Chapter 2

Quadratic Equations

1. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is **[AIEEE-2009]**

(1) Less than $4ab$ (2) Greater than $-4ab$

(3) Less than $-4ab$ (4) Greater than $4ab$

2. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[AIEEE-2010]**

(1) –2 (2) –1

(3) 1 (4) 2

3. Let for $a \neq a_1 \neq 0$,

$f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$.

If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(2)$ is **[AIEEE-2011]**

(1) 6 (2) 18

(3) 3 (4) 9

4. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4, 3). Rahul made a mistake in writing down coefficient of x to get roots (3, 2). The correct roots of equation are: **[AIEEE-2011]**

(1) –6, –1 (2) –4, –3

(3) 6, 1 (4) 4, 3

5. The real number k for which the equation $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1] **[JEE (Main)-2013]**

(1) Lies between 1 and 2

(2) Lies between 2 and 3

(3) Lies between –1 and 0

(4) Does not exist

6. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$, have a common root, then $a : b : c$ is **[JEE (Main)-2013]**

(1) 1 : 2 : 3 (2) 3 : 2 : 1

(3) 1 : 3 : 2 (4) 3 : 1 : 2

7. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is equal to **[JEE (Main)-2015]**

(1) 6 (2) –6

(3) 3 (4) –3

8. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is **[JEE (Main)-2016]**

(1) – 4 (2) 6

(3) 5 (4) 3

9. If, for a positive integer n, the quadratic equation, $x(x+1) + (x+1)(x+2) + ... + (x+\overline{n-1})(x+n) = 10n$ has two consecutive integral solutions, then n is equal to **[JEE (Main)-2017]**

(1) 9 (2) 10

(3) 11 (4) 12

10. Let $S = \{x \in R : x \geq 0$ and $2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0\}$. Then S **[JEE (Main)-2018]**

(1) Is an empty set

(2) Contains exactly one element

(3) Contains exactly two elements

(4) Contains exactly four elements

11. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1, 5] then m lies in the interval **[JEE (Main)-2019]**

(1) (–5, –4) (2) (3, 4)

(3) (4, 5) (4) (5, 6)

12. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is **[JEE (Main)-2019]**

(1) 4 (2) 5

(3) 2 (4) 3

13. Consider the quadratic equation $(c-5)x^2 - 2cx + (c-4) = 0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is **[JEE (Main)-2019]**

(1) 11 (2) 18

(3) 12 (4) 10

14. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3-\lambda)x + 2 = \lambda$ has the least value is **[JEE (Main)-2019]**

(1) 2 (2) 1

(3) $\frac{15}{8}$ (4) $\frac{4}{9}$

15. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is **[JEE (Main)-2019]**

(1) –300 (2) 144

(3) –81 (4) 100

16. Let α and β the roots of the quadratic equation

$x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$

$(0 < \theta < 45°)$, and $\alpha < \beta$. Then

$\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$ (2) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

(3) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$ (4) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

17. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m-4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is

[JEE (Main)-2019]

(1) $4-2\sqrt{3}$ (2) $4-3\sqrt{2}$

(3) $2-\sqrt{3}$ (4) $-2+\sqrt{2}$

18. The number of integral values of m for which the quadratic expression, $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, $x \in R$, is always positive, is

[JEE (Main)-2019]

(1) 8 (2) 3

(3) 6 (4) 7

19. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is **[JEE (Main)-2019]**

(1) 4 (2) 5

(3) 3 (4) 2

20. The sum of the solutions of the equation $\left|\sqrt{x}-2\right| + \sqrt{x}\left(\sqrt{x}-4\right) + 2 = 0, (x > 0)$ is equal to

[JEE (Main)-2019]

(1) 4 (2) 10

(3) 9 (4) 12

21. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?

[JEE (Main)-2019]

(1) d, e, f are in A.P.

(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(4) d, e, f are in G.P.

22. The number of integral values of m for which the equation $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real root is : **[JEE (Main)-2019]**

(1) Infinitely many (2) 3

(3) 2 (4) 1

23. Let $p, q \in R$. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then

[JEE (Main)-2019]

(1) $q^2 - 4p - 16 = 0$ (2) $p^2 - 4q + 12 = 0$

(3) $p^2 - 4q - 12 = 0$ (4) $q^2 + 4p + 14 = 0$

24. If m is chosen in the quadratic equation $(m^2+1)x^2 - 3x + (m^2+1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is **[JEE (Main)-2019]**

(1) $8\sqrt{3}$ (2) $10\sqrt{5}$

(3) $4\sqrt{3}$ (4) $8\sqrt{5}$

25. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r + \lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ is equal to **[JEE (Main)-2019]**

(1) $\frac{21}{346}$ (2) $\frac{7}{116}$

(3) $\frac{29}{358}$ (4) $\frac{1}{12}$

26. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to

[JEE (Main)-2019]

(1) 0 (2) $\alpha\gamma$

(3) $\beta\gamma$ (4) $\alpha\beta$

27. Let α and β be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}\cdot\lambda\tan x = (1-k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is **[JEE (Main)-2020]**

(1) 10 (2) $10\sqrt{2}$

(3) 5 (4) $5\sqrt{2}$

28. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true?

[JEE (Main)-2020]

(1) $p_3 = p_5 - p_4$

(2) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(3) $p_5 = 11$

(4) $p_5 = p_2 \cdot p_3$

29. Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S

[JEE (Main)-2020]

(1) Contains at least four elements

(2) Is a singleton

(3) Contains exactly two elements

(4) Is an empty set

30. The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is **[JEE (Main)-2020]**

(1) 4 (2) 2

(3) 3 (4) 1

31. Let $a, b \in R$, $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to **[JEE (Main)-2020]**

(1) 25 (2) 24

(3) 26 (4) 28

32. Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, $n = 1, 2, 3, ...$, then **[JEE (Main)-2020]**

(1) $5S_6 + 6S_5 = 2S_4$ (2) $6S_6 + 5S_5 + 2S_4 = 0$

(3) $6S_6 + 5S_5 = 2S_4$ (4) $5S_6 + 6S_5 + 2S_4 = 0$

33. Let $f(x)$ be a quadratic polynomial such that $f(-1) + f(2) = 0$. If one of the roots of $f(x) = 0$ is 3, then its other root lies in **[JEE (Main)-2020]**

(1) $(-1, 0)$ (2) $(-3, -1)$

(3) $(0, 1)$ (4) $(1, 3)$

34. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to

[JEE (Main)-2020]

(1) $\frac{9}{4}(9 - q^2)$ (2) $\frac{9}{4}(9 + p^2)$

(3) $\frac{9}{4}(9 + q^2)$ (4) $\frac{9}{4}(9 - p^2)$

35. The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval $(0, 1)$ is **[JEE (Main)-2020]**

(1) $(-3, -1)$ (2) $(2, 4]$

(3) $(0, 2)$ (4) $(1, 3]$

36. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to **[JEE (Main)-2020]**

(1) 18 (2) 9

(3) 27 (4) 36

37. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is **[JEE (Main)-2020]**

(1) $\frac{25}{9}$ (2) $\frac{25}{81}$

(3) $\frac{5}{9}$ (4) $\frac{5}{27}$

38. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to **[JEE (Main)-2020]**

(1) $\frac{1}{24}$ (2) $\frac{27}{32}$

(3) $\frac{3}{8}$ (4) $\frac{27}{16}$

39. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is **[JEE (Main)-2020]**

(1) 3 (2) 2

(3) 4 (4) 1

40. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to **[JEE (Main)-2020]**

(1) $2\alpha^2$

(2) $-2\alpha(\alpha + 1)$

(3) $2\alpha(\alpha - 1)$

(4) $2\alpha(\alpha + 1)$

41. The least positive value of 'a' for which the equation, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is ___________. **[JEE (Main)-2020]**

42. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is : **[JEE (Main)-2021]**

(1) 2 (2) 3

(3) 4 (4) 0

43. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is : **[JEE (Main)-2021]**

(1) 2 (2) 4

(3) 3 (4) 1

44. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and $p_{n+1} = 29$ for some integer $n \geq 1$. Then, the value of p_n^2 is ___________.

