

Chapter 21

Integrals

1. If the integral

$$\int \frac{5 \tan x}{\tan x - 2} dx = x + a \ln |\sin x - 2 \cos x| + k,$$

then a is equal to

[AIEEE-2012]

- (1) -2 (2) 1
(3) 2 (4) -1

2. If $\int f(x) dx = \psi(x)$, then $\int x^5 f(x^3) dx$ is equal to

[JEE (Main)-2013]

(1) $\frac{1}{3} [x^3 \psi(x^3) - \int x^2 \psi(x^3) dx] + C$

(2) $\frac{1}{3} x^3 \psi(x^3) - 3 \int x^3 \psi(x^3) dx + C$

(3) $\frac{1}{3} x^3 \psi(x^3) - \int x^2 \psi(x^3) dx + C$

(4) $\frac{1}{3} [x^3 \psi(x^3) - \int x^3 \psi(x^3) dx] + C$

3. Let the population of rabbits surviving at a time t be governed by the differential equation

$$\frac{dp(t)}{dt} = \frac{1}{2} p(t) - 200. \text{ If } p(0) = 100, \text{ then } p(t) \text{ equals}$$

[JEE (Main)-2014]

- (1) $600 - 500 e^{t/2}$ (2) $400 - 300 e^{-t/2}$
(3) $400 - 300 e^{t/2}$ (4) $300 - 200 e^{-t/2}$

4. The integral $\int \frac{dx}{x^2(x^4 + 1)^{3/4}}$ equals

[JEE (Main)-2015]

- (1) $\left(\frac{x^4 + 1}{x^4} \right)^{\frac{1}{4}} + c$ (2) $(x^4 + 1)^{\frac{1}{4}} + c$
(3) $-(x^4 + 1)^{\frac{1}{4}} + c$ (4) $-\left(\frac{x^4 + 1}{x^4} \right)^{\frac{1}{4}} + c$

5. The integral $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3}$ is equal to

[JEE (Main)-2016]

(1) $\frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$

(2) $\frac{x^5}{2(x^5 + x^3 + 1)^2} + C$

(3) $\frac{-x^{10}}{2(x^5 + x^3 + 1)^2} + C$

(4) $\frac{-x^5}{(x^5 + x^3 + 1)^2} + C$

where C is an arbitrary constant.

6. Let $I_n = \int \tan^n x dx, (n > 1)$.

If $I_4 + I_6 = a \tan^5 x + b x^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to

[JEE (Main)-2017]

(1) $\left(\frac{1}{5}, 0 \right)$ (2) $\left(\frac{1}{5}, -1 \right)$

(3) $\left(-\frac{1}{5}, 0 \right)$ (4) $\left(-\frac{1}{5}, 1 \right)$

7. The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

is equal to

[JEE (Main)-2018]

(1) $\frac{1}{3(1 + \tan^3 x)} + C$ (2) $\frac{-1}{3(1 + \tan^3 x)} + C$

(3) $\frac{1}{1 + \cot^3 x} + C$ (4) $\frac{-1}{1 + \cot^3 x} + C$

(where C is a constant of integration)

8. For $x^2 \neq n\pi + 1$, $n \in \mathbb{N}$ (the set of natural numbers),

the integral $\int x \sqrt{\frac{2 \sin(x^2 - 1) - \sin 2(x^2 - 1)}{2 \sin(x^2 - 1) + \sin 2(x^2 - 1)}} dx$ is

equal to

(where c is a constant of integration)

[JEE (Main)-2019]

(1) $\frac{1}{2} \log_e |\sec(x^2 - 1)| + c$

(2) $\frac{1}{2} \log_e \left| \sec^2 \left(\frac{x^2 - 1}{2} \right) \right| + c$

(3) $\log_e \left| \cos \left(\frac{x^2 - 1}{2} \right) \right| + c$

(4) $\log_e \left| \frac{1}{2} \sec^2(x^2 - 1) \right| + c$

9. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$, ($x \geq 0$), and $f(0) = 0$,

then the value of $f(1)$ is [JEE (Main)-2019]

(1) $\frac{1}{2}$

(2) $\frac{1}{4}$

(3) $-\frac{1}{2}$

(4) $-\frac{1}{4}$

10. Let $n \geq 2$ be a natural number and $0 < \theta < \pi/2$.

Then $\int \frac{(\sin^n \theta - \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$ is equal to

(where C is a constant of integration)

[JEE (Main)-2019]

(1) $\frac{n}{n^2 - 1} \left(1 - \frac{1}{\sin^{n+1} \theta} \right)^{\frac{n+1}{n}} + C$

(2) $\frac{n}{n^2 + 1} \left(1 - \frac{1}{\sin^{n-1} \theta} \right)^{\frac{n+1}{n}} + C$

(3) $\frac{n}{n^2 - 1} \left(1 - \frac{1}{\sin^{n-1} \theta} \right)^{\frac{n+1}{n}} + C$

