sim q \vee p)] \vee \sim r$

$\equiv (\sim q \vee p) \vee \sim r$

Its negation is $\sim p \wedge q \wedge r$.

77. Answer (4)

Truth Table

					A	B	C	D
p	q	$\sim p$	$\sim q$	$(\sim p) \vee q$	$((\sim p) \vee q) \to p$	$p \to ((\sim p) \vee q)$	$(\sim p) \vee q \to q$	$q \to ((\sim p) \vee q)$
T	T	F	F	T	T	T	T	T
T	F	F	T	F	T	F	T	T
F	T	T	F	T	F	T	T	T
F	F	T	T	T	F	T	F	T

78. Answer (4)

$\sim (p \Leftrightarrow \sim q) \wedge q$

$= (p \Leftrightarrow q) \wedge q$

p	q	$p \leftrightarrow q$	$(p \leftrightarrow q) \wedge q$	$(p \to q)$	$(p \to q) \wedge q$	$(p \to q) \wedge p$
T	T	T	T	T	T	T
T	F	F	F	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	F	F

$\therefore$ $(\sim (p \Leftrightarrow \sim q)) \wedge q$ is equivalent to $(p \Rightarrow q) \wedge p$.

79. Answer (3)

The truth table

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$p \wedge q \Leftrightarrow p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

Clearly $p \wedge q \Leftrightarrow p \wedge \sim q \equiv \sim p$

$\therefore$ $r = q$

80. Answer (4)

$\underbrace{P \wedge (\sim R)}_{X} \to \underbrace{((\sim R) \wedge Q)}_{Y} = \text{False}$

$X \to Y = \text{False}$

X	Y	$X \to Y$
F	F	T
T	T	T
F	T	T
T	F	F

$P \wedge \sim R = \text{T}$ and $(\sim R) \wedge Q = \text{F}$

$\Rightarrow$ $P = \text{T}$

$\sim R = \text{T} \Rightarrow R = \text{F}$

$\Rightarrow$ $P = \text{T}$, $Q = \text{F}$ and $R = \text{F}$

$\text{T} \wedge Q = \text{F}$

$\Rightarrow Q = \text{F}$

Now $\sim (R \vee Q) \to \sim P$

$\sim(\text{F} \vee \text{F}) \to \text{F}$

$\text{F} \to \text{F} = \text{False}$

81. Answer (2)

$*, \odot \in \{\wedge, \vee\}$

Now for $(p * q) \odot (p \odot \sim q)$ is tautology

(1) $(\vee, \wedge) : (p \vee q) \wedge (p \wedge \sim q)$ not a tautology

(2) $(\vee, \vee) : (p \vee q) \vee (p \vee \sim q)$

$= P \vee T$ is tautology

(3) $(\wedge, \wedge) : (p \wedge q) \wedge (p \wedge \sim q)$

$= (p \wedge p) \wedge (q \wedge \sim q) = p \wedge F$ not a tautology (Fallasy)

(4) $(\wedge, \vee) : (p \wedge q) \vee (p \vee \sim q)$ not a tautology

82. Answer (4)

p : Ramesh listens to music

q : Ramesh is out of his village

r : It is Sunday

s : It is Saturday

$p \to q$ conveys the same p only if q

Statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday"

$p \Rightarrow ((\sim q) \wedge (r \vee s))$

83. Answer (2)

(1) $(p \wedge (\sim r)) \Rightarrow q$

$\sim (p \wedge \sim r) \vee q$

$\equiv (\sim p \vee r) \vee q$

$\equiv \sim p \vee (r \vee q)$

$\equiv p \to (q \vee r)$

$\equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

(3) $p \Rightarrow (q \vee r)$

$\equiv \sim p \vee (q \vee r)$

$\equiv (\sim p \vee q) \vee (\sim p \vee r)$

$\equiv (p \to q) \vee (p \to r)$

(4) $(p \wedge \sim q) \Rightarrow r$

$\equiv p \Rightarrow (q \vee r)$

$\equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

84. Answer (4)

P : Ramu is intelligent

Q : Ramu is rich

R : Ramu is not honest

Given statement, “Ramu is intelligent and honest if and only if Ramu is not rich”

$= (P \wedge \sim R) \Leftrightarrow \sim Q$

So, negation of the statement is

$\sim [(P \wedge \sim R) \Leftrightarrow \sim Q]$

$= \sim [\{\sim (P \wedge \sim R) \vee \sim Q\} \wedge \{Q \vee (P \wedge \sim R)\}]$

$= ((P \wedge \sim R) \wedge Q) \vee (\sim Q \wedge (\sim P \vee R))$

85. Answer (4)

p	q	r	$p \wedge q$ (A)	$p \wedge r$ (B)	$A \to B$	$q \to B$	$p \to B$	$B \to A$	$A \to r$
T	T	T	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T	F	T
F	T	T	F	F	T	F	T	T	T
F	F	T	F	F	T	T	T	T	T
T	T	F	T	F	F	F	F	T	F
T	F	F	F	F	T	T	F	T	T
F	T	F	F	F	T	F	T	T	T
F	F	F	F	F	T	T	T	T	T

$(p \wedge q) \Rightarrow (p \wedge r)$ is equivalent to $(p \wedge q) \Rightarrow r$

86. Answer (4)

$p \Leftrightarrow (q \Rightarrow p)$

$\sim (p \Leftrightarrow (q \Leftrightarrow p))$

$\equiv p \Leftrightarrow \sim (q \Rightarrow p)$

$\equiv p \Leftrightarrow (q \wedge \sim p)$

$\equiv (p \Rightarrow (q \wedge \sim p)) \wedge ((q \wedge \sim p) \Rightarrow p))$

$\equiv (\sim p \vee (q \wedge \sim p)) \wedge ((\sim q \vee p) \vee p))$

$\equiv ((\sim p \vee q) \wedge \sim p) \wedge (\sim q \vee p)$

$\equiv \sim p \wedge (\sim q \vee p)$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge p)$

$\equiv (\sim p \wedge \sim q) \vee c$

$\equiv (\sim p \wedge \sim q)$