[JEE (Main)-2021]

45. The value of $4 + \cfrac{1}{5 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{4 + \ldots\infty}}}}$ is :

[JEE (Main)-2021]

(1) $2 + \frac{4}{\sqrt{5}}\sqrt{30}$ (2) $4 + \frac{4}{\sqrt{5}}\sqrt{30}$

(3) $2 + \frac{2}{5}\sqrt{30}$ (4) $5 + \frac{2}{5}\sqrt{30}$

46. The value of $3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots\infty}}}}$ is equal to

(1) $2 + \sqrt{3}$

(2) $3 + 2\sqrt{3}$

(3) $4 + \sqrt{3}$

(4) $1.5 + \sqrt{3}$

[JEE (Main)-2021]

47. The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ is

[JEE (Main)-2021]

(1) 1 (2) 2

(3) 6 (4) 4

48. If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2}\right)$ is equal to _______.

[JEE (Main)-2021]

49. The number of real solutions of the equation, $x^2 - |x| - 12 = 0$ is **[JEE (Main)-2021]**

(1) 4 (2) 2

(3) 1 (4) 3

50. If a + b + c = 1, ab + bc + ca = 2 and abc = 3, then the value of $a^4 + b^4 + c^4$ is equal to ____.

[JEE (Main)-2021]

51. Let α, β be two roots of the equation $x^2 + (20)^{\frac{1}{4}}x + (5)^{\frac{1}{2}} = 0$. Then $\alpha^8 + \beta^8$ is equal to

[JEE (Main)-2021]

(1) 160 (2) 10
(3) 50 (4) 100

52. Let $\alpha = \max\limits_{x \in \mathbb{R}} \{8^{2\sin 3x} \cdot 4^{4\cos 3x}\}$ and

$\beta = \min\limits_{x \in \mathbb{R}} \{8^{2\sin 3x} \cdot 4^{4\cos 3x}\}$.

If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{\frac{1}{5}}$ and $\beta^{\frac{1}{5}}$, then the value of c – b is equal to: **[JEE (Main)-2021]**

(1) 43 (2) 42
(3) 50 (4) 47

53. The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to ____.

[JEE (Main)-2021]

54. The sum of all integral values of $k (k \neq 0)$ for which the equation $\dfrac{2}{x-1} - \dfrac{1}{x-2} = \dfrac{2}{k}$ in x has no real roots, is______. **[JEE (Main)-2021]**

55. Let $\lambda \neq 0$ be in **R**. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of the equation $3x^2 - 10x + 27\lambda = 0$, then $\dfrac{\beta\gamma}{\lambda}$ is equal to ________. **[JEE (Main)-2021]**

56. The set of all values of k > –1, for which the equation $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$ has real roots, is **[JEE (Main)-2021]**

(1) $\left(\frac{1}{2}, \frac{3}{1}\right] - \{1\}$ (2) $\left[-\frac{1}{2}, 1\right)$

(3) [2, 3) (4) $\left(1, \frac{5}{2}\right]$

57. cosec18° is a root of the equation

[JEE (Main)-2021]

(1) $x^2 - 2x + 4 = 0$ (2) $x^2 + 2x - 4 = 0$
(3) $x^2 - 2x - 4 = 0$ (4) $4x^2 + 2x - 1 = 0$

58. The number of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of this equation, is :

[JEE (Main)-2021]

(1) 8 (2) 4
(3) 6 (4) 2

59. Let $f(x)$ be a polynomial of degree 3 such that $f(\text{k}) = -\dfrac{2}{\text{k}}$ for k = 2, 3, 4, 5. Then the value of $52 - 10f(10)$ is equal to ______.

[JEE (Main)-2021]

60. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\dfrac{\alpha^{12} + \beta^{12}}{(\alpha^{-12} + \beta^{-12}) \cdot (\alpha - \beta)^{24}}$ is equal to

[JEE (Main)-2021]

(1) $\dfrac{2^{12}}{(\sin\theta - 8)^6}$ (2) $\dfrac{2^{12}}{(\sin\theta - 4)^{12}}$

(3) $\dfrac{2^6}{(\sin\theta + 8)^{12}}$ (4) $\dfrac{2^{12}}{(\sin\theta + 8)^{12}}$

61. If for some $p, q, r \in \boldsymbol{R}$, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\dfrac{q^2 + r^2}{p^2}$ is equal to __________.

[JEE (Main)-2022]

62. If α, β are the roots of the equation

$$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right) + 3\left(3^{(\log_3 5)^{\frac{1}{2}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0,$$

then the equation, whose roots are $\alpha + \dfrac{1}{\beta}$ and $\beta + \dfrac{1}{\alpha}$, is **[JEE (Main)-2022]**

(1) $3x^2 - 20x - 12 = 0$ (2) $3x^2 - 10x - 4 = 0$
(3) $3x^2 - 10x + 2 = 0$ (4) $3x^2 - 20x + 16 = 0$

63. Let a, b be the roots of the equation $x^2 - \sqrt{2}x + \sqrt{6} = 0$ and $\dfrac{1}{\alpha^2} + 1, \dfrac{1}{\beta^2} + 1, \dfrac{1}{\beta^2} + 1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are **[JEE (Main)-2022]**

(1) non-real complex number
(2) real and both negative
(3) real and both positive
(4) real and exactly one of them is positive

64. Let $f(x) = ax^2 + bx + c$ be such that $f(1) = 3$, $f(-2) = \lambda$ and $f(3) = 4$. If $f(0) + f(1) + f(-2) + f(3) = 14$, then λ is equal to **[JEE (Main)-2022]**

(1) -4 (2) $\frac{13}{2}$

(3) $\frac{23}{2}$ (4) 4

65. Let a, $b(a > b)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n$, $n \in \mathbb{N}$, then $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to _______. **[JEE (Main)-2022]**

66. If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to :

(1) 18 (2) 24

(3) 36 (4) 96

[JEE (Main)-2022]

67. The sum of all the real roots of the equation $(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$ is

(1) $\log_e 3$ (2) $-\log_e 3$

(3) $\log_e 6$ (4) $-\log_e 6$

[JEE (Main)-2022]

68. Let $a, b \in R$ be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to

(1) 37 (2) 58

(3) 68 (4) 92

[JEE (Main)-2022]

69. The sum of the cubes of all the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$ is _______. **[JEE (Main)-2022]**

70. Let p and q be two real numbers such that $p + q = 3$ and $p^4 + q^4 = 369$. Then $\left(\frac{1}{p} + \frac{1}{q}\right)^{-2}$ is equal to _______.

[JEE (Main)-2022]

71. If the sum of all the roots of the equation $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ is $\log_e p$, then p is equal to _____.

[JEE (Main)-2022]

72. Let α, β be the roots of the equation $x^2 - 4\lambda x + 5 = 0$ and α, γ be the roots of the equation $x^2 - \left(3\sqrt{2} + 2\sqrt{3}\right)x + 7 + 3\lambda\sqrt{3} = 0$, $\lambda > 0$. If $\beta + \lambda = 3\sqrt{2}$, then $(\alpha + 2\beta + \gamma)^2$ is equal to _______.

[JEE (Main)-2022]

73. The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ is _____.

74. Let $f(x)$ be a quadratic polynomial such that $f(-2) + f(3) = 0$. If one of the roots of $f(x) = 0$ is -1, then the sum of the roots of $f(x) = 0$ is equal to:

(1) $\frac{11}{3}$ (2) $\frac{7}{3}$

(3) $\frac{13}{3}$ (4) $\frac{14}{3}$

[JEE (Main)-2022]

75. Let $f(x)$ and $g(x)$ be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$ and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is ___________ .

[JEE (Main)-2022]

76. Let $f(x)$ be a quadratic polynomial with leading coefficient 1 such that $f(0) = p$, $p \neq 0$, and $f(1) = \frac{1}{3}$. If the equations $f(x) = 0$ and $fofofo\ f(x) = 0$ have a common real root, then $f(-3)$ is equal to ______.

[JEE (Main)-2022]

77. The sum of all real value of x for which $\frac{3x^2 - 9x + 17}{x^2 + 3x + 10} = \frac{5x^2 - 7x + 19}{3x^2 + 5x + 12}$ is equal to ________. **[JEE (Main)-2022]**

78. The minimum value of the sum of the squares of the roots of $x^2 + (3 - a)x + 1 = 2a$ is **[JEE (Main)-2022]**

(1) 4 (2) 5

(3) 6 (4) 8

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Chapter 2

Quadratic Equations

1. Answer (2)

$bx^2 + cx + a = 0$

Roots are imaginary $c^2 - 4ab < 0$

$f(x) = 3b^2x^2 + 6bcx + 2c^2$

$D = 36b^2c^2 - 24b^2c^2 = 12b^2c^2$

$\because$ $3b^2 > 0$

$\therefore$ $f(x) \geq \left(-\frac{D}{4a}\right)$

$f(x) \geq -c^2$

Now $c^2 - 4ab < 0$

$c^2 < 4ab$

$-c^2 > -4ab$

$\therefore$ $f(x) > -4ab.$

2. Answer (3)

α and β are roots of the equation $x^2 - x + 1 = 0.$

$\Rightarrow$ $\alpha + \beta = 1$, $\alpha\beta = 1$

$\Rightarrow$ $x = \frac{1 \pm \sqrt{3}i}{2}, \frac{1+\sqrt{3}i}{2}, \frac{1-\sqrt{3}i}{2}$

$\Rightarrow$ $x = -\omega$ or ω^2

Thus, $\alpha = -\omega^2$, then $\beta = -\omega$

$\alpha = -\omega$, then $\beta = -\omega^2$ where $\omega^3 = 1$

$\alpha^{2009} + \beta^{2009} = (-\omega)^{2009} + (-\omega^2)^{2009}$

$= -[(\omega^3)^{669}.\omega^2 + (\omega^3)^{1337}.\omega]$

$= -[\omega^2 + \omega] = -(-1) = 1$

3. Answer (2)

$p(x) = 0 \Rightarrow (a - a_1)x^2 + (b - b_1)x + (c - c_1)$

Let $p(x) = \lambda_1 x^2 + \lambda_2 x + \lambda_3$

$p(-1) = 0 \Rightarrow \lambda_1 - \lambda_2 + \lambda_3 = 0$ ...(i)

$p'(-1) = 0 \Rightarrow -2\lambda_1 + \lambda_2 = 0$ ...(ii)

$p(-2) = 2 \Rightarrow 4\lambda_1 - 2\lambda_2 + \lambda_3 = 2$ ...(iii)

(ii) $\times$ 2 + (iii)

$\lambda_3 = 2$

$\lambda_1 = 2$

$\lambda_2 = 4$

$p(x) = 2x^2 + 4x + 2$

$p(2) = 2.2^2 + 4.2 + 2$

$= 8 + 8 + 2$

$= 18$

4. Answer (3)

Coeff. of $x = -7$

Constant term = 6

$\therefore$ The quadratic eqaution is $x^2 - 7x + 6 = 0$

$\Rightarrow$ $x = 1, 6$

5. Answer (4)

Let $f(x) = 2x^3 + 3x + k$

$f'(x) = 6x^2 + 3 > 0, \ \forall x \in R$

$\therefore$ $f(x)$ is strictly increasing function for all real values of k.