(4) $\frac{n}{n^2 - 1} \left(1 + \frac{1}{\sin^{n-1} \theta} \right)^{\frac{n+1}{n}} + C$

11. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then $f(x)$ is equal to

[JEE (Main)-2019]

(1) $4x^3 + 1$

(2) $-4x^3 - 1$

(3) $-2x^3 + 1$

(4) $-2x^3 - 1$

12. If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x) \left(\sqrt{1-x^2} \right)^m + C$, for a

suitable chosen integer m and a function $A(x)$, where C is a constant of integration, then $(A(x))^m$ equals [JEE (Main)-2019]

(1) $\frac{-1}{3x^3}$

(2) $\frac{1}{27x^6}$

(3) $\frac{1}{9x^4}$

(4) $\frac{-1}{27x^9}$

13. If $\int \frac{x+1}{\sqrt{2x-1}} dx = f(x) \sqrt{2x-1} + C$, where C is a constant of integration, then $f(x)$ is equal to

[JEE (Main)-2019]

(1) $\frac{1}{3}(x+1)$

(2) $\frac{1}{3}(x+4)$

(3) $\frac{2}{3}(x-4)$

(4) $\frac{2}{3}(x+2)$

14. The integral $\int \cos(\log_e x) dx$ is equal to (where C is a constant of integration)

[JEE (Main)-2019]

(1) $x [\cos(\log_e x) - \sin(\log_e x)] + C$

(2) $\frac{x}{2} [\cos(\log_e x) + \sin(\log_e x)] + C$

(3) $\frac{x}{2} [\sin(\log_e x) - \cos(\log_e x)] + C$

(4) $x [\cos(\log_e x) + \sin(\log_e x)] + C$

15. The integral $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$ is equal to

(where C is a constant of integration)

[JEE (Main)-2019]

(1) $\frac{x^4}{6(2x^4 + 3x^2 + 1)^3} + C$

(2) $\frac{x^{12}}{(2x^4 + 3x^2 + 1)^3} + C$

(3) $\frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$

(4) $\frac{x^4}{(2x^4 + 3x^2 + 1)^3} + C$

16. $\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx$ is equal to

(where c is a constant of integration)

[JEE (Main)-2019]

(1) $2x + \sin x + 2 \sin 2x + c$

(2) $x + 2 \sin x + 2 \sin 2x + c$

(3) $x + 2 \sin x + \sin 2x + c$

(4) $2x + \sin x + \sin 2x + c$

17. If $\int \frac{dx}{x^3(1+x^6)^{\frac{2}{3}}} = xf(x)(1+x^6)^{\frac{1}{3}} + C$ where C is a constant of integration, then the function $f(x)$ is equal to:

[JEE (Main)-2019]

(1) $-\frac{1}{2x^3}$

(2) $\frac{3}{x^2}$

(3) $-\frac{1}{6x^3}$

(4) $-\frac{1}{2x^2}$

18. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ is equal to (Here C is a constant of integration)

[JEE (Main)-2019]

(1) $-3 \cot^{-1/3} x + C$

(2) $-3 \tan^{-1/3} x + C$

(3) $-\frac{3}{4} \tan^{-4/3} x + C$

(4) $3 \tan^{-1/3} x + C$

19. If $\int e^{\sec x} (\sec x \tan x f(x) + \sec x \tan x + \sec^2 x) dx = e^{\sec x} f(x) + C$, then a possible choice of $f(x)$ is

[JEE (Main)-2019]

(1) $\sec x - \tan x - \frac{1}{2}$

(2) $\sec x + \tan x + \frac{1}{2}$

(3) $\sec x + x \tan x - \frac{1}{2}$

(4) $x \sec x + \tan x + \frac{1}{2}$

20. If $\int \frac{dx}{(x^2 - 2x + 10)^2}$

$= A \left(\tan^{-1} \left(\frac{x-1}{3} \right) + \frac{f(x)}{x^2 - 2x + 10} \right) + C$

where C is a constant of integration, then

[JEE (Main)-2019]

(1) $A = \frac{1}{81}$ and $f(x) = 3(x-1)$

(2) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

(3) $A = \frac{1}{27}$ and $f(x) = 9(x-1)$

(4) $A = \frac{1}{54}$ and $f(x) = 9(x-1)^2$

21. If $\int x^5 e^{-x^2} dx = g(x) e^{-x^2} + c$, where c is a constant of integration, then $g(-1)$ is equal to

[JEE (Main)-2019]

(1) -1

(2) $-\frac{1}{2}$

(3) 1

(4) $-\frac{5}{2}$

22. The integral $\int \frac{2x^3 - 1}{x^4 + x} dx$ is equal to

(Here C is a constant of integration)

[JEE (Main)-2019]

(1) $\log_e \left| \frac{x^3 + 1}{x} \right| + C$

(2) $\frac{1}{2} \log_e \frac{(x^3 + 1)^2}{|x^3|} + C$

(3) $\frac{1}{2} \log_e \frac{|x^3 + 1|}{x^2} + C$

(4) $\log_e \frac{|x^3 + 1|}{x^2} + C$

23. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral

$\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx =$

$A(x) \cos 2\alpha + B(x) \sin 2\alpha + C$, where C is a constant of integration, then the functions $A(x)$ and $B(x)$ are respectively

