Chapter 15

Statistics

1. **Statement-1 :** The variance of first n even natural numbers is $\frac{n^2-1}{4}$.

Statement-2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$.

[AIEEE-2009]

(1) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

2. If the mean deviation of the numbers 1, 1 + d, 1 + 2d, ..., 1 + 100d from their mean is 255, then the d is equal to **[AIEEE-2009]**

(1) 20.0 (2) 10.1

(3) 20.2 (4) 10.0

3. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

[AIEEE-2010]

(1) $\frac{5}{2}$ (2) $\frac{11}{2}$

(3) 6 (4) $\frac{13}{2}$

4. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standard deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively **[AIEEE-2011]**

(1) 28, 2 (2) 28, 4

(3) 32, 2 (4) 32, 4

5. Let x_1, x_2 ..., x_n be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be their variance.

Statement-1: Variance of 2x_1, 2x_2, ..., 2x_n is 4 σ^2.

Statement-2: Arithmetic mean of 2x_1, 2x_2, ..., 2x_n is 4 $\bar{x}$. **[AIEEE-2012]**

(1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(2) Statement-1 is true, statement-2 is true, statement-2 is **not** a correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is false.

(4) Statement-1 is false, statement-2 is true.

6. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given?

[JEE (Main)-2013]

(1) Mean (2) Median

(3) Mode (4) Variance

7. The variance of first 50 even natural numbers is **[JEE (Main)-2014]**

(1) 437 (2) $\frac{437}{4}$

(3) $\frac{833}{4}$ (4) 833

8. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is **[JEE (Main)-2015]**

(1) 16.8 (2) 16.0

(3) 15.8 (4) 14.0

9. If the standard deviation of the number 2, 3, a and 11 is 3.5, then which of the following is true? **[JEE (Main)-2016]**

(1) $3a^2 - 32a + 84 = 0$ (2) $3a^2 - 34a + 91 = 0$

(3) $3a^2 - 23a + 44 = 0$ (4) $3a^2 - 26a + 55 = 0$

10. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is **[JEE (Main)-2018]**

(1) 9 (2) 4
(3) 2 (4) 3

11. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is **[JEE (Main)-2019]**

(1) 18 (2) 20
(3) 22 (4) 16

12. A data consists of n observations $x_1, x_2, ..., x_n$. If $\sum_{i=1}^{n}(x_i+1)^2=9n$ and $\sum_{i=1}^{n}(x_i-1)^2=5n$, then the standard deviation of this data is **[JEE (Main)-2019]**

(1) $\sqrt{7}$ (2) 5
(3) $\sqrt{5}$ (4) 2

13. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is **[JEE (Main)-2019]**

(1) 4 : 9 (2) 6 : 7
(3) 10 : 3 (4) 5 : 8

14. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, ..., x_5$ and –50 is equal to **[JEE (Main)-2019]**

(1) 586.5 (2) 582.5
(3) 509.5 (4) 507.5

15. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2}-d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2}+d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals **[JEE (Main)-2019]**

(1) $\sqrt{2}$ (2) $\frac{\sqrt{5}}{2}$
(3) $\frac{2}{3}$ (4) 2

16. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is **[JEE (Main)-2019]**

(1) 31 (2) 30
(3) 50 (4) 51

17. The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then the absolute value of the difference of the other two observations, is **[JEE (Main)-2019]**

(1) 5 (2) 7
(3) 3 (4) 1

18. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is **[JEE (Main)-2019]**

(1) 45 (2) 40
(3) 48 (4) 49

19. A student scores the following marks in five tests : 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is **[JEE (Main)-2019]**

(1) $\frac{100}{\sqrt{3}}$ (2) $\frac{10}{\sqrt{3}}$
(3) $\frac{100}{3}$ (4) $\frac{10}{3}$

20. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$, where $k > 0$, then k is equal to **[JEE (Main)-2019]**

(1) $\sqrt{6}$ (2) $2\sqrt{6}$
(3) $2\sqrt{\frac{10}{3}}$ (4) $4\sqrt{\frac{5}{3}}$

21. The mean and the median of the following ten numbers in increasing order

10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to **[JEE (Main)-2019]**

(1) $\frac{7}{3}$ (2) $\frac{8}{3}$
(3) $\frac{7}{2}$ (4) $\frac{9}{4}$

22. If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequency	$(x+1)^2$	$2x-5$	x^2-3x	x

Then the mean of the marks is

[JEE (Main)-2019]

(1) 3.2 (2) 3.0

(3) 2.5 (4) 2.8

23. If both the mean and the standard deviation of 50 observations $x_1, x_2, \ldots x_{50}$ are equal to 16, then the mean of $(x_1-4)^2, (x_2-4)^2, \ldots (x_{50}-4)^2$ is

[JEE (Main)-2019]

(1) 380 (2) 480

(3) 400 (4) 525

24. If the data $x_1, x_2, \ldots, x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is

[JEE (Main)-2019]

(1) $2\sqrt{2}$ (2) 4

(3) 2 (4) $\sqrt{2}$

25. The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \neq 0$ and $q \neq 0$. If the new mean and new s.d. become half of their original values, then q is equal to **[JEE (Main)-2020]**

(1) –10 (2) –20

(3) –5 (4) 10

26. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is **[JEE (Main)-2020]**

(1) 3.98 (2) 4.02

(3) 3.99 (4) 4.01

27. Let the observations $x_i (1 \le i \le 10)$ satisfy the equations, $\sum_{i=1}^{10}(x_i-5)=10$ and $\sum_{i=1}^{10}(x_i-5)^2=40$. If μ and λ are the mean and the variance of the observations, $x_1-3, x_2-3, \ldots, x_{10}-3$, then the ordered pair (μ, λ) is equal to **[JEE (Main)-2020]**

(1) (6, 3) (2) (3, 6)

(3) (3, 3) (4) (6, 6)

28. Let $X = \{x \in N : 1 \le x \le 17\}$ and $Y \{ax + b : x \in X$ and $a, b \in R, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then $a + b$ is equal to **[JEE (Main)-2020]**

(1) 7 (2) –27

(3) 9 (4) –7

29. For the frequency distribution :

Variate (x) : $x_1 \quad x_2 \quad x_3 \ldots x_{15}$

Frequency (f) : $f_1 \quad f_2 \quad f_3 \ldots f_{15}$

where $0 < x_1 < x_2 < x_3 < \ldots < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be

[JEE (Main)-2020]

(1) 1 (2) 6

(3) 2 (4) 4

30. Let x_i $(1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i-p)=3$ and $\sum_{i=1}^{10}(x_i-p)^2=9$ where $0 \neq 0\ p \in R$, then the standard deviation of these observations is **[JEE (Main)-2020]**

(1) $\frac{7}{10}$ (2) $\frac{9}{10}$

(3) $\sqrt{\frac{3}{5}}$ (4) $\frac{4}{5}$

31. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is

[JEE (Main)-2020]

(1) 9 (2) 3

(3) 7 (4) 5

32. The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is

[JEE (Main)-2020]

(1) 2 (2) 4

(3) 3 (4) 1

33. If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation

[JEE (Main)-2020]

(1) $x^2 - 20x + 18 = 0$ (2) $2x^2 - 20x + 19 = 0$

(3) $x^2 - 10x + 18 = 0$ (4) $x^2 - 10x + 19 = 0$

34. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, $(n, a > 1)$ then the standard deviation of n observations x_1, x_2, ..., x_n is **[JEE (Main)-2020]**

(1) a – 1 (2) $n\sqrt{a-1}$

(3) $\sqrt{n(a-1)}$ (4) $\sqrt{a-1}$

35. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then $m + n$ is equal to ________.