$\therefore$ No real k exists such that equation has two distinct roots in [0, 1].

6. Answer (1)

$\because$ The equation $x^2 + 2x + 3 = 0$ has complex roots and coefficients of both equations are real.

$\therefore$ Both roots are common.

$\therefore$ $\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$

7. Answer (3)

From equation,

$\alpha + \beta = 6$

$\alpha\beta = -2$

The value of $\frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} + \beta^{10} + \alpha\beta(\alpha^8 + \beta^8)}{2(\alpha^9 + \beta^9)}$

$= \frac{\alpha^9(\alpha + \beta) + \beta^9(\alpha + \beta)}{2(\alpha^9 + \beta^9)}$

$= \frac{\alpha + \beta}{2} = \frac{6}{2} = 3$

8. Answer (4)

$x^2 - 5x + 5 = 1$

$\Rightarrow x = 1, 4$

or $x^2 - 5x + 5 = -1$

$\Rightarrow x = 2, 3$

or $x^2 + 4x - 60 = 0$

$\Rightarrow x = -10, 6$

$\therefore$ $x = 3$ will be rejected as L.H.S. becomes -1

So, sum of value of x = 1 + 4 + 2 – 10 + 6 = 3

9. Answer (3)

Rearranging equation, we get

$$nx^2 + \{1+3+5+....+(2n-1)\}x$$

$$+\{1\cdot2+2\cdot3+...+(n-1)n\} = 10n$$

$$\Rightarrow nx^2 + n^2x + \frac{(n-1)n(n+1)}{3} = 10n$$

$$\Rightarrow x^2 + nx + \left(\frac{n^2-31}{3}\right) = 0$$

Given difference of roots = 1

$\Rightarrow$ $|\alpha - \beta| = 1$

$\Rightarrow$ $D = 1$

$$\Rightarrow \quad n^2 - \frac{4}{3}(n^2 - 31) = 1$$

So, $n = 11$

10. Answer (3)

$$2|\sqrt{x}-3| + \sqrt{x}(\sqrt{x}-6) + 6 = 0$$

$$2|\sqrt{x}-3| + (\sqrt{x}-3+3)(\sqrt{x}-3-3) + 6 = 0$$

$$2|\sqrt{x}-3| + (\sqrt{x}-3)^2 - 3 = 0$$

$$(\sqrt{x}-3)^2 + 2|\sqrt{x}-3| - 3 = 0$$

$$(|\sqrt{x}-3|+3)(|\sqrt{x}-3|-1) = 0$$

$$\Rightarrow \quad |\sqrt{x}-3| = 1, |\sqrt{x}-3|+3 \neq 0$$

$$\Rightarrow \quad \sqrt{x}-3 = \pm 1$$

$$\Rightarrow \quad \sqrt{x} = 4, 2$$

$x = 16, 4$

11. Answer (3)

Given quadratic equation is : $x^2 - mx + 4 = 0$

Both the roots are real and distinct.

$\therefore$ $m^2 - 4 \cdot 1 \cdot 4 > 0$

$\therefore$ $(m-4)(m+4) > 0$

$\therefore$ $m \in (-\infty, -4) \cup (4, \infty)$ ...(i)

$\because$ both roots lies in [1, 5]

$\therefore$ $-\frac{-m}{2} \in (1, 5)$

$\Rightarrow$ $m \in (2, 10)$ ...(ii)

and $1 \cdot (1 - m + 4) > 0$ $\Rightarrow$ $m < 5$

$\therefore$ $m \in (-\infty, 5)$ ...(iii)

and $1 \cdot (25 - 5m + 4) > 0$ $\Rightarrow m < \frac{29}{5}$

$\therefore$ $m \in \left(-\infty, \frac{29}{5}\right)$ ...(iv)

From (i), (ii), (iii) and (iv), $m \in (4, 5)$

12. Answer (4)

The roots of $6x^2 - 11x + \alpha = 0$ are rational numbers.

$\therefore$ Discriminant D must be perfect square number.

$D = (-11)^2 - 4 \cdot 6 \cdot \alpha$

$= 121 - 24\alpha$ must be a perfect square

$\therefore$ $\alpha = 3, 4, 5.$

$\therefore$ 3 positive integral values are possible.

13. Answer (1)

$f(0).f(3) > 0$ and $f(0).f(2) < 0$

$\Rightarrow$ $(c-4)(4c-49) > 0$ and $(c-4)(c-24) < 0$

$\Rightarrow$ $c \in (-\infty, 4) \cup \left(\frac{49}{4}, \infty\right)$ and $c \in (4, 24)$

$\Rightarrow$ $c \in \left(\frac{49}{4}, 24\right)$

$\therefore$ $S = \{13, 14, ..., 23\}$

14. Answer (1)

Sum of roots = $\alpha + \beta = \lambda - 3$

Product of roots = $\alpha\beta = 2 - \lambda$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (\lambda - 3)^2 - 2(2 - \lambda)$

$= \lambda^2 - 4\lambda + 5$

$= (\lambda - 2)^2 + 1$

$\lambda = 2$ for least $(\alpha^2 + \beta^2)$.

15. Answer (1)

$81x^2 + kx + 256 = 0 <^{\alpha}_{\beta}$

Given $(\alpha)^{\frac{1}{3}} = \beta$

$\alpha = \beta^3$

So $(\alpha)(\beta) = \frac{256}{81}$

$\Rightarrow \quad \beta^4 = \left(\frac{4}{3}\right)^4 \quad \Rightarrow \quad \beta = \frac{4}{3}$

Now $\quad \alpha = \frac{64}{27}$

Now $\alpha + \beta = -\frac{k}{81} \quad \Rightarrow \quad \frac{4}{3} + \frac{64}{27} = -\frac{k}{81}$

$k = -300$

16. Answer (2)

$x^2\sin\theta - x(\sin\theta \cdot \cos\theta + 1) + \cos\theta = 0.$

$x^2\sin\theta - x\sin\theta \cdot \cos\theta - x + \cos\theta = 0.$

$x\sin\theta(x - \cos\theta) - 1(x - \cos\theta) = 0.$

$(x - \cos\theta)(x\sin\theta - 1) = 0.$

$\therefore x = \cos\theta, \operatorname{cosec}\theta, \theta \in = (0, 45^\circ)$

$\therefore \alpha = \cos\theta, \beta = \operatorname{cosec}\theta$

$$\sum_{n=0}^{\infty} \alpha^n = 1 + \cos\theta + \cos^2\theta + \ldots\ldots \infty = \frac{1}{1 - \cos\theta}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\beta^n} = 1 - \frac{1}{\operatorname{cosec}\theta} + \frac{1}{\operatorname{cosec}^2\theta} - \frac{1}{\operatorname{cosec}^3\theta} + \ldots\ldots \infty$$

$= 1 - \sin\theta + \sin^2\theta - \sin^3\theta + \ldots\ldots \infty.$

$= \frac{1}{1 + \sin\theta}$

$$\therefore \sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$$

$$= \sum_{n=0}^{\infty} \alpha^n + \sum_{n=0}^{\infty} \frac{(-1)^n}{\beta^n}$$

$$= \frac{1}{1 - \cos\theta} + \frac{1}{1 + \sin\theta}.$$

17. Answer (2)

Let roots are α, β.