[JEE (Main)-2019]

(1) $x - \alpha$ and $\log_e |\cos(x - \alpha)|$

(2) $x + \alpha$ and $\log_e |\sin(x - \alpha)|$

(3) $x - \alpha$ and $\log_e |\sin(x - \alpha)|$

(4) $x + \alpha$ and $\log_e |\sin(x + \alpha)|$

24. If $\int \frac{\cos x dx}{\sin^3 x (1 + \sin^6 x)^{1/3}} = f(x)(1 + \sin^6 x)^{1/3} + c$

where c is a constant of integration, then

$\lambda f\left(\frac{\pi}{3}\right)$ is equal to **[JEE (Main)-2020]**

- (1) $\frac{9}{8}$ (2) $-\frac{9}{8}$
(3) -2 (4) 2

25. The integral $\int \frac{dx}{(x+4)^{7/6} (x-3)^{7/6}}$ is equal to

(where C is a constant of integration)

[JEE (Main)-2020]

- (1) $\frac{1}{2} \left(\frac{x-3}{x+4} \right)^{3/7} + C$ (2) $-\frac{1}{13} \left(\frac{x-3}{x+4} \right)^{-13/7} + C$
(3) $\left(\frac{x-3}{x+4} \right)^{1/7} + C$ (4) $-\left(\frac{x-3}{x+4} \right)^{-1/7} + C$

26. If $\int \frac{d\theta}{\cos^2 \theta (\tan 2\theta + \sec 2\theta)} =$

$\lambda \tan \theta + 2 \log_e |f(\theta)| + C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to

[JEE (Main)-2020]

- (1) $(1, 1 - \tan \theta)$ (2) $(-1, 1 + \tan \theta)$
(3) $(-1, 1 - \tan \theta)$ (4) $(1, 1 + \tan \theta)$

27. If $\int \sin^{-1} \left(\sqrt{\frac{x}{1+x}} \right) dx = A(x) \tan^{-1}(\sqrt{x}) + B(x) + C$,

where C is a constant of integration, then the ordered pair $(A(x), B(x))$ can be

[JEE (Main)-2020]

- (1) $(x+1, -\sqrt{x})$ (2) $(x+1, \sqrt{x})$
(3) $(x-1, -\sqrt{x})$ (4) $(x-1, \sqrt{x})$

28. The integral $\int \left(\frac{x}{x \sin x + \cos x} \right)^2 dx$ is equal to

(where C is a constant of integration)

[JEE (Main)-2020]

- (1) $\tan x - \frac{x \sec x}{x \sin x + \cos x} + C$
(2) $\sec x - \frac{x \tan x}{x \sin x + \cos x} + C$
(3) $\tan x + \frac{x \sec x}{x \sin x + \cos x} + C$
(4) $\sec x + \frac{x \tan x}{x \sin x + \cos x} + C$

29. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$ ($x \geq 0$). Then $f(3) - f(1)$

is equal to **[JEE (Main)-2020]**

- (1) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$ (2) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$
(3) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

30. If $\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x + e^{-x})} dx$

$= g(x)e^{(e^x + e^{-x})} + c$, where c is a constant of integration, then $g(0)$ is equal to

[JEE (Main)-2020]

- (1) e^2 (2) 1
(3) 2 (4) e

31. If $\int \frac{\cos \theta}{5 + 7 \sin \theta - 2 \cos^2 \theta} d\theta = A \log_e |B(\theta)| + C$,

where C is a constant of integration, then $\frac{B(\theta)}{A}$

can be **[JEE (Main)-2020]**

- (1) $\frac{2 \sin \theta + 1}{5(\sin \theta + 3)}$ (2) $\frac{2 \sin \theta + 1}{\sin \theta + 3}$
(3) $\frac{5(2 \sin \theta + 1)}{\sin \theta + 3}$ (4) $\frac{5(\sin \theta + 3)}{2 \sin \theta + 1}$

32. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1} \left(\frac{\sin x + \cos x}{b} \right) + c$,

where c is a constant of integration, then the ordered pair (a, b) is equal to :

[JEE (Main)-2021]

- (1) $(-1, 3)$ (2) $(3, 1)$
(3) $(1, -3)$ (4) $(1, 3)$

33. The value of the integral

$$\int \frac{\sin \theta \cdot \sin 2\theta (\sin^6 \theta + \sin^4 \theta + \sin^2 \theta)}{\sqrt{2 \sin^4 \theta + 3 \sin^2 \theta + 6}} \cdot \frac{1}{1 - \cos 2\theta} d\theta$$

(where c is a constant of integration)

[JEE (Main)-2021]