[JEE (Main)-2020]

36. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then x.y is equal to __________.

[JEE (Main)-2020]

37. If the variance of the terms in an increasing A.P., b_1, b_2, b_3, ..., b_{11} is 90, then the common difference of this A.P. is ________.

[JEE (Main)-2020]

38. If the variance of the following frequency distribution

Class	:	10–20	20–30	30–40
Frequency	:	2	x	2

is 50, then x is equal to ________.

[JEE (Main)-2020]

39. Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ..., ${}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to _________ . **[JEE (Main)-2020]**

40. If the variance of 10 natural numbers 1, 1, 1, ..., 1, k is less than 10, then the maximum possible value of k is ________.

[JEE (Main)-2021]

41. Let X_1, X_2, ..., X_{18} be eighteen observations such that $\sum_{i=1}^{18}(X_i = \alpha) = 36$ and $\sum_{i=1}^{18}(X_i = \beta)^2 = 90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is _________.

[JEE (Main)-2021]

42. Consider three observations a, b and c such that b = a + c. If the standard deviation of a + 2, b + 2, c + 2 is d, then which of the following is true?

[JEE (Main)-2021]

(1) $b^2 = a^2 + c^2 + 3d^2$ (2) $b^2 = 3(a^2 + c^2) - 9d^2$

(3) $b^2 = 3(a^2 + c^2) + 9d^2$ (4) $b^2 = 3(a^2 + c^2 + d^2)$

43. Consider the statistics of two sets of observations as follows:

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to ________.

[JEE (Main)-2021]

44. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to ______.

[JEE (Main)-2021]

45. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is ____. **[JEE (Main)-2021]**

46. Let in a series of 2n observations, half of them are equal to a and remaining half are equal to –a. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to : **[JEE (Main)-2021]**

(1) 925 (2) 650

(3) 425 (4) 250

47. Consider the following frequency distribution :

Class :	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency :	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha - \beta|$ is equal to _______.

[JEE (Main)-2021]

48. The first of the two samples in group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is

[JEE (Main)-2021]

(1) 5 (2) 6

(3) 4 (4) 8

49. Let the mean and variance of the frequency distribution

$x: \quad x_1 = 2 \quad x_2 = 6 \quad x_3 = 8 \quad x_4 = 9$
$f: \quad 4 \quad 4 \quad \alpha \quad \beta$

be 6 and 6.8 respectively. If x_3 is changed from 8 to 7, then the mean for the new data will be : **[JEE (Main)-2021]**

(1) 5 (2) 4

(3) $\frac{17}{3}$ (4) $\frac{16}{3}$

50. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is **[JEE (Main)-2021]**

(1) (11, 25) (2) (11, 26)

(3) (10.5, 25) (4) (10.5, 26)

51. Let the mean and variance of four numbers 3, 7, x and y ($x > y$) be 5 and 10 respectively. Then the mean of four numbers $3 + 2x$, $7 + 2y$, $x + y$ and $x - y$ is ________. **[JEE (Main)-2021]**

52. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is **[JEE (Main)-2021]**

(1) $\frac{536}{25}$ (2) $\frac{134}{5}$

(3) $\frac{112}{5}$ (4) $\frac{92}{5}$

53. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are: **[JEE (Main)-2021]**

(1) 1, 20 (2) 10, 11

(3) 3, 18 (4) 8, 13

54. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of $|a - b|$ is equal to **[JEE (Main)-2021]**

(1) 7 (2) 1

(3) 11 (4) 9

55. Consider the following frequency distribution :

Class :	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency :	a	b	12	9	5

If mean $= \frac{309}{22}$ and median = 14, then the value $(a - b)^2$ is equal to _______. **[JEE (Main)-2021]**

56. If the mean and variance of the following data :
6, 10, 7, 13, a, 12, b, 12
are 9 are $\frac{37}{4}$ respectively, then $(a - b)^2$ is equal to **[JEE (Main)-2021]**

(1) 32 (2) 12

(3) 24 (4) 16

57. An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to _______. **[JEE (Main)-2021]**

58. If the mean deviation about the mean of the numbers 1, 2, 3, …. n, where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to ___________. **[JEE (Main)-2022]**

59. The mean of the numbers a, b, 8, 5, 10 is 6 and their variance is 6.8. If M is the mean deviation of the numbers about the mean, then 25 M is equal to:

(1) 60 (2) 55

(3) 50 (4) 45

[JEE (Main)-2022]

60. The mean and standard deviation of 50 observations are 15 and 2 respectively. It was found that one incorrect observation was taken such that the sum of correct and incorrect observations is 70. If the correct mean is 16, then the correct variance is equal to : **[JEE (Main)-2022]**

(1) 10 (2) 36

(3) 43 (4) 60

61. The mean and variance of the data 4, 5, 6, 6, 7, 8, x, y, where $x < y$, are 6 and $\frac{9}{4}$ respectively. Then $x^4 + y^2$ is equal to **[JEE (Main)-2022]**

(1) 162 (2) 320

(3) 674 (4) 420

62. The mean and standard deviation of 15 observations are found to be 8 and 3 respectively. On rechecking it was found that, in the observations, 20 was misread as 5. Then, the correct variance is equal to _______. **[JEE (Main)-2022]**

63. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is __________. **[JEE (Main)-2022]**

64. Let the mean and the variance of 5 observations x_1, x_2, x_3, x_4, x_5 be $\frac{24}{5}$ and $\frac{194}{25}$ respectively. If the mean and variance of the first 4 observation are $\frac{7}{2}$ and a respectively, then $(4a + x_5)$ is equal to

(1) 13 (2) 15

(3) 17 (4) 18

[JEE (Main)-2022]

65. The number of values of $a \in N$ such that the variance of 3, 7, 12, a, 43 – a is a natural number is :

(1) 0 (2) 2

(3) 5 (4) Infinite

[JEE (Main)-2022]

66. The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If σ is the standard deviation of the data after omitting the two wrong observations from the data, then $38\sigma^2$ is equal to ______. [JEE (Main)-2022]

67. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are : **[JEE (Main)-2022]**

(1) 1, 20 (2) 10, 11

(3) 3, 18 (4) 8, 13

68. Let the mean and the variance of 20 observations x_1, x_2,...., x_{20} be 15 and 9, respectively. For $a \in \mathbf{R}$, if the mean of $(x_1 + \alpha)^2$, $(x_2 + \alpha)^2$,....,$(x_{20} + \alpha)^2$ is 178, then the square of the maximum value of α is equal to ___________. **[JEE (Main)-2022]**

69. If the mean deviation about median for the number 3, 5, 7, 2k, 12, 16, 21, 24 arranged in the ascending order, is 6 then the median is

[JEE (Main)-2022]

(1) 11.5 (2) 10.5

(3) 12 (4) 11

70. The mean and variance of 10 observation were calculated as 15 and 15 respectively by a student who took by mistake 25 instead of 15 for one observation. Then, the correct standard deviation is ________. **[JEE (Main)-2022]**

❑ ❑ ❑

Chapter 15

Statistics

1. Answer (3)

Statement (2) is true.

$$\text{var } x = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$= \frac{4\,n\,(n+1)\,(2n+1)}{6n} - (n+1)^2$$

$$= \frac{2}{3}\,(n+1)\,(2n+1) - (n+1)^2$$

$$= \frac{(n+1)}{3}\,\{4n + 2 - 3n - 3)$$

$$= \frac{(n+1)\,(n-1)}{3}$$

$$= \frac{n^2 - 1}{3}$$

∴ Statement (1) is false.