Given, $\lambda = \frac{\alpha}{\beta}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1$

$\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = 1$

As, $\alpha + \beta = \frac{m(4 - m)}{3m^2} = \frac{4 - m}{3m}$, $\alpha\beta = \frac{2}{3m^2}$

$$\frac{\left(\frac{4 - m}{3m}\right)^2}{\frac{2}{3m^2}} = 3$$

$\Rightarrow \quad (m - 4)^2 = 18$

$m = 4 \pm \sqrt{18}$

Least value is $4 - \sqrt{18} = 4 - 3\sqrt{2}$

18. Answer (4)

Given quadratic expression

$(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, is positive for all $x \in R$, then

$1 + 2m > 0 \qquad$ ...(i)

$D < 0$

$\Rightarrow \quad 4(1 + 3m)^2 - 4(1 + 2m)4(1 + m) < 0$

$\Rightarrow \quad 1 + 9m^2 + 6m - 4[1 + 2m^2 + 3m] < 0$

$\Rightarrow \quad m^2 - 6m - 3 < 0$

$m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$

$\therefore \quad m > -\frac{1}{2}$

So $m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$

So integral values of $m = \{0, 1, 2, 3, 4, 5, 6\}$

Number of integral values of $m = 7$

19. Answer (1)

$x^2 - 2x + 2 = 0$

Roots of this equation are $\frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$

Then $\frac{\alpha}{\beta} = \frac{1 + i}{1 - i} = \frac{(1 + i)^2}{1 - i^2} = i$

or $\frac{\alpha}{\beta}=\frac{1-i}{1+i}=\frac{(1-i)^2}{1-i^2}=-i$

So, $\frac{\alpha}{\beta}=\pm i$

Now, $\left(\frac{\alpha}{\beta}\right)^n=1 \Rightarrow (\pm i)^n=1$

$\Rightarrow$ n must be a multiple of 4

minimum value of n = 4

20. Answer (2)

Let $\sqrt{x}=t$

$|t-2|+t(t-4)+2=0$

$\Rightarrow |t-2|+t^2-4t+4-2=0$

$\Rightarrow |t-2|+(t-2)^2-2=0$

Let $|t-2|=z$ (Clearly $z\geq 0$)

$\Rightarrow z+z^2-2=0$

$\Rightarrow z=1$ or -2 (rejected)

$\Rightarrow |t-2|=1 \Rightarrow t=1, 3$

If $\sqrt{x}=1 \Rightarrow x=1$

If $\sqrt{x}=3 \Rightarrow x=9$

Sum of solutions = 10

21. Answer (3)

Since a, b, c are in G.P.

$\Rightarrow$ $b^2=ac$

Given, $ax^2+2bx+c=0$

$\Rightarrow$ $ax^2+2\sqrt{ac}\,x+c=0$

$\Rightarrow$ $(\sqrt{a}\,x+\sqrt{c})^2=0$

$\Rightarrow$ $x=-\sqrt{\frac{c}{a}}$

$\because$ $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have common root

$\Rightarrow$ $x=-\sqrt{\frac{c}{a}}$ must satisfy $dx^2+2ex+f=0$

$\Rightarrow$ $d\cdot\frac{c}{a}+2e\left(-\sqrt{\frac{c}{a}}\right)+f=0$

$\frac{d}{a}-\frac{2e}{\sqrt{ac}}+\frac{f}{c}=0$

$\Rightarrow$ $\frac{d}{a}-\frac{2e}{b}+\frac{f}{c}=0$

$\Rightarrow$ $\frac{2e}{b}=\frac{d}{a}+\frac{f}{c}$

$\Rightarrow$ $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in A.P.

22. Answer (1)

$(1+m^2)x^2-2(1+3m)x+(1+8m)=0$

equation has no real solution

$\Rightarrow D<0$

$4(1+3m)^2<4(1+m^2)(1+8m)$

$1+9m^2+6m<1+8m+m^2+8m^3$

$8m^3-8m^2+2m>0$

$2m(4m^2-4m+1)>0$

$2m(2m-1)^2>0$

$m>0,\ m\neq\frac{1}{2}$

$\Rightarrow$ number of integral values of m are infinitely many.

23. Answer (3)

p, q are rational numbers.

$\therefore$ $2+\sqrt{3}$ in the other root

Now, $p=-4$, $q=1$

$\Rightarrow$ $p^2-4q-12=16-4-12=0$

Note:- (Erratum) p, q, should be given as rational numbers instead of real numbers

24. Answer (4)

Sum of roots = $\frac{3}{m^2+1}$

For maximum $m=0$

Hence equation becomes $x^2-3x+1=0$

$\alpha+\beta=3,\quad \alpha\beta=1,\ |\alpha-\beta|=\sqrt{5}$

$|\alpha^3-\beta^3|=|(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta)|$

$=\sqrt{5}(9-1)$

$=8\sqrt{5}$

25. Answer (4)

$375x^2 - 25x - 2 = 0$

$$\alpha+\beta=\frac{25}{375},\ \alpha\beta=\frac{-2}{375}$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\left(\alpha^r+\beta^r\right)$$

$$=\left(\alpha+\alpha^2+\alpha^3+\ldots\infty\right)+\left(\beta+\beta^2+\beta^3+\ldots\infty\right)$$

$$=\frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta}$$

$$=\frac{\alpha+\beta-2\alpha\beta}{1-(\alpha+\beta)+\alpha\beta}$$

$$=\frac{\frac{25}{375}+\frac{4}{375}}{1-\frac{25}{375}-\frac{2}{375}}$$

$$=\frac{29}{375-25-2}$$

$$=\frac{29}{348}=\frac{1}{12}$$

26. Answer (3)

$\beta^2 = \alpha\gamma$ so roots of the equation $\alpha x^2 + 2\beta x + \gamma = 0$

are $\dfrac{-2\beta\pm 2\sqrt{\beta^2-\alpha\gamma}}{2\alpha}=-\dfrac{\beta}{\alpha}$

This root satisfy the equation $x^2 + x - 1 = 0$

$\beta^2 - \alpha\beta - \alpha^2 = 0$

$\Rightarrow$ $\alpha\gamma - \alpha\beta - \alpha^2 = 0$

$\Rightarrow$ $\alpha + \beta = \gamma$

Now, $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$

$= \alpha\beta + \beta^2$

$= (\alpha + \beta)\beta$

$= \beta\gamma$

27. Answer (1)

$\tan\alpha$ and $\tan\beta$ are roots of $(k + 1)x^2 - \sqrt{2}\lambda x - (1 - k) = 0$

$$\therefore\quad \tan\alpha+\tan\beta=\frac{\sqrt{2}\lambda}{k+1}$$

$$\tan\alpha\tan\beta=\frac{k-1}{k+1}$$

$$\text{Now }\tan(\alpha+\beta)=\frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\left(\frac{k-1}{k+1}\right)}=\frac{\sqrt{2}\lambda}{2}=\frac{\lambda}{\sqrt{2}}$$

$$\frac{\lambda^2}{2}=50$$

$\therefore$ $\lambda = 10$

28. Answer (4)

$\because$ α, β are roots of $x^2 - x - 1 = 0$ ...(i)

$\therefore$ $\alpha^2 - \alpha - 1 = 0$

$\Rightarrow$ $\alpha^{n+2} - \alpha^{n+1} - \alpha^n = 0$ ...(ii)

Similarly, $\beta^{n+2} - \beta^{n+1} - \beta^n = 0$ ...(iii)

From eq. (ii) + (iii), we get

$$\alpha^{n+2}+\beta^{n+2}=\left(\alpha^{n+1}+\beta^{n+1}\right)+\left(\alpha^n+\beta^n\right)$$

$\therefore$ $p_{n+2} = p_{n+1} + p_n$

For $n = 0$, $p_0 = \alpha^0 + \beta^0 = 2$

For $n = 1$, $p_1 = \alpha + \beta = 1$

and $p_2 = p_0 + p_1 = 2 + 1 = 3$

$p_3 = p_2 + p_1 = 3 + 1 = 4$

$p_4 = p_3 + p_2 = 4 + 3 = 7$

$p_5 = p_4 + p_3 = 7 + 4 = 11$

29. Answer (2)

$3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$

Case I: $0 < 3^x < 1 \Rightarrow -\infty < x < 0$

$\Rightarrow$ $(3^x)^2 - 3^x + 2 = 1 - 3^x + 2 - 3^x$

$\Rightarrow$ $(3^x)^2 + 3^x - 1 = 0 \Rightarrow 3^x=\dfrac{-1+\sqrt{5}}{2}<1$

$\Rightarrow$ One real solution

Case II: $1 < 3^x < 2 \Rightarrow 0 < x < \log_2 3$

$\Rightarrow$ $(3^x)^2 - 3^x + 2 = 3^x - 1 + 2 - 3^x$

$\Rightarrow$ $(3^x)^2 - 3^x + 1 = 0$

$\Rightarrow$ No solution $\quad\therefore$ Discriminant is negative

Case III: $2 < 3^x < \infty$

$\Rightarrow$ $(3^x)^2 - 3^x + 2 = 2.3^x - 3$

$\Rightarrow$ $(3^x)^2 - 3.(3^x) + 5 = 0$

$\Rightarrow$ No solution $\quad\therefore$ Discriminant is negative

30. Answer (4)

$\because$ $(e^{4x} - 2e^{2x} + 1) + (e^{3x} - 2e^{2x} + e^x) = 0$

$\Rightarrow$ $(e^{2x} - 1)^2 + e^x(e^x - 1)^2 = 0$

$$\Rightarrow\ (e^x-1)^2\ [\underbrace{(e^x+1)^2+e^x}_{\text{Always positive terms}}]=0$$