- (1) $\frac{1}{18} [9 - 2 \cos^6 \theta - 3 \cos^4 \theta - 6 \cos^2 \theta]^{\frac{3}{2}} + c$
(2) $\frac{1}{18} [11 - 18 \cos^2 \theta + 9 \cos^4 \theta - 2 \cos^6 \theta]^{\frac{3}{2}} + c$
(3) $\frac{1}{18} [9 - 2 \sin^6 \theta - 3 \sin^4 \theta - 6 \sin^2 \theta]^{\frac{3}{2}} + c$
(4) $\frac{1}{18} [11 - 18 \sin^2 \theta + 9 \sin^4 \theta - 2 \sin^6 \theta]^{\frac{3}{2}} + c$

34. The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$,
 $x > 0$, is equal to : **[JEE (Main)-2021]**

(1) $4\log_e |x^2 + 5x - 7| + c$

(2) $\log_e |x^2 + 5x - 7| + c$

(3) $\frac{1}{4}\log_e |x^2 + 5x - 7| + c$

(4) $\log_e \sqrt{x^2 + 5x - 7} + c$

35. For real numbers α, β, γ and δ , if

$$\int \frac{(x^2 - 1) + \tan^{-1}\left(\frac{x^2 + 1}{x}\right)}{(x^4 + 3x^2 + 1)\tan^{-1}\left(\frac{x^2 + 1}{x}\right)} dx$$

$$= \alpha \log_e \left(\tan^{-1}\left(\frac{x^2 + 1}{x}\right) \right) + \beta \tan^{-1}\left(\frac{\gamma(x^2 - 1)}{x}\right)$$

$$+ \delta \tan^{-1}\left(\frac{x^2 + 1}{x}\right) + C$$

Where C is an arbitrary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to _____.

[JEE (Main)-2021]

36. The integral $\int \frac{(2x - 1)\cos\sqrt{(2x - 1)^2 + 5}}{\sqrt{4x^2 - 4x + 6}} dx$ is equal

to :

(where c is a constant of integration)

[JEE (Main)-2021]

(1) $\frac{1}{2}\sin\sqrt{(2x + 1)^2 + 5} + c$

(2) $\frac{1}{2}\sin\sqrt{(2x - 1)^2 + 5} + c$

(3) $\frac{1}{2}\cos\sqrt{(2x + 1)^2 + 5} + c$

(4) $\frac{1}{2}\cos\sqrt{(2x - 1)^2 + 5} + c$

37. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$, ($x \geq 0$), $f(0) = 0$ and
 $f(1) = \frac{1}{K}$, then the value of K is _____.

[JEE (Main)-2021]

38. If $\int \frac{dx}{(x^2 + x + 1)^2} = a \tan^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right) + b\left(\frac{2x + 1}{x^2 + x + 1}\right) + C$,
 $x > 0$ where C is the constant of integration, then
the value of $9(\sqrt{3}a + b)$ is equal to _____.

[JEE (Main)-2021]

39. If $\int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}} dx = \frac{1}{14}(ux + v\log_e(4e^x + 7e^{-x})) +$
C, where C is a constant of integration, then $u + v$
is equal to _____.

[JEE (Main)-2021]

40. The integral $\int \frac{1}{\sqrt[4]{(x - 1)^3(x + 2)^5}} dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{3}{4}\left(\frac{x + 2}{x - 1}\right)^{\frac{5}{4}} + C$ (2) $\frac{4}{3}\left(\frac{x - 1}{x + 2}\right)^{\frac{5}{4}} + C$

(3) $\frac{3}{4}\left(\frac{x + 2}{x - 1}\right)^{\frac{1}{4}} + C$ (4) $\frac{4}{3}\left(\frac{x - 1}{x + 2}\right)^{\frac{1}{4}} + C$

[JEE (Main)-2021]

41. If $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx = \alpha \log_e |1 + \tan x| + \beta \log_e$
 $|1 - \tan x + \tan^2 x| + \gamma \tan^{-1}\left(\frac{2 \tan x - 1}{\sqrt{3}}\right) + C$,
when C is constant of integration, then the value of
 $18(\alpha + \beta + \gamma^2)$ is _____.

[JEE (Main)-2021]

42. If $\cos x \frac{dy}{dx} - y \sin x = 6x$, $\left(0 < x < \frac{\pi}{2}\right)$ and

$y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

[JEE (Main)-2021]

(1) $-\frac{\pi^2}{2}$

(2) $-\frac{\pi^2}{4\sqrt{3}}$

(3) $\frac{\pi^2}{2\sqrt{3}}$

(4) $-\frac{\pi^2}{2\sqrt{3}}$

43. The integral $\int \left(1 + x - \frac{1}{x}\right) e^{x + \frac{1}{x}} dx$ is equal to

[JEE (Main)-2021]

(1) $(x + 1)e^{x + \frac{1}{x}} + c$

(2) $-xe^{x + \frac{1}{x}} + c$

(3) $(x - 1)e^{x + \frac{1}{x}} + c$

(4) $xe^{x + \frac{1}{x}} + c$

44. If $\int \frac{1}{x} \sqrt{\frac{1-x}{1+x}} dx = g(x) + c$, $g(1) = 0$, then $g\left(\frac{1}{2}\right)$ is equal to

[JEE (Main)-2022]