Statement (2) is true.

2. Answer (2)

$$\bar{x} = \frac{1 + (1+d) + (1+2d) + ...(1+100d)}{101}$$

$$\bar{x} = \frac{101 + d(1+2+3+.....100)}{101}$$

$$\bar{x} = \frac{101 + d \times \dfrac{100 \times 101}{2}}{101}$$

$$\bar{x} = 1 + 50d$$

Mean deviation

$$= \frac{|1+50d-1| + |1+50d-1-d| + ...|1+50d-1-100d|}{101}$$

$$= \frac{50d + 49d + 48d + ...d + 0 + d + 2d + ...50d}{101}$$

$$= \frac{2 \times d \times \left(\dfrac{50 \times 51}{2}\right)}{101}$$

$$\Rightarrow \frac{50 \times 51 \times d}{101} = 255$$

$\Rightarrow d = 10.1$

3. Answer (2)

$E(X^2) - (E(X))^2 = 4$

$\therefore \quad E(X^2) = 4 + 4 = 8$

$\sum X_i^2 = 40$

$E(Y^2) - (E(Y))^2 = 5$

$E(Y^2) = 5 + 16 = 21$

$\therefore \quad \sum Y_i^2 = 105$

$\sum X_i = 10, \sum Y_i = 20$

$\therefore \quad \sum (X_i + Y_i) = 30$

$\sum (X_i^2 + Y_i^2) = 145$

$\therefore$ Variance(combined data) $= \dfrac{145}{10} - 9 = \dfrac{55}{10} = \dfrac{11}{2}$

4. Answer (3)

Since weight of each fish is measured 2 gm lesser

∴ actual mean : 30 + 2 = 32

But standard deviation will remain unaffected as each data has been decreased by a constant.

5. Answer (3)

6. Answer (4)

With increase in data, mean will also increase by the same, hence variance will remain unchanged.

7. Answer (4)

Variance = $\dfrac{\Sigma x_i^2}{N} - (\bar{x})^2$

$\Rightarrow \quad \sigma^2 = \dfrac{2^2+4^2+\ldots+100^2}{50} - \left(\dfrac{2+4+\ldots+100}{50}\right)^2$

$= \dfrac{4(1^2+2^2+3^2+\ldots.+50^2)}{50} - (51)^2$

$= 4\left(\dfrac{50\times 51\times 101}{50\times 6}\right) - (51)^2$

= 3434 – 2601

$\Rightarrow \quad \sigma^2 = 833$

8. Answer (4)

Mean = 16

Sum = 16 × 16 = 256

New sum = 256 – 16 + 3 + 4 + 5 = 252

Mean = $\dfrac{252}{18}$ = 14

9. Answer (1)

$Var = \sigma^2 = \dfrac{\Sigma x_1^2}{n} - \left(\bar{x}\right)^2$

Standard Deviation =

$$\sqrt{\dfrac{2^2+3^2+a^2+11^2}{4} - \left(\dfrac{2+3+a+11}{4}\right)^2} = 3.5$$

$\Rightarrow \quad \dfrac{134+a^2}{4} - \left(\dfrac{16+a}{4}\right)^2 = (3.5)^2$

$\dfrac{4\left(134+a^2\right)}{16} - \dfrac{\left(16^2+a^2+32a\right)}{16} = (3.5)^2$

$536 + 4a^2 - 256 - a^2 - 32a = 196$

$3a^2 - 32a + 84 = 0$

10. Answer (3)

Standard deviation of $x_i - 5$ is

$$\sigma = \sqrt{\dfrac{\sum_{i=1}^{9}(x_i-5)^2}{9} - \left(\dfrac{\sum_{i=1}^{9}(x_i-5)}{9}\right)^2}$$

$\Rightarrow \quad \sigma = \sqrt{5-1} = 2$

As, standard deviation remains constant if observations are added/subtracted by a fixed quantity.

So, σ of x_i is 2

11. Answer (2)

Given, Variance = $\sigma^2 = \dfrac{\sum x_i^2}{N} - (\bar{x})^2$

$\Rightarrow 18 = \dfrac{\sum x_i^2}{5} - (150)^2$

$\Rightarrow \sum x_i^2 = 112590$

$V_{New} = \dfrac{112590+(156)^2}{6} - \left(\dfrac{750+156}{6}\right)^2$

= 22821 – 22801

= 20

12. Answer (3)

$\sigma^2 = \dfrac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\dfrac{1}{n}\sum_{i=1}^{n} x_i\right)^2$

$\sigma^2 = \dfrac{1}{n}A - \dfrac{1}{n^2}B^2$ ...(i)

$\because \sum_{i=1}^{n}(x_i+1)^2 = 9n$

$\Rightarrow \quad A + n + 2B = 9n \Rightarrow A + 2B = 8n$ ...(ii)

$\because \sum_{i=1}^{n}(x_i-1)^2 = 5n$

$\Rightarrow \quad A + n - 2B = 5n \Rightarrow A - 2B = 4n$ ...(iii)

From (ii) and (iii),

$A = 6n,\ B = n$

$\Rightarrow \quad \sigma^2 = \dfrac{1}{n}\times 6n - \dfrac{1}{n^2}\times n^2 = 6-1 = 5$

$\Rightarrow \quad \sigma = \sqrt{5}$

13. Answer (1)

$x_1 + x_2 + x_3 + x_5 = 25$

$x_1 + x_2 + x_3 = 1 + 3 + 8 = 12$

$\Rightarrow \quad x_4 + x_5 = 25 - 12 = 13$ ...(1)

$\dfrac{\sum_{i=1}^{5} x_i^2}{5} - (5)^2 = 9.2 \quad \Rightarrow \quad \sum_{i=1}^{5} x_i^2 = 5(25+9.2)$

= 125 + 46 = 171

$\Rightarrow \quad (1)^2 + (3)^2 + (8)^2 + x_4^2 + x_5^2 = 171$

$\Rightarrow \quad x_4^2 + x_5^2 = 97$ ...(2)

$\therefore \quad 2_4 x_5 = 13^2 - 97 = 72 \quad \Rightarrow \quad x_4 x_5 = 36$ ...(3)