Hence $e^x - 1 = 0$

$\Rightarrow$ $x = 0$ is the only solution

31. Answer (1)

The given equations are

$ax^2 - 2bx + 5 = 0$ and $x^2 - 2bx - 10 = 0$

$$A/Q \quad \left.\begin{matrix} 2\alpha = \dfrac{2b}{a} \\ \alpha^2 = \dfrac{5}{a} \end{matrix}\right] \quad \text{and} \quad \begin{matrix} \alpha + \beta = 2b \\ \alpha\beta = -10 \end{matrix}$$

and $4b^2 = 20a \Rightarrow b^2 = 5a$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4b^2 + 20$

As 'α' is a root of $x^2 - 2bx - 10 = 0$

$\therefore \ \alpha^2 - 2b\alpha = 10$

$\Rightarrow \ \dfrac{5}{a} - 2b \cdot \dfrac{b}{a} = 10$

$\Rightarrow \ 5 - 2b^2 = 10a$

$\Rightarrow \ 5 - 10a = 10a$

$\Rightarrow \ a = \dfrac{1}{4}$

Now, $\alpha^2 + \beta^2 = 2(5 - 10a) + 20$

$= 30 - 20a$

$= 25$

32. Answer (1)

$\because$ α is a root of given equation, then

$5\alpha^2 + 6\alpha = 2$

$\Rightarrow \ 5\alpha^6 + 6\alpha^5 = 2\alpha^4$ ...(1)

Similarly $5\beta^6 + 6\beta^5 = 2\beta^4$ ...(2)

Adding (1) and (2), we get

$\boxed{5S_6 + 6S_5 = 2S_4}$

33. Answer (1)

Let $f(x) = ax^2 + bx + c$

Let roots are 3 and α

and $f(-1) + f(2) = 0$

$4a + 2b + c + a - b + c = 0$

$5a + b + 2c = 0$ ...(i)

$\because \ f(3) = 0 \Rightarrow 9a + 3b + c = 0$ ...(ii)

From equation (i) and (ii)

$\dfrac{a}{1-6} = \dfrac{b}{18-5} = \dfrac{c}{15-9} \Rightarrow \dfrac{a}{-5} = \dfrac{b}{13} = \dfrac{c}{6}$

$\therefore \ f(x) = k(-5x^2 + 13x + 6)$

$= -k(5x + 2)(x - 3)$

$\therefore$ Roots are 3 and $-\dfrac{2}{5}$

$\therefore \ -\dfrac{2}{5}$ lies in interval $(-1, 0)$

34. Answer (4)

$\alpha \cdot \beta = 2$ and $\alpha + \beta = -p$ also $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = -q$

$\Rightarrow \ p = 2q$

Now $\left(\alpha - \dfrac{1}{\alpha}\right)\left(\beta - \dfrac{1}{\beta}\right)\left(\alpha + \dfrac{1}{\beta}\right)\left(\beta + \dfrac{1}{\alpha}\right)$

$= \left[\alpha\beta + \dfrac{1}{\alpha\beta} - \dfrac{\alpha}{\beta} - \dfrac{\beta}{\alpha}\right]\left[\alpha\beta + \dfrac{1}{\alpha\beta} + 1 + 1\right]$

$= \dfrac{9}{2}\left[\dfrac{5}{2} - \dfrac{\alpha^2 + \beta^2}{2}\right] = \dfrac{9}{4}\left[5 - \left(p^2 - 4\right)\right]$

$= \dfrac{9}{4}\left(9 - p^2\right)$

35. Answer (4)

$\because$ Equation is : $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$

$\because$ One root in interval $(0, 1)$

$\therefore \ f(0) \cdot f(1) < 0$

$2 \cdot (\lambda^2 + 1 - 4\lambda + 2) < 0$

$(\lambda - 3)(\lambda - 1) < 0$

$\therefore \ \lambda \in (1, 3)$

If $\lambda = 3$, then roots are 1 and $\dfrac{1}{5}$

$\therefore \ \lambda \in (1, 3]$

36. Answer (1)

Roots of $x^2 - x + 2\lambda = 0$ are α and β

and roots of $3x^2 - 10x + 27\lambda = 0$ are α and γ

Here,

$3\alpha^2 - 10\alpha + 27\lambda = 0$ ...(i)

$3\alpha^2 - 3\alpha + 6\lambda = 0$ ...(ii)

$\therefore$ $\alpha = 3\lambda$

Now,

$3\lambda + \beta = 1$ and $3\lambda \cdot \beta = 2\lambda$

and, $3\lambda + \gamma = \frac{10}{3}$ and $3\lambda \cdot \gamma = 9\lambda$

$\therefore$ $\gamma = 3, \alpha = \frac{1}{3}$ and $\beta = \frac{2}{3}, \lambda = \frac{1}{9}$

$\frac{\beta\gamma}{\lambda} = 18$

37. Answer (2)

Let $|x| = t$ we have

$9t^2 - 18t + 5 = 0$

$9t^2 - 15t - 3t + 5 = 0$

$(3t - 1)(3t - 5) = 0$

$\Rightarrow$ $t = \frac{1}{3}$ or $\frac{5}{3}$ $\Rightarrow |x| = \frac{1}{3}$ or $\frac{5}{3}$

Roots are $\pm\frac{1}{3}$ and $\pm\frac{5}{3}$

Product = $\frac{25}{81}$

38. Answer (4)

$7x^2 - 3x - 2 = 0 \Rightarrow \alpha + \beta = \frac{3}{7}, \alpha\beta = \frac{-2}{7}$

Now $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$

$$= \frac{\alpha - \alpha\beta(\alpha+\beta) + \beta}{1-(\alpha^2+\beta^2)+(\alpha\beta)^2} = \frac{(\alpha+\beta) - \alpha\beta(\alpha+\beta)}{1-(\alpha+\beta)^2 + 2\alpha\beta + (\alpha\beta)^2}$$

$$= \frac{\frac{3}{7} + \frac{2}{7} \times \frac{3}{7}}{1 - \frac{9}{49} + 2 \times \frac{-2}{7} + \frac{4}{49}} = \frac{21+6}{49-9-28+4} = \frac{27}{16}$$

39. Answer (2)

$$\frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}} = \frac{\alpha+\beta}{(\alpha\beta)^{5/8}}$$

For $x^2 - 64x + 256 = 0$

$\alpha + \beta = 64$

$\alpha\beta = 256$

$$\therefore \quad \frac{\alpha+\beta}{(\alpha\beta)^{5/8}} = \frac{64}{(2^8)^{5/8}} = \frac{64}{32} = 2$$

40. Answer (2)

$\alpha + \beta = -\frac{1}{2} \Rightarrow -1 = 2\alpha + 2\beta$

and $4\alpha^2 + 2\alpha - 1 = 0$

$\Rightarrow$ $4\alpha^2 + 2\alpha + 2\alpha + 2\beta = 0$

$\Rightarrow$ $\beta = -2\alpha(\alpha + 1)$

41. Answer (08.00)

Real roots $D \geq 0$

$\therefore$ $(a-10)^2 - 4(2)\left(\frac{33}{2} - 2a\right) \geq 0$

$a^2 - 20a - 32 + 16a \geq 0$

$\Rightarrow$ $a^2 - 4a - 32 \geq 0$

$\Rightarrow$ $a^2 - 8a + 4a - 32 \geq 0$

$\Rightarrow$ $(a - 8)(a + 4) \geq 0$

$a \in (-\infty, -4] \cup [8, \infty)$

Minimum positive integral value is 8

42. Answer (2)

$x^2 - 2(3x - 1)x + 8k^2 - 7 > 0, \ \forall x \in R$

Here D < 0

$4(3k-1)^2 - 4 \cdot 1 \cdot (8k^2 - 7) < 0$

$9k^2 - 6k + 1 - 8k^2 + 7 < 0$

$k^2 - 6k + 8 < 0$

$(k - 2)(k - 4) < 0$

$k \in (2, 4)$

43. Answer (1)

α, β are roots of $x^2 - 6x - 2 = 0$

$\therefore$ $\alpha^2 - 6\alpha - 2 = 0$

$\Rightarrow$ $\alpha^2 - 2 = 6\alpha$

Similarly $\beta^2 - 2 = 6\beta$

$$\frac{a_{10} - 2a_8}{3a_9} = \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{(\alpha^{10} - 2\alpha^8) - (\beta^{10} - 2\beta^8)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{3(\alpha^9 - \beta^9)} = \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{6(\alpha^9 - \beta^9)}{3(\alpha^9 - \beta^9)} = 2$$

44. Answer (324)

$\because$ $\alpha + \beta = 1$ and $\alpha\beta = -1$

$\therefore$ Equation $x^2 - x = 0$ has two roots α and β.