- (1) $\log_e \left(\frac{\sqrt{3}-1}{\sqrt{3}+1} \right) + \frac{\pi}{3}$ (2) $\log_e \left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \right) + \frac{\pi}{3}$
 (3) $\log_e \left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \right) - \frac{\pi}{3}$ (4) $\frac{1}{2} \log_e \left(\frac{\sqrt{3}-1}{\sqrt{3}+1} \right) - \frac{\pi}{6}$

45. If $\int \frac{(x^2+1)e^x}{(x+1)^2} dx = f(x)e^x + C$, where C is a

constant, then $\frac{d^3 f}{dx^3}$ at $x = 1$ is equal to :

[JEE (Main)-2022]

- (1) $-\frac{3}{4}$
 (2) $\frac{3}{4}$
 (3) $-\frac{3}{2}$
 (4) $\frac{3}{2}$

46. For $I(x) = \int \frac{\sec^2 x - 2022}{\sin^{2022} x} dx$, if $I\left(\frac{\pi}{4}\right) = 2^{1011}$, then

[JEE (Main)-2022]

- (1) $3^{1010} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$
 (2) $3^{1010} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$
 (3) $3^{1011} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$
 (4) $3^{1011} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

47. Let $g : (0, \infty) \rightarrow \mathbb{R}$ be a differentiable function such

that $\int \left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2} \right) dx = \frac{xg(x)}{e^x + 1} + c$, for all $x > 0$, where c is an arbitrary constant. Then :

- (1) g is decreasing in $\left(0, \frac{\pi}{4}\right)$
 (2) g' is increasing in $\left(0, \frac{\pi}{4}\right)$
 (3) $g + g'$ is increasing in $\left(0, \frac{\pi}{2}\right)$
 (4) $g - g'$ is increasing in $\left(0, \frac{\pi}{2}\right)$

[JEE (Main)-2022]

48. The integral $\int \frac{\left(1 - \frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(1 + \frac{2}{\sqrt{3}} \sin 2x\right)} dx$ is equal to

[JEE (Main)-2022]

- (1) $\frac{1}{2} \log_e \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)}{\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)} \right| + C$
 (2) $\frac{1}{2} \log_e \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)}{\tan\left(\frac{x}{2} + \frac{\pi}{3}\right)} \right| + C$
 (3) $\log_e \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)}{\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)} \right| + C$
 (4) $\frac{1}{2} \log_e \left| \frac{\tan\left(\frac{x}{2} - \frac{\pi}{12}\right)}{\tan\left(\frac{x}{2} - \frac{\pi}{6}\right)} \right| + C$

Chapter 21

Integrals

1. Answer (3)

2. Answer (3)

$$\begin{aligned} & \int x^5 \cdot f(x^3) dx \\ &= \frac{1}{3} \int x^3 \cdot f(x^3) \cdot 3x^2 dx \\ &= \frac{1}{3} \cdot x^3 \int f(x^3) \cdot 3x^2 dx - \frac{1}{3} \int (3x^2 \cdot \int f(x^3) \cdot 3x^2 dx) dx \\ &= \frac{1}{3} \cdot x^3 \psi(x^3) - \int x^2 \cdot \psi(x^3) dx + C \end{aligned}$$

3. Answer (3)

$$\frac{dp(t)}{dt} = \frac{1}{2} p(t) - 200$$

$$\int \frac{d(p(t))}{\left(\frac{1}{2} p(t) - 200\right)} = \int dt$$

$$2 \log\left(\frac{p(t)}{2} - 200\right) = t + cx$$

$$\frac{p(t)}{2} - 200 = e^{\frac{t}{2}k}$$

Using given condition $p(t) = 400 - 300 e^{t/2}$

4. Answer (4)

$$I = \int \frac{dx}{x^2(x^4 + 1)^{3/4}} = \int \frac{dx}{x^5 \left(1 + \frac{1}{x^4}\right)^{3/4}}$$

$$\text{Let } 1 + \frac{1}{x^4} = t \Rightarrow \frac{-4}{x^5} dx = dt$$

$$\text{So, } I = \frac{-1}{4} \int \frac{dt}{t^{3/4}} = \frac{-1}{4} \int t^{-3/4} dt$$

$$= \frac{-1}{4} \left(\frac{t^{1/4}}{1/4} \right) + c$$

$$= - \left(1 + \frac{1}{x^4} \right)^{1/4} + c$$

where c is an arbitrary constant.

So, option (4) is right answer.