(1) and (3) $\Rightarrow \quad x_4 : x_5 = \dfrac{4}{9}$ or $\dfrac{9}{4}$

14. Answer (4)

$$\sum_{i=1}^{5} x_i = 10 \times 5 = 50 \Rightarrow \sum_{i=1}^{6} x_i = 50 - 50 = 0$$

$$\frac{\sum_{i=1}^{5} x_i^2}{5} - (10)^2 = 3^2 = 9 \Rightarrow \sum_{i=1}^{5} x_i^2 = 545$$

$$\Rightarrow \sum_{i=1}^{6} x_i^2 = 545 + (-50)^2 = 3045$$

$$\text{Variance} = \frac{\sum_{i=1}^{6} x_i^2}{6} - \left(\frac{\sum_{i=1}^{6} x_i}{6}\right)^2$$

$$= \frac{3045}{6} - 0 = 507.5$$

15. Answer (1)

Outcomes are $\left(\frac{1}{2}-d\right), \left(\frac{1}{2}-d\right), \ldots,$ 10 times, $\frac{1}{2}, \frac{1}{2}, \ldots,$ 10 times, $\frac{1}{2}+d, \frac{1}{2}+d, \ldots,$ 10 times

$$\text{Mean} = \frac{1}{30}\left(\frac{1}{2} \times 30\right) = \frac{1}{2}$$

$$\sigma^2 = \frac{1}{30}\Sigma x_i^2 - (\bar{x})^2$$

$$= \frac{1}{30}\left[\left(\frac{1}{2}-d\right)^2 \times 10 + \left(\frac{1}{2}\right)^2 \times 10 + \left(\frac{1}{2}+d\right)^2 \times 10\right] - \frac{1}{4}$$

$$\Rightarrow \frac{4}{3} = \frac{1}{30}\left[30 \times \frac{1}{4} + 20d^2\right] - \frac{1}{4}$$

$$\Rightarrow \frac{4}{3} = \frac{1}{4} + \frac{2}{3}d^2 - \frac{1}{4}$$

$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$

$\Rightarrow$ Option (1) is correct.

16. Answer (1)

Given, $\sum(x_i - 30) = 50$

$$\sum x_i - 50(30) = 50$$

$$\Rightarrow \sum x_i = 1550$$

$$\text{Mean, } \bar{x} = \frac{\sum x_i}{N}$$

$$= \frac{1550}{50} = 31$$

17. Answer (2)

Let two observations are x_1 and x_2

$$\frac{x_1 + x_2 + 3 + 4 + 4}{5} = 4$$

$\Rightarrow x_1 + x_2 = 9$ ...(i)

$$\text{Variance} = \frac{\sum x_i^2}{N} - (\bar{x})^2$$

$$5 \cdot 20 = \frac{9 + 16 + 16 + x_1^2 + x_2^2}{5} - 16$$

$(21 \cdot 20)5 = 41 + x_1^2 + x_2^2$

$x_1^2 + x_2^2 = 65$ ...(ii)

From (i) and (ii);

$x_1 = 8,\ x_2 = 1$

$|x_1 - x_2| = 7$

18. Answer (3)

Let the remaining numbers are x and y.

$$\text{Mean } (\bar{x}) = \frac{\sum x_i}{N} = \frac{2+4+10+12+14+x+y}{7} = 8$$

$\Rightarrow x + y = 14$ ...(i)

$$\text{Variance } (\sigma^2) = \frac{\sum x_i^2}{N} - (\bar{x})^2 = 16$$

$$\Rightarrow \frac{2^2+4^2+10^2+12^2+14^2+x^2+y^2}{7} - (8)^2 = 16$$

$\Rightarrow x^2 + y^2 = 100$ ...(ii)

From (i) and (ii),

$(x, y) = (6, 8)$ or $(8, 6)$

$xy = 48$

19. Answer (2)

$$\bar{x} = \frac{41+45+54+57+43+x}{6} = 48$$

x + 240 = 288

x = 48

$$\sigma^2 = \frac{1}{6}\begin{bmatrix}(48-41)^2 + (48-45)^2 + (48-54)^2 \\ +(48-57)^2 + (48-43)^2 + (48-48)^2\end{bmatrix}$$

$$= \frac{1}{6}(49 + 9 + 36 + 81 + 25)$$

$$= \frac{200}{6} = \frac{100}{3}$$

$$\sigma = \frac{10}{\sqrt{3}}$$

20. Answer (2)

Mean of given observation = $\frac{k}{4}$

$\therefore$ σ^2 = 5(given) ...(i)

Also,

$$\sigma^2 = \frac{\left(\frac{k}{4}+1\right)^2 + \left(\frac{k}{4}\right)^2 + \left(\frac{k}{4}-1\right)^2 + \left(\frac{3k}{4}\right)^2}{4} \quad ...(ii)$$

$\therefore$ From (i) and (ii),

$$\frac{\frac{12k^2}{16}+2}{4} = 5$$

$$\Rightarrow \frac{12k^2}{16} = 18$$

$\Rightarrow$ k^2 = 24

$\Rightarrow$ $k = 2\sqrt{6}$

21. Answer (1)

$$\text{Mean} = \frac{\sum x_i}{n} = \frac{x+y+300}{10} = 42 \Rightarrow x+y=120$$

$$\text{Median} = \frac{T_5+T_6}{2} = 35 = \frac{34+x}{2} \Rightarrow x = 36 \ \& \ y = 84$$

Hence, $\frac{y}{x} = \frac{84}{36} = \frac{7}{3}$

22. Answer (4)

Number of students

$\Rightarrow$ $(x + 1)^2 + (2x - 5) + (x^2 - 3x) + x = 20$

$\Rightarrow$ $2x^2 + 2x - 4 = 20$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x = 3$

So,

Marks	2	3	5	7
No. of students	16	1	0	3

Average marks = $\frac{32+3+21}{20} = \frac{56}{20} = 2.8$

23. Answer (3)

$$\frac{x_1+x_2+...x_{50}}{50} = 16$$

$$16^2 = \frac{x_1^2+x_2^2...x_{50}^2}{50} - 16^2$$

$$2(16)^2 50 = x_1^2 + x_2^2 + ... x_{50}^2$$

Required mean

$$= \frac{(x_1-4)^2 + (x_2-4)^2 + ...(x_{50}-4)^2}{50}$$

$$= \frac{16^2(100) + 4^2(50) - 8(16 \times 50)}{50}$$

$= 16^2(2) + 16 - 8(16)$

$= 400$

24. Answer (3)

$$\frac{x_1+x_2+x_3+x_4}{4} = 11 \text{ and } x_1+x_2+x_3+x_4 = 44$$

$$\frac{x_5+x_6+...+x_{10}}{6} = 16 \Rightarrow x_5+x_6+...+x_{10} = 96$$

$$x_1^2 + x_2^2 + ... + x_{10}^2 = 2000$$

$$\sigma^2 = \frac{\Sigma x_i^2}{N} - (\bar{x})^2$$

$$= \frac{2000}{10} - \left(\frac{140}{10}\right)^2 = 4$$

$\Rightarrow$ $\sigma = 2$

25. Answer (2)

$\mu = 20$, $\sigma^2 = 2$

and $p\mu - q = \frac{20}{2}$ $\Rightarrow$ $20p - q = 10$

and also

$|p|\sigma^2 = |p|.2 = 1 \Rightarrow p = \pm\frac{1}{2}$

if $p = \frac{1}{2}$ $\Rightarrow$ q = 0 (rejected)

& $p = -\frac{1}{2}$ $\Rightarrow$ q = –20

26. Answer (3)

$\sum x_i = 200$ and $\sum x_i^2 = 2080$

Now Actual Mean = $\dfrac{200+11-9}{20} = \dfrac{202}{20}$

$\therefore$ Actual variance = $\dfrac{2080-81+121}{20} - \left(\dfrac{202}{20}\right)^2$

$106 - (10.1)^2 = 106 - 102.01 = 3.99$

27. Answer (3)

Let $(x_i - 5) = y_i$

So $\bar{y} = \dfrac{\sum y_i}{10} = \dfrac{10}{10} = 1$

and $\mathrm{Var}(y) = \dfrac{\sum y_i^2}{10} - (\bar{y})^2 = 3$

Now mean of $(x_i - 3) = (y_i + 2)$ is $\bar{y}+2$, which is 3.