$\therefore$ $\alpha^2 - \alpha = 1$ and $\beta^2 - \beta = 1$

$\Rightarrow$ $\alpha^{n+1} - \alpha^n = \alpha^{n-1}$ and $\beta^{n+n} - \beta^n = \beta^{n-1}$

$\Rightarrow$ $\alpha^{n+1} + \beta^{n+1} - \alpha^n - \beta^n = \alpha^{n-1} + \beta^{n-1}$

$\Rightarrow$ $P_{n+1} - P_n = P_{n-1}$

$\Rightarrow$ $P_n = 29 - 11$

$\Rightarrow$ $(P_n)^2 = 18^2 = 324$

45. Answer (3)

$$\text{Let } k = 4 + \cfrac{1}{5 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{4 + ...\infty}}}}$$

$$\Rightarrow k = 4 + \cfrac{1}{5 + \cfrac{1}{k}}$$

$\Rightarrow$ $5k^2 - 20k - 4 = 0$

$\Rightarrow$ $k = 2 + \frac{2\sqrt{30}}{5}$ (taking positive value)

46. Answer (4)

$$\text{Let } y = 3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + ...}}}$$

$$\Rightarrow y = 3 + \cfrac{1}{4 + \cfrac{1}{y}}$$

$\Rightarrow$ $(y - 3)(4y + 1) = y$

$\Rightarrow$ $4y^2 - 11y - 3 = y$

$\Rightarrow$ $4y^2 - 12y - 3 = 0$

$$4\left(y - \frac{3}{2}\right)^2 = 12$$

$$\Rightarrow y = \sqrt{3} + \frac{3}{2}$$

47. Answer (2)

Let $f(x) = e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1$

if $e^x = t$ here t must be positive

$f(x) = t^6 - t^4 - 2t^3 - 12t^2 + t + 1$

Using Descartes rule atmost 2 values of t can be positive.

So $f(x) = 0$ can have atmost 2 roots.

$\because$ $f(0) = -12$ and $\lim\limits_{x\to\infty} f(x) = \infty$, $\lim\limits_{x\to-\infty} f(x) = 1$

hence $f(x) = 0$ must have only 2 roots.

48. Answer (1)

$\because$ $P_n + 5\sqrt{2}\, P_{n-1} = -10P_{n-2}$

$$\frac{P_{17}\left(P_{20} + 5\sqrt{2}\, P_{19}\right)}{P_{18}\left(P_{19} + 5\sqrt{2}\, P_{18}\right)} = \frac{P_{17}\cdot\left(-10\, P_{18}\right)}{P_{18}\cdot\left(-10\, P_{17}\right)} = 1$$

49. Answer (2)

$x^2 - |x| - 12 = 0$

$x^2 - 4|x| + 3|x| - 12 = 0$

$(|x| - 4)(|x| + 3) = 0$

$|x| = 4$ or -3 (rejected)

$x = \pm 4$ 2 solutions

50. Answer (13)

$\because$ $a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$

$= 1 - 4 \ -3$

and $a^2b^2 + b^2c^2 + c^2a^2 = (ab + bc + ca)^2$

$- 2abc(a + b + c) = 4 - 6 = -2$

So $a^4 + b^4 + c^4 = (a^2 + b^2 + c^2)$

$- 2(a^2b^2 + b^2c^2 + c^2a^2)$

$= 9 + 4 = 13$

51. Answer (3)

$$x^2 + (20)^{\frac{1}{4}} x + (5)^{\frac{1}{2}} = 0$$

$$\therefore \quad \alpha + \beta = -(20)^{\frac{1}{4}}, \alpha\cdot\beta = (5)^{\frac{1}{2}}$$

$\alpha^8 + \beta^8 = (\alpha^4 + \beta^4)^2 - 2\alpha^4\beta^4$

$$= \left\{\left(\alpha^2 + \beta^2\right)^2 - 2\alpha^2\beta^2\right\}^2 - 2\alpha^4\beta^4$$

$$= \left[\left\{\left(\alpha + \beta\right)^2 - 2\alpha\beta\right\}^2 - 2\alpha^2\beta^2\right]^2 - 2\alpha^4\beta^4$$

$$= \left[\left\{20^{\frac{1}{2}} - 2.5^{\frac{1}{2}}\right\}^2 - 2.5\right]^2 - 2.5^2$$

$= (0 - 10)^2 - 50$

$= 50$

52. Answer (2)

$\alpha = \max\{2^{6\sin 3x + 8\cos 3x}\} = 2^{10}$

$\beta = \min\{2^{6\sin 3x + 8\cos 3x}\} = 2^{-10}$

$\alpha^{\frac{1}{5}} = 4$ and $\beta^{\frac{1}{5}} = \frac{1}{4}$

Sum of roots $= \frac{17}{4}$ & Product of roots = 1

$\frac{-b}{8} = \frac{17}{4} \Rightarrow b = -34$ & $\frac{c}{8} = 1 \Rightarrow c = 8$

c – b = 8 + 34 = 42

53. Answer (2)

Let $e^x = t$, (t > 0)

$t^4 - t^3 - 4t^2 - t = 1 = 0$

$\left(t^2 + \frac{1}{t^2}\right) - \left(t^3 + t\right) - 4 = 0$

$\left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 6 = 0$

Let $t + \frac{1}{t} = u \quad (u > 2)$

$u^2 - u - 6 = 0$

$(u - 3)(u + 2) = 0$

u = 3, –2 (rejected)

u = 3

$t + \frac{1}{t} = 3 \qquad \Rightarrow t^2 - 3t + 1 = 0$

$t = \frac{3 \pm \sqrt{5}}{2} = e^x$

$x = \ln\frac{3+\sqrt{5}}{2},\ \ln\frac{3-\sqrt{5}}{2}$

54. Answer (66)

$\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$

$\Rightarrow \frac{2x - 4 - x + 1}{(x-1)(x-2)} = \frac{2}{k}$

$\Rightarrow 2x^2 - 6x + 4 = k(x - 3)$

$\Rightarrow 2x^2 - x(6 + k) + (4 + 3k) = 0$

This equation has no solution then

$(6 + k)^2 < 4 \cdot 2(4 + 3k)$

$\Rightarrow k^2 - 12k + 4 < 0$

$\Rightarrow k \in \left(6 - 4\sqrt{2},\ 6 + 4\sqrt{2}\right)$

$\Rightarrow k = 1, 2, 3, \ldots\ldots, 11$

Sum of all values of k

$11\left(\frac{11+1}{2}\right) = 66$

55. Answer (18)

$x^2 - x + 2\lambda = 0 \begin{cases}\alpha \\ \beta\end{cases} \Rightarrow \alpha \cdot \beta = 2\lambda$

$3x^2 - 10x + 27\lambda = 0 \begin{cases}\alpha \\ \gamma\end{cases} \Rightarrow \alpha \cdot \gamma = \frac{27}{3} = 9\lambda$

Both equations have a common root α.

$\frac{\alpha^2}{-27\lambda + 20\lambda} = \frac{\alpha}{6\lambda - 27\lambda} = \frac{1}{-10+3}$

$\frac{\alpha^2}{-7\lambda} = \frac{\alpha}{-19\lambda} = \frac{1}{-7}$

$\alpha^2 = \lambda$

Now, $(\alpha\beta) \cdot (\alpha \cdot \gamma) = (2\lambda)(9\lambda)$

$\frac{\beta \cdot \gamma}{\lambda} = 2 \times 9 \cdot \frac{\lambda}{\alpha^2} = 18$

56. Answer (4)

$3x^2 + 4x + 2 > 0 \quad \forall x \in R \quad (\because D < 0)$

$(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$

$\Rightarrow \left(\frac{3x^2 + 4x + 3}{3x^2 + 4x + 2}\right)^2 - (k+1)\left(\frac{3x^2 + 4x + 3}{3x^2 + 4x + 2}\right) + k = 0 \quad \ldots(i)$

Let $\frac{3x^2 + 4x + 3}{3x^2 + 4x + 2} = t$

$t = \frac{3x^2 + 4x + 2 + 1}{3x^2 + 4x + 2} = 1 + \frac{1}{3x^2 + 4x + 2}$

$3x^2 + 4x + 2 \in \left[\frac{2}{3}, \infty\right)$

$\frac{1}{3x^2 + 4x + 2} \in \left(0, \frac{3}{2}\right]$

$t = 1 + \frac{1}{3x^2 + 4x + 2} \in \left(1, \frac{5}{2}\right]$

$\Rightarrow t^2 - (k + 1)t + k = 0$ where $t \in \left(1, \frac{5}{2}\right] \quad \ldots(ii)$

(ii) should have at least one root in $\left(1, \frac{5}{2}\right]$

$(t - 1)(t - k) = 0$

$t = 1,\ t = k$

$\therefore \quad k \in \left(1, \frac{5}{2}\right]$

57. Answer (3)

We know that $\operatorname{cosec}18^\circ = \frac{4}{\sqrt{5}-1}$

As equation is with real coefficients other root will be $\frac{4(-\sqrt{5}+1)}{4} = -\sqrt{5}+1$

$\therefore$ Sum of root $\sqrt{5}+1-\sqrt{5}+1=2$

Product of roots = 1 – 5 = –4

$\therefore$ Equation is $x^2 - 2x - 4 = 0$

58. Answer (3)

Let α, β are the roots of a quadratic, then

$\alpha = \beta^2 - 2$ and $\beta = \alpha^2 - 2$

$\Rightarrow (\alpha^2 - 2)^2 - 2 = \alpha \Rightarrow \alpha^4 - 4\alpha^2 - \alpha + 2 = 0$

$\Rightarrow (\alpha + 1)(\alpha - 2)(\alpha^2 + \alpha - 1) = 0$

$\Rightarrow (\alpha, \beta) = (-1, -1), (-1, 1), (2, 2), (2, -2), (-1, 2)$

and $\left(\frac{\sqrt{5}-1}{2}, -\frac{\sqrt{5}+1}{2}\right)$

Hence there will be 6 possible values of (a, b).