5. Answer (1)

$$\int \frac{2x^{12} + 5x^9}{(x^2 + x^3 + 1)} dx$$

$$= \int \frac{\left(\frac{2}{x^3} + \frac{5}{x^6}\right) dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$$

$$1 + \frac{1}{x^2} + \frac{1}{x^5} = t$$

$$\left(-\frac{2}{x^3} - \frac{5}{x^6}\right) dx = dt$$

$$= \int \frac{-dt}{t^3} = \frac{1}{2t^2} + C$$

$$= \frac{1}{2 \left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3} + C = \frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$$

6. Answer (1)

$$I_n = \int \tan^n x dx, n > 1$$

$$I_4 + I_6 = \int (\tan^4 x + \tan^6 x) dx$$

$$= \int \tan^4 x \sec^2 x dx$$

Let $\tan x = t$

$$\sec^2 x dx = dt$$

$$= \int t^4 dt = \frac{t^5}{5} + C = \frac{1}{5} \tan^5 x + C$$

$$a = \frac{1}{5}, b = 0$$

7. Answer (2)

$$I = \int \frac{\sin^2 x \cdot \cos^2 x dx}{\left\{(\sin^2 x + \cos^2 x)(\sin^3 x + \cos^3 x)\right\}^2}$$

Dividing the numerator and denominator by $\cos^6 x$

$$\Rightarrow I = \int \frac{\tan^2 x \sec^2 x \, dx}{(1 + \tan^3 x)^2}$$

$$\text{Let, } \tan^3 x = z$$

$$\Rightarrow 3 \tan^2 x \cdot \sec^2 x \, dx = dz$$

$$I = \frac{1}{3} \int \frac{dz}{z^2} = \frac{-1}{3z} + C$$

$$= \frac{-1}{3(1 + \tan^3 x)} + C$$

8. Answer (2)

$$I = \int x \sqrt{\frac{2 \sin(x^2 - 1) - 2 \sin(x^2 - 1) \cos(x^2 - 1)}{2 \sin(x^2 - 1) + 2 \sin(x^2 - 1) \cos(x^2 - 1)}} \, dx$$

$$I = \int x \sqrt{\frac{1 - \cos(x^2 - 1)}{1 + \cos(x^2 - 1)}} \, dx$$

$$I = \int x \left| \tan \left(\frac{x^2 - 1}{2} \right) \right| \, dx, \quad \text{Now let } \frac{x^2 - 1}{2} = t$$

$$\frac{2x}{2} \, dx = dt$$

$$I = \int |\tan(t)| \, dt$$

$$I = \ln \left| \sec \left(\frac{x^2 - 1}{2} \right) \right| + c = \frac{1}{2} \ln \left| \sec^2 \left(\frac{x^2 - 1}{2} \right) \right| + c$$

9. Answer (2)

$$f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} \, dx, \quad x \geq 0$$

$$= \int \frac{5x^8 + 7x^6}{x^{14} (x^{-5} + x^{-7} + 2)^2} \, dx$$

$$= \int \frac{5x^{-6} + 7x^{-8}}{(2 + x^{-7} + x^{-5})^2} \, dx$$

$$\text{Let } 2 + x^{-7} + x^{-5} = t$$

$$(-7x^{-8} - 5x^{-6}) \, dx = dt$$

$$f(x) = \int \frac{-dt}{t^2} = \int -t^{-2} \, dt = t^{-1} + c$$

$$f(x) = \frac{1}{2 + x^{-7} + x^{-5}} + c, \quad f(0) = 0 \Rightarrow c = 0$$

$$\therefore f(1) = \frac{1}{4}$$

10. Answer (3)

$$I = \int \frac{(\sin^n \theta - \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} \, d\theta$$

$$\text{Let } \sin \theta = t$$

$$\cos \theta \, d\theta = dt$$

$$I = \int \frac{(t^n - t)^{\frac{1}{n}}}{t^{n+1}} \, dt$$

$$= \int \frac{\left(1 - \frac{1}{t^{n-1}}\right)^{\frac{1}{n}}}{t^n} \, dt = \int t^{-n} (1 - t^{1-n})^{\frac{1}{n}} \, dt$$

$$\text{Let } 1 - t^{1-n} = u$$

$$-(1 - n)t^{-n} \, dt = du$$

$$t^{-n} \, dt = \frac{du}{n-1}$$

$$I = \int u^{\frac{1}{n}} \cdot \frac{du}{n-1} = \frac{1}{n-1} \cdot \frac{u^{\frac{1}{n}+1}}{\frac{1}{n}+1}$$

$$= \frac{n}{n^2-1} u^{\frac{n+1}{n}} + C$$

$$= \frac{n}{n^2-1} \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$$

11. Answer (2)

$$I = \int x^5 e^{-4x^3} \, dx$$

$$\text{Put } -4x^3 = t$$

$$\Rightarrow -12x^2 \, dx = dt$$

$$\Rightarrow x^2 \, dx = -\frac{dt}{12}$$

$$\Rightarrow I = \int \frac{1}{48} t e^t \, dt = \frac{1}{48} [t e^t - e^t] + C$$