And variance remains same.

28. Answer (4)

$\because \quad \bar{x} = \dfrac{\sum_{r=1}^{17} r}{17} = \dfrac{17\times 18}{17\times 2} = 9$

$\because \quad \bar{y} = a\bar{x} + b = 17 \Rightarrow 9a + b = 17 \quad \ldots(1)$

$\because \quad Var(x) = \dfrac{\sum_{r=1}^{17} r^2}{17} - (\bar{x})^2 = \dfrac{17\times 18\times 35}{17\times 6} - 9^2 = 24$

and Var(y) = a^2.Var(x) $\Rightarrow 24a^2 = 216 \Rightarrow a = 3$ …(2)

Clearly b = –10

hence a + b = –7

29. Answer (2)

If variate varries from a to b then variance

$\mathrm{var}(x) \le \left(\dfrac{b-a}{2}\right)^2$

$\Rightarrow \quad \mathrm{var}(x) < \left(\dfrac{10-0}{2}\right)^2$

$\Rightarrow$ var(x) < 25

$\Rightarrow$ standard deviation < 5

Clearly standard deviation cann't be 6.

30. Answer (2)

Here n = 10

So $S.D. = \sqrt{\dfrac{\sum(x_i-p)^2}{n} - \left(\dfrac{\sum x_i - p}{n}\right)^2}$

$= \sqrt{\dfrac{9}{10} - \dfrac{9}{100}} = \sqrt{\dfrac{90-9}{100}} = \dfrac{9}{10}$

31. Answer (3)

Let the two remaining observations be x and y.

$\because \quad \bar{x} = 10 = \dfrac{5+7+10+12+14+15+x+y}{8}$

$\Rightarrow$ x + y = 17 ...(1)

$\because \quad \mathrm{var}(x) = 13.5 = \dfrac{25+49+100+144+196+225+x^2+y^2}{8} - (10)^2$

$\Rightarrow$ $x^2 + y^2 = 169$ ...(2)

From (1) and (2)

(x, y) = (12, 5) or (5, 12)

So |x – y| = 7

32. Answer (1)

Let two remaining observations are x, y

So $\bar{x} = \dfrac{2+4+10+12+14+x+y}{7} = 8$ (given)

$\Rightarrow$ x + y = 14 …(1)

Now also $\sigma^2 = \dfrac{\Sigma x_i^2}{N} - \left(\dfrac{\Sigma x_i}{N}\right)^2 = 16$ (given)

$= \dfrac{4+16+100+144+196+x^2+y^2}{7} - 64 = 16$

$\Rightarrow$ $460 + x^2 + y^2 = (16 + 64) \times 7$

$\Rightarrow$ $x^2 + y^2 = 100$ …(2)

Now $(x + y)^2 = x^2 + y^2 + 2xy \Rightarrow xy = 48$ …(3)

Now $(x - y)^2 = (x + y)^2 - 4xy = 196 - 192 = 4$

$\Rightarrow$ x – y = 2 $\Rightarrow$ |x – y| = 2

33. Answer (4)

Mean $= \dfrac{3+5+7+a+b}{5} = 5 \Rightarrow a+b = 10$

Variance $= \dfrac{3^2+5^2+7^2+a^2+b^2}{5} - (5)^2 = 4$

$\Rightarrow$ $a^2 + b^2 = 62$

$\Rightarrow$ $(a + b)^2 - 2ab = 62$

$\Rightarrow$ ab = 19

So a and b are the roots of the equation

$x^2 - 10x + 19 = 0$

34. Answer (4)

Standard deviation = $\sqrt{\dfrac{\sum x_i^{\,2}}{N} - \left(\dfrac{\sum x_i}{N}\right)^2}$

$= \sqrt{\dfrac{\sum (x_i - a)^2}{N} - \left(\dfrac{\sum (x_i - a)}{N}\right)^2}$

$= \sqrt{\dfrac{na}{n} - \left(\dfrac{n}{n}\right)^2}$

$= \sqrt{a-1}$

35. Answer (18)

As $\sigma^2 = \dfrac{\sum x_i^2}{N} - (\bar{x})^2$

$\Rightarrow \quad 10 = \dfrac{1^2 + 2^2 + ... n^2}{n} - \left(\dfrac{n(n+1)}{2n}\right)^2$

$\therefore \quad 10 = \dfrac{(n+1)(2n+1)}{6} - \dfrac{(n+1)^2}{4}$

Solving, n = 11

Also, $16 = \dfrac{2^2 + 4^2 + ... (2m)^2}{m} - (m+1)^2$

$\Rightarrow \quad 16 = \dfrac{2(m+1)(2m+1)}{3} - (m+1)^2$

$\Rightarrow \quad m^2 = 49$

$\Rightarrow \quad m = 7$

$\therefore \quad m + n = 18$

36. Answer (54)

$\dfrac{x+y+64}{8} = 10$

$\Rightarrow \quad x + y = 16 \qquad \ldots(1)$

Also $\quad 25 = \dfrac{\Sigma x_i^2}{8} - 100$

$\Rightarrow \quad \Sigma x_i^2 = 1000$

$x^2 + y^2 = 148 \qquad \ldots(2)$

From (1) & (2); $\quad \Rightarrow \quad xy = 54$

37. Answer (3)

Variance $= \dfrac{\sum_{i=1}^{11} b_i^2}{11} - \left(\dfrac{\sum_{i=1}^{11} b_i}{11}\right)^2$

$= \dfrac{\sum_{r=0}^{10} (b_1 + rd)^2}{11} - \left(\dfrac{\sum_{r=0}^{10} (b_1 + rd)}{11}\right)^2$

$= \dfrac{11 b_1^2 + 2b_1 d\left(\dfrac{10\times 11}{2}\right) + d^2\left(\dfrac{10\times 11\times 21}{6}\right)}{11} - \left(\dfrac{11 b_1 + \dfrac{10\times 11}{2} d}{11}\right)^2$

$= \left(b_1^2 + 10 b_1 d + 35 d^2\right) - (b_1 + 5d)^2 = 10d^2$

$\because \quad$ Variance = 90

$\Rightarrow \quad 10d^2 = 90$

$\Rightarrow \quad d = 3$

38. Answer (4)

x_i	15	25	35
f_i	2	x	2

$\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{30 + 70 + 25x}{4 + x} = 25$

Given $\sigma^2 = 50 = \dfrac{\sum f_i x_i^{\,2}}{\sum f_i} - (\bar{x})^2$

$\Rightarrow \quad 50 = \dfrac{450 + 625x + 2450}{4+x} - 625$

$\Rightarrow \quad 675 = \dfrac{2900 + 625x}{4+x}$

$\Rightarrow \quad 50x = 200$

$\therefore \quad x = 4$

39. Answer (06)

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i}$$

$$= \frac{0.{}^nC_0 + 2.{}^nC_1 + 2^2.{}^nC_2 + \ldots + 2^n.{}^nC_n}{{}^nC_0 + {}^nC_1 + \ldots + {}^nC_n}$$

For finding sum of numerator consider

$(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n$ ...(1)