59. Answer (26)

Let P(k) = kf(k) + 2

So kf(k) + 2 = a$(x-2)(x-3)(x-4)(x-5)$

If k = 0,

2 = a(–2)(–3)(–4)(–5)

$a = \frac{1}{60}$

kf(k) + 2 $= \frac{1}{60}(x-2)(x-3)(x-4)(x-5)$

Putting k = 10

$10f(10) + 2 = \frac{1}{60}\cdot 8\cdot 7\cdot 6\cdot 5$

= 28

$10f(10) = 26$

$52 - 10f(10) = 26$

60. Answer (4)

Given $\alpha + \beta = -\sin\theta$ and $\alpha\beta = -2\sin\theta$

$$\frac{(\alpha^{12}+\beta^{12})\alpha^{12}\beta^{12}}{(\alpha^{12}+\beta^{12})(\alpha-\beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$$

$$|\alpha-\beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{\sin^2\theta + 8\sin\theta}$$

Hence required quantity

$$\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}} = \frac{(2\sin\theta)^{12}}{\sin^{12}\theta(\sin\theta+8)^{12}} = \frac{2^{12}}{(\sin\theta+8)^{12}}$$

61. Answer (272)

Let roots of $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2$ $= 0 <^{\alpha}_{\beta}$

$\therefore$ $\alpha + \beta > 0$ and $\alpha\beta > 0$

Also, it has a common root with $x^2 + 2x - 8 = 0$

$\therefore$ The common root between above two equations is 4.

$\Rightarrow 16(p^2 + q^2) - 8q(p + r) + q^2 + r^2 = 0$

$\Rightarrow (16p^2 - 8pq + q^2) + (16q^2 - 8qr + r^2) = 0$

$\Rightarrow (4p - q)^2 + (4q - r)^2 = 0$

$\Rightarrow q = 4p$ and $r = 16p$

$\therefore \frac{q^2+r^2}{p^2} = \frac{16p^2+256p^2}{p^2} = 272$

62. Answer (2)

$$3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}} = 3^{\sqrt{\log_3 5}} - \left(3^{\log_3 5}\right)^{\sqrt{\log_5 3}}$$

= 0

$$3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} = 5^{(\log_5 3)^{\frac{2}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}}$$

= 0

Note : In the given equation 'x' is missing.

So $x^2 - 5x + 3(-1) = 0 <^{\alpha}_{\beta}$

$$\alpha+\beta+\frac{1}{\alpha}+\frac{1}{\beta} = (\alpha+\beta) + \frac{\alpha+\beta}{\alpha\beta}$$

$$= 5 - \frac{5}{3} = \frac{10}{3}$$

$$\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = 2 + \alpha\beta + \frac{1}{\alpha\beta} = 2 - 3 - \frac{1}{3}$$

$$= \frac{-4}{3}$$

So Equation must be option (2)

63. Answer (2)

$$\alpha + \beta = \sqrt{2},\ \alpha\beta = \sqrt{6}$$

$$\frac{1}{\alpha^2} + 1 + \frac{1}{\beta^2} + 1 = 2 + \frac{\alpha^2 + \beta^2}{6}$$

$$= 2 + \frac{2 - 2\sqrt{6}}{6} = -a$$

$$\left(\frac{1}{\alpha^2} + 1\right)\left(\frac{1}{\beta^2} + 1\right) = 1 + \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\alpha^2\beta^2}$$

$$= \frac{7}{6} + \frac{2 - 2\sqrt{6}}{6} = b$$

$$\Rightarrow \quad a + b = \frac{-5}{6}$$

So, equation is $x^2 + \frac{17x}{6} + \frac{7}{6} = 0$

OR $6x^2 + 17x + 7 = 0$

Both roots of equation are –ve and distinct

64. Answer (4)

$f(1) = a + b + c = 3$ ...(i)

$f(3) = 9a + 3b + c = 4$ ...(ii)

$f(0) + f(1) + f(-2) + f(3) = 14$

OR $c + 3 + (4a - 2b + c) + 4 = 14$

OR $4a - 2b + 2c = 7$ ...(iii)

From (i) and (ii) $8a + 2b = 1$ ...(iv)

From (iii) – (2) × (i)

$\Rightarrow$ $2a - 4b = 1$ ...(v)

From (iv) and (v) $a = \frac{1}{6},\ b = \frac{-1}{6}$ and $c = 3$

$f(-2) = 4a - 2b + c$

$$= \frac{4}{6} + \frac{2}{6} + 3 = 4$$

65. Answer (16)

$x^2 - x - 4 = 0 \begin{smallmatrix} \nearrow \alpha \\ \searrow \beta \end{smallmatrix}$ and $P_n = \alpha^n - \beta^n$

$$\therefore\ I = \frac{(P_{15} - P_{14})P_{16} - P_{15}(P_{15} - P_{14})}{P_{13}\,P_{14}} = \frac{(P_{16} - P_{15})(P_{15} - P_{14})}{P_{13}\,P_{14}}$$

$$\Rightarrow \quad I = \frac{(\alpha^{16} - \beta^{16} - \alpha^{15} + \beta^{15})(\alpha^{15} - \beta^{15} - \alpha^{14} + \beta^{14})}{(\alpha^{13} - \beta^{13})(\alpha^{14} - \beta^{14})}$$

$$\Rightarrow \quad I = \frac{(\alpha^{15}(\alpha - 1) - \beta^{15}(\beta - 1))(\alpha^{14}(\alpha - 1) - \beta^{14}(\beta - 1))}{(\alpha^{13} - \beta^{13})(\alpha^{14} - \beta^{14})}$$

As $\alpha^2 - \alpha = 4 \quad \Rightarrow \alpha - 1 = \frac{4}{\alpha}$ and $\beta - 1 = \frac{4}{\beta}$

$$\Rightarrow \quad I = \frac{\left(\alpha^{15}.\frac{4}{\alpha} - \beta^{15}.\frac{4}{\beta}\right)\left(\alpha^{14}.\frac{4}{\alpha} - \beta^{14}.\frac{4}{\beta}\right)}{(\alpha^{13} - \beta^{13})(\alpha^{14} - \beta^{14})}$$

$$= \frac{16(\alpha^{14} - \beta^{14})(\alpha^{13} - \beta^{13})}{(\alpha^{14} - \beta^{14})(\alpha^{13} - \beta^{13})} = 16$$

66. Answer (2)

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15 \Rightarrow \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2\beta^2} = 15$$

$$\Rightarrow \quad \frac{\frac{\lambda^2}{9} + \frac{2}{3}}{\frac{1}{9}} = 15$$

$$\Rightarrow \quad \frac{\lambda^2}{9} = 1 \Rightarrow \lambda^2 = 9$$

$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$$= \left(\frac{-\lambda}{3}\right)\left(\frac{\lambda^2}{9} - 3\left(\frac{-1}{3}\right)\right) = \left(\frac{-\lambda}{3}\right)\left(\frac{\lambda^2}{9} + 1\right) = \frac{-2\lambda}{3}$$

$$6(\alpha^3 + \beta^3)^2 = 6 \cdot \frac{4\lambda^2}{9} = 24$$

67. Answer (2)

Given equation : $(e^{2x} - 4)(6e^{2x} - 5e^{x} + 1) = 0$

$\Rightarrow$ $e^{2x} - 4 = 0$ or $6e^{2x} - 5e^{x} + 1 = 0$

$\Rightarrow$ $e^{2x} = 4$ or $6(e^{x})^2 - 3e^{x} - 2e^{x} + 1 = 0$

$\Rightarrow$ $2x = \ln 4$ or $(3e^{x} - 1)(2e^{x} - 1) = 0$

$\Rightarrow$ $\boxed{x = \ln 2}$ or $e^{x} = \frac{1}{3}$ or $e^{x} = \frac{1}{2}$

or $x = \ln\left(\frac{1}{3}\right)$, $-\ln 2$

Sum of all real roots = $\ln 2 - \ln 3 - \ln 2$

$= -\ln 3$

68. Answer (2)

$ax^2 - 2bx + 15 = 0$ has repeated root so $b^2 = 15a$

and $\alpha = \frac{15}{b}$

$\because$ α is a root of $x^2 - 2bx + 21 = 0$

So $\frac{225}{b^2} = 9 \Rightarrow b^2 = 25$

Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4b^2 - 42 = 100 - 42$

$= 58$

69. Answer (36)

$x^4 - 3x^3 - x^2 - x^2 + 3x + 1 = 0$

$(x^2 - 1)(x^2 - 3x - 1) = 0$

Let the root of $x^2 - 3x - 1 = 0$ be α and β and other two roots of given equation are 1 and –1