$$I = \frac{1}{48} e^{-4x^3} (-4x^3 - 1) + C$$

$$\Rightarrow f(x) = -4x^3 - 1$$

12. Answer (4)

$$A(x) \left(\sqrt{1-x^2} \right)^m + C = \int \frac{\sqrt{1-x^2}}{x^4} dx$$

$$= \int \frac{\sqrt{\frac{1}{x^2} - 1}}{x^3} dx$$

$$\text{Let } \frac{1}{x^2} - 1 = t^2$$

$$\Rightarrow -\frac{2}{x^3} = \frac{2t dt}{dx}$$

$$\frac{dx}{x^3} = -\frac{2t}{2} dt$$

$$A(x) \left(\sqrt{1-x^2} \right)^m + C = \int (-t^2) dt = -\frac{t^3}{3} + C$$

$$= -\frac{1}{3} \left(\frac{1}{x^2} - 1 \right)^{\frac{3}{2}} + C$$

$$= -\frac{1}{3} \cdot \frac{1}{x^3} \cdot (1-x^2)^{\frac{3}{2}} + C$$

$$= -\frac{1}{3x^3} \left(\sqrt{1-x^2} \right)^3 + C$$

$$\therefore A(x) = -\frac{1}{3x^3}$$

$$\therefore (A(x))^3 = \frac{-1}{27x^9}$$

13. Answer (2)

$$\text{Let } I = \int \frac{x+1}{\sqrt{2x-1}} dx$$

$$\text{Let } \sqrt{2x-1} = t$$

$$\therefore 2x-1 = t^2$$

$$\Rightarrow dx = t dt$$

$$I \int \frac{(t^2+3)}{2} dt = \frac{t^3}{6} + \frac{3t}{2} + C$$

$$= \frac{(2x-1)^{\frac{3}{2}}}{6} + \frac{3}{2} (2x-1)^{\frac{1}{2}} + C$$

$$= \sqrt{2x-1} \left(\frac{x+4}{3} \right) + C$$

$$= f(x) \cdot \sqrt{2x-1} + C$$

$$\text{where } f(x) = \frac{x+4}{3}$$

14. Answer (2)

$$I = \int \cos(\ln x) dx$$

$$I = \cos(\ln x) \cdot x - \int \frac{-\sin(\ln x)}{x} \cdot x dx$$

$$= x \cos(\ln x) + \int \sin(\ln x) dx$$

$$= x \cos(\ln x) + \sin(\ln x) \cdot x - \int \frac{\cos(\ln x)}{x} \cdot x dx$$

$$2I = x(\cos(\ln x) + \sin(\ln x)) + C$$

$$I = \frac{x}{2} [\cos(\ln x) + \sin(\ln x)] + C$$

15. Answer (3)

$$I = \int \frac{3x^{13} + 2x^{11}}{x^{16} \left(2 + \frac{3}{x^2} + \frac{1}{x^4} \right)^4} dx$$

$$I = \int \frac{\frac{3}{x^3} + \frac{2}{x^5}}{\left(2 + \frac{3}{x^2} + \frac{1}{x^4} \right)^4} dx$$

$$\text{Let } 2 + \frac{3}{x^2} + \frac{1}{x^4} = t, \quad -2 \left(\frac{3}{x^3} + \frac{2}{x^5} \right) dx = dt$$

$$I = \int \frac{-\frac{dt}{2}}{t^4} = -\frac{1}{2} \frac{t^{-4+1}}{-4+1} + C$$

$$I = \frac{-1}{2} \times \frac{1}{(-3)} \frac{1}{\left(2 + \frac{3}{x^2} + \frac{1}{x^4} \right)^3} + C$$

$$I = \frac{1}{6} \frac{x^{12}}{\left(2x^4 + 3x^2 + 1 \right)^3} + C$$

16. Answer (3)

$$\int \frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)} dx = \int \frac{2\cos\frac{x}{2} \cdot \sin\frac{5x}{2}}{2\cos\frac{x}{2} \cdot \sin\frac{x}{2}} dx$$

$$= \int \frac{\sin 3x + \sin 2x}{\sin x} dx$$

Now use $\sin 2x = 2\sin x \cos x$ and $\sin 3x = 3\sin x - 4\sin^3 x$

$$= \int (1 + 2\cos x + 2\cos 2x) dx$$

$$= x + 2\sin x + \sin 2x + c$$

17. Answer (1)

$$I = \int \frac{dx}{x^3(1+x^6)^{2/3}} = \int \frac{dx}{x^7(1+x^{-6})^{2/3}}$$

Put $1 + x^{-6} = t^3$

$$\Rightarrow -6x^{-7}dx = 3t^2 dt$$

$$\Rightarrow \frac{dx}{x^7} = -\frac{1}{2}t^2 dt$$

$$\Rightarrow I = \int -\frac{1}{2} \frac{t^2 dt}{t^2}$$

$$= -\frac{1}{2}t + C$$

$$= -\frac{1}{2}(1+x^{-6})^{1/3} + C$$

$$= -\frac{1}{2} \frac{(1+x^6)^{1/3}}{x^2} + C$$

$$= -\frac{1}{2x^3} x(1+x^6)^{1/3} + C$$

$$\Rightarrow f(x) = -\frac{1}{2x^3}$$

18. Answer (2)

$$I = \int \sec^{\frac{2}{3}} x \cdot \operatorname{cosec}^{\frac{4}{3}} x dx$$

$$I = \int \frac{\sec^2 x dx}{\tan^{\frac{4}{3}} x}$$

Put $\tan x = t$

$$\Rightarrow \sec^2 x dx = dt$$

$$\Rightarrow I = \frac{t^{-1/3}}{\left(\frac{-1}{3}\right)} + C \Rightarrow I = -3(\tan x)^{-1/3} + C$$