Put $x = 2 \Rightarrow 3^n - 1 = 2.{}^nC_1 + 2^2.{}^nC_2 + \ldots + 2^n.{}^nC_n$

For sum of denominator

Put x = 1 in (1)

$2^n = {}^nC_0 + {}^nC_1 + \ldots {}^nC_n$

$$\therefore \frac{3^n - 1}{2^n} = \frac{728}{2^n} \Rightarrow 3^n = 729 \Rightarrow n = 6$$

40. Answer (11)

$$\sigma^2 = \frac{9 + k^2}{10} - \left(\frac{9 + k}{10}\right)^2 < 10$$

$10(k^2 + 9) - (k^2 + 18k + 81) < 1000$

$9k^2 - 18k + 9 < 1000$

$9(k - 1)^2 < 1000$

$$|k - 1| < \frac{10\sqrt{10}}{3} = \frac{10 \times 3.162}{3} = 10.54$$

$-10.54 < k - 1 < 10.54$

$-9.54 < k < 11.54$

But $k \in N$, $\therefore k_{max} = 11$

41. Answer (4)

$$\because \sum_{i=1}^{18}(x_i - \beta)^2 = 90$$

$$\text{and } \sum_{i=1}^{18}(x_i - \beta) = \sum_{i=1}^{18}(x_i - \alpha) + 18(\alpha - \beta)$$

$= 36 + 18(\alpha - \beta)$

So

$$\text{Var}(x_i) = \text{Var}(x_i - \beta) = \frac{\sum(x_i - \beta)^2}{18} - \left(\frac{\sum(x_i - \beta)}{18}\right)^2$$

$$\Rightarrow 1 = \frac{90}{18} - (2 + \alpha - \beta)^2$$

$\Rightarrow 2 + \alpha - \beta = \pm 2$

$\Rightarrow \alpha - \beta = 0, -4$

$\because \alpha$ and β are distinct, so $|\alpha - \beta| = 4$

42. Answer (2)

$\because$ Standard deviation of a, b, c is d.

$$d^2 = \frac{a^2 + b^2 + c^2}{3} - \left(\frac{a + b + c}{3}\right)^2$$

$\Rightarrow 9d^2 = 3(a^2 + b^2 + c^2) - 4b^2$

$\Rightarrow b^2 = 3(a^2 + c^2) - 9d^2$

43. Answer (5)

$$\overline{x_1} = 2,\ \overline{x_2} = 3,\ \overline{x} = \frac{3n + 20}{n + 10}$$

$$d_1^2 = \left(\overline{x} - \overline{x_1}\right)^2 = \frac{n^2}{(n + 10)^2},\ d_2^2 = \left(\overline{x} - \overline{x_2}\right)^2 = \frac{100}{(n + 10)^2}$$

$$\sigma_1^2 = 2,\ \sigma_2^2 = 1,\ \sigma^2 = \frac{17}{9}$$

$$(n_1 + n_2)\sigma^2 = n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)$$

$$(n + 10) \times \frac{17}{9} = 10\left(2 + \frac{n^2}{(n + 10)^2}\right) + n\left(1 + \frac{100}{(n + 10)^2}\right)$$

$$(n + 10)17 = \left[20 + n + \frac{10n^2 + 100n}{(n + 10)^2}\right] \times 9$$

$(8n - 10)(n+10)^2 = 90n^2 + 900n$

$(8n - 10)(n^2 + 20n + 100) = 90n^2 + 900n$

$(4n - 5)(n^2 + 20n + 100) = 45n^2 + 450n$

$2n^3 + 15n^2 - 75n - 250 = 0$

$(n - 5)(n + 10)(2n + 5) = 0$

$n = 5$

44. Answer (68)

Let $x_1, x_2, \ldots\ldots\ldots\ldots x_{3n}$ be the given numbers.

$$\overline{x} = \frac{6 \cdot 2n + 3 \cdot n}{3n} = 5$$

$$4 = \frac{\Sigma x_i^2}{3n} - 25 \Rightarrow \frac{\Sigma x_i^2}{3n} = 29 \quad .....(i)$$

Let $y_i = x_i + 1$ for $1 \le i \le 2n$

and $y_i = x_i - 1$ for $2n + 1 \le i \le 3n$

$$\text{So } \overline{y} = \frac{\Sigma y_i}{3n} = \frac{\Sigma x_i + n}{3n} = 5 + \frac{1}{3} = \frac{16}{3}$$

$$\text{Now } k = \frac{\Sigma y_i^2}{3n} - \left(\frac{16}{3}\right)^2 = \frac{\Sigma x_i^2}{3n} + 2\left(\frac{\sum_{i=1}^{2n} x_i}{3n}\right)$$

$$-2\left(\frac{\sum_{i=2n+1}^{3n} x_i}{3n}\right)+1-\frac{256}{9}$$

$$k=29+8-2+1-\frac{256}{9}=36-\frac{256}{9}=\frac{68}{9}$$

45. Answer (35)

$x_1 + x_2 + ... + x_{25} = 25 \times 40$ ...(i)

Let age of new teacher is A

then $(x_1 + x_2 + ... + x_{25}) - 60 + A = 25 \times 39$

$\Rightarrow$ A = 975 + 60 – 1000 = 35 years

46. Answer (3)

Old mean $= \frac{\sum x_i}{n} = 0$

New mean = 0 + b = 5

$\Rightarrow$ b = 5

Old S.D $= \sqrt{\frac{\sum (x_i-\bar{x})^2}{n}} = \sqrt{\frac{\sum a^2}{n}} = a$

New S.D = old S.D = a = 20

$a^2 + b^2 = 425$

47. Answer (164)

C.I.	x_i	f_i	$x_i \cdot f_i$	C.F.
10-20	15	α	15α	α
20-30	25	110	2750	$110+\alpha$
30-40	35	54	1890	$164+\alpha$
40-50	45	30	1350	$194+\alpha$
50-60	55	β	55β	$194+\alpha+\beta$
		$194+\alpha+\beta$	$5990+15\alpha+55\beta$	

median class

$\because$ $\alpha + \beta = 584 - 194$

$\Rightarrow$ $\alpha + \beta = 390$ ...(1)

and median $= 40+\left(\frac{194+\alpha-292}{30}\right)10 = 45$

$\Rightarrow$ $\alpha = 113$ ...(2)

So $\beta = 277$

48. Answer (3)

$n_1 = 100$, $n_2 = 150$,

$(n_1+n_2)\bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$

$250\times 15.6 = 100\times 15 + 150\times \bar{x}_2$

$\bar{x}_2 = 16 \quad d_1^2 = (\bar{x}-\bar{x}_1)^2 = 0.36$

$(n_1+n_2)\sigma^2 = n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)$

$250\times 13.44 = 100(9.36)+150\left(\sigma_2^2+0.16\right)$

$\sigma_2^2 = 16$

49. Answer (3)

$\because \quad \bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{8+24+8\alpha+9\beta}{8+\alpha+\beta} = 6 \Rightarrow 2\alpha+3\beta = 16$ ...(i)

$\sigma^2 = \frac{\Sigma x_i^2 f_i}{\Sigma f_i} - (\bar{x})^2 \Rightarrow \frac{16+144+64\alpha+81\beta}{8+\alpha+\beta} = 42.8$