So sum of cubes of roots $= 1^3 + (-1)^3 + \alpha^3 + \beta^3$

$= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= (3)^3 - 3(-1)(3)$

$= 36$

70. Answer (4)

$\because$ $p + q = 3$ ...(i)

and $p^4 + q^4 = 369$ ...(ii)

$\{(p + q)^2 - 2pq\}^2 - 2p^2q^2 = 369$

or $(9 - 2pq)^2 - 2(pq)^2 = 369$

or $(pq)^2 - 18pq - 144 = 0$

$\therefore$ $pq = -6$ or 24

But $pq = 24$ is not possible

$\therefore$ $pq = -6$

Hence, $\left(\frac{1}{p} + \frac{1}{q}\right)^{-2} = \left(\frac{pq}{p+q}\right)^2 = (-2)^2 = 4$

71. Answer (45)

Let $e^x = t$ then equation reduces to

$t^2 - 11t - \frac{45}{t} + \frac{81}{2} = 0$

$\Rightarrow 2t^3 - 22t^2 + 81t - 45 = 0$ ...(i)

if roots of $e^{2x} - 11e^{x} - 45e^{-x} + \frac{81}{2} = 0$ are α, β, γ then roots of (i) will be $e^{\alpha_1}e^{\alpha_2}e^{\alpha_3}$ using product of roots

$e^{\alpha_1 + \alpha_2 + \alpha_3} = 45$

$\Rightarrow \alpha_1 + \alpha_2 + \alpha_3 = \ln 45 \Rightarrow p = 45$

72. Answer (98)

$\because$ α, β are roots of $x^2 - 4\lambda x + 5 = 0$

$\therefore$ $\alpha + \beta = 4\lambda$ and $\alpha\beta = 5$

Also, α, γ are roots of

$x^2 - \left(3\sqrt{2} + 2\sqrt{3}\right)x + 7 + 3\sqrt{3}\lambda = 0, \lambda > 0$

$\therefore$ $\alpha + \gamma = 3\sqrt{2} + 2\sqrt{3}$, $\alpha\gamma = 7 + 3\sqrt{3}\lambda$

$\because$ α is common root

$\therefore$ $\alpha^2 - 4\lambda\alpha + 5 = 0$ ...(i)

and $\alpha^2 - (3\sqrt{2} + 2\sqrt{3})\alpha + 7 + 3\sqrt{3}\lambda = 0$ ...(ii)

From (i) – (ii): we get $\alpha = \frac{2 + 3\sqrt{3}\lambda}{3\sqrt{2} + 2\sqrt{3} - 4\lambda}$

$\because$ $\beta + \gamma = 3\sqrt{2}$

$\therefore$ $4\lambda + 3\sqrt{2} + 2\sqrt{3} - 2\alpha = 3\sqrt{2}$

$\Rightarrow$ $3\sqrt{2} = 4\lambda + 3\sqrt{2} + 2\sqrt{3} - \frac{4 + 6\sqrt{3}\lambda}{3\sqrt{2} + 2\sqrt{3} - 4\lambda}$

$\Rightarrow$ $8\lambda^2 + 3(\sqrt{3} - 2\sqrt{2})\lambda - 4 - 3\sqrt{6} = 0$

$\therefore \quad \lambda = \dfrac{6\sqrt{2}-3\sqrt{3} \pm \sqrt{9(11-4\sqrt{6})+32(4+3\sqrt{6})}}{16}$

$\therefore \quad \lambda = \sqrt{2}$

$\therefore \quad (\alpha + 2\beta + \gamma)^2 = (\alpha + \beta + \beta + \gamma)^2$

$= \left(4\sqrt{2}+3\sqrt{2}\right)^2$

$= \left(7\sqrt{2}\right)^2$

$= 98$

73. Answer (2)

Dividing by e^{2x}

$e^{2x} + 4e^{x} - 58 + 4e^{-x} + e^{-2x} = 0$

$\Rightarrow \quad (e^{x} + e^{-x})^2 + 4(e^{x} + e^{-x}) - 60 = 0$

Let $e^{x} + e^{-x} = t \in [2, \infty)$

$\Rightarrow \quad t^2 + 4t - 60 = 0$

$\Rightarrow \quad t = 6$ is only possible solution

$e^{x} + e^{-x} = 6 \Rightarrow e^{2x} - 6e^{x} + 1 = 0$

Let $e^{x} = p$,

$p^2 - 6p + 1 = 0$

$\Rightarrow \quad p = \dfrac{3+\sqrt{5}}{2}$ OR $\dfrac{3-\sqrt{5}}{2}$

So $x = \ln\left(\dfrac{3+\sqrt{5}}{2}\right)$ OR $\ln\left(\dfrac{3-\sqrt{5}}{2}\right)$

74. Answer (1)

$\because \quad x = -1$ be the roots of $f(x) = 0$

$\therefore \quad$ let $f(x) = A(x + 1)(x - b) \quad \ldots$(i)

Now, $f(-2) + f(3) = 0$

$\Rightarrow \quad A[-1(-2 - b) + 4(3 - b)] = 0$

$b = \dfrac{14}{3}$

$\therefore \quad$ Second root of $f(x) = 0$ will be $\dfrac{14}{3}$

$\therefore \quad$ Sum of roots $= \dfrac{14}{3} - 1 = \dfrac{11}{3}$

75. Answer (18)

$f(g(x)) = 8x^2 - 2x$

$g(f(x)) = 4x^2 + 6x + 1$

let $\quad f(x) = cx^2 + dx + e$

$g(x) = ax + b$

$f(g(x)) = c(ax + b)^2 + d(ax + b) + e \equiv 8x^2 - 2x$

$g(f(x)) = a(cx^2 + dx + e) + b \equiv 4x^2 + 6x + 1$

$\therefore \quad ac = 4 \qquad ad = 6 \qquad ae + b = 1$

$a^2c = 8 \qquad 2abc + ad = -2 \qquad cb^2 + bd + e = 0$

By solving

$a = 2 \qquad b = -1$

$c = 2 \qquad d = 3 \qquad e = 1$

$\therefore \quad f(x) = 2x^2 + 3x + 1$

$g(x) = 2x - 1$

$f(2) + g(2) = 2(2)^2 + 3(2) + 1 + 2(2) - 1$

$= 18$

76. Answer (25)

Let $f(x) = (x - \alpha)(x - \beta)$

It is given that $f(0) = p \Rightarrow \alpha\beta = p$

and $f(1) = \dfrac{1}{3} \quad \Rightarrow \quad (1-\alpha)(1-\beta) = \dfrac{1}{3}$

Now, let us assume that α is the common root of $f(x) = 0$ and $fofofof(x) = 0$

$fofofof(x) = 0$

$\Rightarrow \quad fofof(0) = 0$

$\Rightarrow \quad fof(p) = 0$

So, $f(p)$ is either α or β.

$(p - \alpha)(p - \beta) = \alpha$

$(\alpha\beta - \alpha)(\alpha\beta - \beta) = \alpha \Rightarrow (\beta - 1)(\alpha - 1)\beta = 1$

$(\because \alpha \neq 0)$

So, $\beta = 3$

$(1 - \alpha)(1 - 3) = \dfrac{1}{3}$

$\alpha = \dfrac{7}{6}$

$f(x) = \left(x - \dfrac{7}{6}\right)(x - 3)$

$f(-3) = \left(-3 - \dfrac{7}{6}\right)(3 - 3) = 25$

77. Answer (06)

$$\frac{3x^2-9x+17}{x^2+3x+10}=\frac{5x^2-7x+19}{3x^2+5x+12}$$

$$\Rightarrow \quad \frac{3x^2-9x+17}{5x^2-7x+19}=\frac{x^2+3x+10}{3x^2+5x+12}$$

$$\frac{-2x^2-2x-2}{5x^2-7x+19}=\frac{-2x^2-2x-2}{3x^2+5x+12}$$

$\Rightarrow$ Either $x^2+x+1=0$ or

No real roots

$5x^2-7x+19$

$= 3x^2+5x+12$

$2x^2-12x+7=0$

sum of roots $=6$

78. Answer (3)

$$x^2+(3-a)x+1=2a \;\begin{array}{l}\nearrow \alpha\\ \searrow \beta\end{array}$$

$\alpha+\beta = a-3$, $\alpha\beta = 1-2a$

$\Rightarrow \quad \alpha^2+\beta^2 = (a-3)^2 - 2(1-2a)$

$= a^2 - 6a + 9 - 2 + 4a$

$= a^2 - 2a + 7$

$= (a-1)^2 + 6$

So, $\alpha^2+\beta^2 \geq 6$

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