$\Rightarrow$ $106\alpha + 191\beta = 912$ ...(ii)

from (i) and (ii), $\alpha = 5$ and $\beta = 2$

Now, correct mean $= \frac{8+24+35+18}{15} = \frac{17}{3}$

50. Answer (4)

$\bar{x} = 10 \quad \sqrt{\frac{\Sigma x_i^2}{20} - (\bar{x})^2} = 2.5$

$\frac{\Sigma x_i^2}{20} - (10)^2 = 6.25 \Rightarrow \Sigma x_i^2 = 20\times 106.25$

$= 2125$

$\Sigma x_{i\,(\text{actual})}^2 = 2125 - 25^2 + 35^2$

$= 2125 + 600 = 2725$

For $\bar{x}_{(\text{actual})} \Rightarrow \frac{\Sigma x}{n} = 10 \Rightarrow \Sigma x = 200$

$\Sigma x_{(\text{actual})} = 200 - 25 + 35 = 210$

$\bar{x}_{(\text{actual})} = \frac{210}{20} = 10.5$

S.D. $= \sqrt{\frac{2725}{20} - (10.5)^2} = \sqrt{136.25 - 110\cdot 5} = \sqrt{26}$

51. Answer (12)

Numbers 3, 7, x, y

$\bar{x} = 5,\ \sigma^2 = 10$

$5 = \frac{3+7+x+y}{4} \Rightarrow x+y = 10$ ...(i)

$10 = \frac{1}{4}((3)^2+(7)^2+(x)^2+(y)^2) - (5)^2$

$140 = 58 + x^2 + y^2 \Rightarrow x^2 + y^2 = 82$ ...(ii)

$(x + y)^2 = x^2 + y^2 + 2xy \Rightarrow 100 = 82 + 2xy$

$xy = 9$

$y = \frac{9}{x} \Rightarrow x + \frac{9}{x} = 10 \Rightarrow \begin{matrix} x & = & 1 & \text{or} & 9 \\ y & = & 9 & \text{or} & 1 \end{matrix}$

Given $x > y \Rightarrow x = 9,\ y = 1$

Now, $3 + 2x$, $7 + 2y$, $x + y$, $x - y$ = 21, 9, 10, 8

$\bar{x} = \frac{21+9+10+8}{4} = \frac{48}{4} = 12$

52. Answer (1)

$$\bar{X}_{old} = 8 \Rightarrow \sum X_{old} = 56$$

$$\frac{\sum X_{old}^2}{7} - (\bar{X}_{old})^2 = 16 \Rightarrow \sum X_{old}^2 = 560$$

Sum of remaining 5 observation

$= \sum X = 56 - 14 = 42$

Sum of squares of 5 observation = $560 - 6^2 - 8^2$

$= 460$

Variance = $\frac{460}{5} - \left(\frac{42}{5}\right)^2 = \frac{536}{25}$

53. Answer (2)

Let the observations be 2, 4, 5, 7, x and y

$\bar{x} = \frac{18 + x + y}{6} = 6.5 \Rightarrow x + y = 21$ ...(i)

and $\sigma^2 = \frac{2^2 + 4^2 + 5^2 + 7^2 + x^2 + y^2}{6} - (6.5)^2 = 10.25$

$\Rightarrow x^2 + y^2 = 221$ ...(ii)

From (i) and (ii), we get

$(x, y) = (10, 11)$ or $(11, 10)$

54. Answer (2)

Given $\frac{7 + 10 + 11 + 15 + a + b}{6} = 10$

$\Rightarrow a + b = 17$ ...(i)

& $\frac{7^2 + 10^2 + 11^2 + 15^2 + a^2 + b^2}{6} - 10^2 = \frac{20}{3}$

$\frac{4095 + a^2 + b^2}{6} = \frac{320}{3}$

$\Rightarrow a^2 + b^2 = 145$ ...(ii)

$\therefore a^2 + b^2 + 2ab = 289$

$\Rightarrow 2ab = 144$

$(a - b)^2 = 145 - 144$

$\therefore (a - b) = 1$

55. Answer (4)

Class Interval	x_i	f_i	x_if_i	C.F.
0 – 6	3	a	3a	a
6 – 12	9	b	9b	a + b
12 – 18	15	12	180	12 + a + b → Median Class
18 – 24	21	9	189	21 + a + b
24 – 30	27	5	135	26 + a + b
		a+b+26	3a+9b+504	

$$\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{3a + 9b + 504}{a + b + 26} = \frac{309}{22} \Rightarrow 81a + 37b = 1018$$

...(1)

$$\text{Median} = 12 + \frac{13 + \frac{a+b}{2} - (a+b)}{12} \times 6 = 14 \Rightarrow a + b = 18$$

...(2)

From (1) and (2), $a = 8$ and $b = 10$

56. Answer (4)

$6 + 10 + 7 + 13 + 12 + 12 + (a + b) = 72$

$\Rightarrow a + b = 12$

and

$$\frac{a^2 + b^2 + 36 + 100 + 49 + 169 + 144 + 144}{8} = \frac{37}{4}$$

$a^2 + b^2 + 642 - 648 = 74$

$a^2 + b^2 = 80$

$\therefore (a + b)^2 = a^2 + b^2 + 2ab \Rightarrow 2ab = 64$

$(a - b)^2 = a^2 + b^2 - 2ab = 16$

57. Answer (25)

Sum of marks of boys $\sum X_B = 240$

Total marks $\Rightarrow \sum X = 750$

So, sum of marks of girls = $510 = \sum X_G$

$$\Rightarrow \frac{\sum X_B^2}{20} - (12)^2 = 2 \text{ and } \frac{\sum X_G^2}{30} - (\bar{X}_G)^2 = 2$$

$$\sum X_B^2 = 2920 \text{ and } \frac{\sum X_B^2}{30} - (17)^2 = 2$$

$$\therefore \quad \sum X_G^2 = 8730$$

$$(\text{variance})_{\text{overall}} = \frac{\sum X_B^2 + X_G^2}{50} - (\bar{X})^2$$

$$= \frac{2920 + 8730}{50} - (15)^2 = 8$$

$\mu = 17$, $\sigma^2 = 8$

58. Answer (21)

$$\text{Mean} = \frac{n\frac{(n+1)}{2}}{n} = \frac{n+1}{2}$$

$$\text{M.D.} = \frac{2\left(\frac{n-1}{2} + \frac{n-3}{2} + \frac{n-5}{2} + ...0\right)}{n} = \frac{5(n+1)}{n}$$

$\Rightarrow$ $((n-1) + (n-3) + (n-5) + \ldots 0) = 5(n+1)$

$$\Rightarrow \left(\frac{n+1}{4}\right) \cdot (n-1) = 5(n+1)$$

So, $n = 21$

59. Answer (1)

$$\because \quad \bar{x} = 6 = \frac{a+b+8+5+10}{5} \Rightarrow a+b=7 \quad \ldots\text{(i)}$$

$$\text{And } \sigma^2 = \frac{a^2+b^2+8^2+5^2+10^2}{5} - 6^2 = 6.8$$

$\Rightarrow$ $a^2 + b^2 = 25$ ...(ii)

From (i) and (ii) $(a, b) = (3, 4)$ or $(4, 3)$

Now mean deviation about mean

$$M = \frac{1}{5}(3+2+2+1+4) = \frac{12}{5}$$

$$\Rightarrow \quad 25M = 60$$

60. Answer (3)

Given $\bar{x} = 15, \sigma = 2 \Rightarrow \sigma^2 = 4$

$\therefore$ $x_2 + x_2 + \ldots\ldots + x_{50} = 15 \times 50 = 750$

$$4 = \frac{x_1^2 + x_2^2 + ... + x_{50}^2}{50} - 225$$

$$\therefore \quad x_1^2 + x_2^2 + ... + x_{50}^2 = 50 \times 229$$

Let a be the correct observation and b is the incorrect observation

then $a + b = 70$

$$\text{and } 16 = \frac{750 - b + a}{50}$$

$\therefore$ $a - b = 50 \Rightarrow a = 60$, $b = 10$

$$\therefore \quad \text{Correct variance} = \frac{50 \times 229 + 60^2 - 10^2}{50} - 256$$

$= 43$

61. Answer (2)

$$\text{Mean} = \frac{4+5+6+6+7+8+x+y}{8} = 6$$

$\therefore$ $x + y = 12$ ...(i)

And variance

$$= \frac{2^2+1^2+0^2+0^2+1^2+2^2+(x-6)^2+(y-6)^2}{8}$$

$$= \frac{9}{4}$$

$\therefore$ $(x-6)^2 + (y-6)^2 = 8$ ...(ii)

From (i) and (ii)

$x = 4$ and $y = 8$

$\therefore$ $x^4 + y^2 = 320$

62. Answer (17)

$$\frac{\Sigma x_i^2}{15} - 8^2 = 9 \Rightarrow \Sigma x_i^2 = 15 \times 73 = 1095$$

Let $\bar{x}_c$ be corrected mean $\bar{x}_c = 9$

$\Sigma x_c^2 = 1095 - 25 + 400 = 1470$

$$\text{Correct variance} = \frac{1470}{15} - (9)^2 = 98 - 81 = 17$$

63. Answer (0)

According to given data

$$\frac{\sum_{i=1}^{7}(x_i - 62)^2}{7} = 20$$

$$\Rightarrow \sum_{i=1}^{7}(x_i - 62)^2 = 140$$

So for any x_i, $(x_i - 62)^2 \le 140$

$\Rightarrow x_i > 50$ □ $i = 1, 2, 3, \ldots 7$

So no student is going to score less than 50.

64. Answer (2)

$$\sum_{i=1}^{5} x_i = 24 \qquad \ldots(i)$$

$$\frac{\sum_{i=1}^{5} x_i^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{25}$$

$$\Rightarrow \sum x_i^2 = 1154 \qquad \ldots(ii)$$

$$\sum_{i=1}^{4} x_i = 14$$

$\Rightarrow x_5 = 10$

$$a = \frac{\sum_{i=1}^{4} x_i^2}{4} - \frac{49}{4} = \frac{54-49}{4} = \frac{5}{4}$$

$\Rightarrow x_5 + 4a = 10 + 5 = 15$

65. Answer (1)

$$\text{Mean} = \frac{3+12+7+a+43-a}{5} = 13$$

Variance

$$= \frac{9+49+144+a^2+(43-a)^2}{5} - 13^2 \in \text{Natural number}$$

$$\frac{2a^2 - a + 1}{5} \in \text{Natural number}$$

$2a^2 - a + 1 = 5n \qquad [n \in N]$

$2a^2 - a + 1 - 5n = 0$

$D = 1 - 4(1 - 5n)2$

$= 40n - 7$

D cannot be a perfect square as all perfect squares will be of the form of 4λ or $4\lambda + 1$

So, a cannot be natural number

$\therefore$ Number of values = 0

66. Answer (238)

$$\mu = \frac{\sum x_i}{40} = 30 \quad \Rightarrow \sum x_i = 1200$$

$$\sigma^2 = \frac{\sum x_i^2}{40} - (30)^2 = 25 \Rightarrow \sum x_i^2 = 37000$$

After omitting two wrong observations

$$\sum y_i = 1200 - 12 - 10 = 1178$$

$$\sum y_i^2 = 37000 - 144 - 100 = 36756$$

$$\text{Now } \sigma^2 = \frac{\sum y_i^2}{38} - \left(\frac{\sum y_i}{38}\right)^2$$

$$= \frac{36756}{38} - \left(\frac{1178}{38}\right)^2 = -31^2$$

$38\sigma^2 = 36756 - 36518 = 238$

67. Answer (2)

Let the observations be 2, 4, 5, 7, x and y

$$\bar{x} = \frac{18+x+y}{6} = 6.5 \Rightarrow x + y = 21 \qquad \ldots(i)$$

$$\text{and } \sigma^2 = \frac{2^2+4^2+5^2+7^2+x^2+y^2}{6} - (6.5)^2 = 10.25$$

$\Rightarrow x^2 + y^2 = 221 \qquad \ldots(ii)$

From (i) and (ii), we get

(x, y) = (10, 11) or (11, 10)

68 Answer (4)

$$\text{Given } \frac{\sum_{i=1}^{20} x_i}{20} = 15 \quad \Rightarrow \sum_{i=1}^{20} x_i = 300 \qquad \ldots(1)$$

$$\text{and } \frac{\sum_{i=1}^{20} x_i^2}{20} - \left(\bar{x}\right)^2 = 9 \Rightarrow \sum_{i=1}^{20} x_i^2 = 4680 \ldots(2)$$

Mean = $\dfrac{(x_i+\alpha)^2+(x_2+\alpha)^2+......+(x_{20}+\alpha)^2}{20}$

= 178

$\Rightarrow \dfrac{\sum_{i=1}^{20} x_i^2+2\alpha\sum_{i=1}^{20} x_i+20\alpha^2}{20}=178$

$\Rightarrow$ 4680 + 600α + 20α² = 3560

$\Rightarrow$ α² + 30α + 56 = 0

$\Rightarrow$ α² + 28α + 2α + 56 = 0

$\Rightarrow$ (α + 28)(α + 2) = 0

$\alpha_{max} = -2 \quad \Rightarrow \quad \alpha_{max}^2 = 4$.

69. Answer (4)

Median = $\dfrac{2k+12}{2}=k+6$

Mean deviation = $\sum\dfrac{|x_i-M|}{n}=6$

$\Rightarrow \dfrac{(k+3)+(k+1)+(k-1)+(6-k)+(6-k)+(10-k)+(15-k)+(18-k)}{8}$

$\therefore \dfrac{58-2k}{8}=6$

$k = 5$

Median = $\dfrac{2\times5+12}{2}=11$

70. Answer (2)

Given $\dfrac{\sum_{i=1}^{10} x_i}{10}=15 \quad \ldots(1) \qquad \Rightarrow \sum_{i=1}^{10} x_i=150$

and $\dfrac{\sum_{i=1}^{10} x_i^2}{10}-15^2=15 \qquad \Rightarrow \sum_{i=1}^{10} x_i^2=2400$

Replacing 25 by 15 we get

$\sum_{i=1}^{9} x_i+25=150 \qquad \Rightarrow \sum_{i=1}^{9} x_i=125$

$\therefore$ Correct mean $=\dfrac{\sum_{i=1}^{9} x_i+15}{10}=\dfrac{125+15}{10}=14$

Similarly, $\sum_{i=1}^{2} x_i^2=2400-25^2=1775$

$\therefore$ correct variance $=\dfrac{\sum_{i=1}^{9} x_i^2+15^2}{10}-14^2$

$=\dfrac{1775+225}{10}-14^2=4$

$\therefore$ correct S.D = $\sqrt{4}=2$

❑ ❑ ❑