19. Answer (2)

$$\int e^{\sec x} (\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x)) dx = e^{\sec x} f(x) + C$$

$\therefore$ We know that

$$\int e^{g(x)} ((g'(x)f(x)) + f'(x)) dx = e^{g(x)} \times f(x) + C$$

$$\therefore f(x) = \int ((\sec x \tan x) + \sec^2 x) dx$$

$$\therefore \boxed{f(x) = \sec x + \tan x + C}$$

20. Answer (2)

$$\int \frac{dx}{(x^2 - 2x + 10)^2} = \int \frac{dx}{((x-1)^2 + 9)^2}$$

Let $(x-1)^2 = 9 \tan^2 \theta \quad \dots(i)$

$$\Rightarrow \tan \theta = \frac{x-1}{3}$$

On Differentiating $\dots(ii)$

$$2(x-1)dx = 18 \tan \theta \sec^2 \theta d\theta$$

$$\therefore I = \int \frac{18 \tan \theta \sec^2 \theta d\theta}{2 \times 3 \tan \theta \times 81 \sec^4 \theta}$$

$$I = \frac{1}{27} \int \cos^2 \theta d\theta = \frac{1}{27} \times \frac{1}{2} \int (1 + \cos 2\theta) d\theta$$

$$I = \frac{1}{54} \left\{ \theta + \frac{\sin 2\theta}{2} \right\} + c$$

$$I = \frac{1}{54} \left[\tan^{-1} \left(\frac{x-1}{3} \right) + \frac{1}{2} \times \frac{2 \left(\frac{x-1}{3} \right)}{1 + \left(\frac{x-1}{3} \right)^2} \right] + c$$

$$I = \frac{1}{54} \left[\tan^{-1} \left(\frac{x-1}{3} \right) + \frac{3(x-1)}{x^2 - 2x + 10} \right] + c$$

So $A = \frac{1}{54}$

$$f(x) = 3(x-1)$$

21. Answer (4)

$$I = \int x^5 \cdot e^{-x^2} dx$$

Put $-x^2 = t \Rightarrow -2x dx = dt$

$$I = \int \frac{t^2 \cdot e^t dt}{(-2)} = \frac{-1}{2} e^t (t^2 - 2t + 2) + c$$

$$\therefore g(x) = \frac{-1}{2} (x^4 + 2x^2 + 2)$$

$$g(-1) = \frac{-5}{2}$$

22. Answer (1)

$$I = \int \frac{(2x^3 - 1)dx}{x^4 + x} = \int \frac{(2x - x^{-2})dx}{x^2 + x^{-1}}$$

Put $x^2 + x^{-1} = t$

$$(2x - x^{-2})dx = dt$$

$$I = \int \frac{dt}{t} = \ln|t| + c$$

$$= \ln|x^2 + x^{-1}| + c$$

$$= \ln\left|\frac{x^3 + 1}{x}\right| + c$$

23. Answer (3)

$$\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx$$

$$= \int \frac{\sin(x + \alpha)}{\sin(x - \alpha)} dx$$

let $x - \alpha = t$

$$\Rightarrow dx = dt$$

$$= \int \frac{\sin(t + 2\alpha)}{\sin t} dt$$

$$= [\cos 2\alpha + \sin 2\alpha \cdot \cot t] dt$$

$$= \cos 2\alpha \cdot t + \sin 2\alpha \cdot \ln |\sin t| + c$$

$$= (x - \alpha) \cdot \cos 2\alpha + \sin 2\alpha \cdot \ln |\sin(x - \alpha)| + c$$

24. Answer (3)

Let $\sin x = t$

$$\Rightarrow \cos x dx = dt$$

$$I = \int \frac{dt}{t^3(1+t^6)^{2/3}} = \int \frac{dt}{t^7 \left(1 + \frac{1}{t^6}\right)^{2/3}}$$

$\therefore$

Put $\frac{1}{t^6} + 1 = k$

$$\Rightarrow \frac{-6}{t^7} dt = dk$$

$$\therefore I = \frac{-1}{6} \int \frac{dk}{k^{2/3}} = \frac{-1}{6} \frac{k^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + c$$

$$= \frac{-1}{2} k^{1/3} + c$$

$$= \frac{-1}{2} (1 + \sin^6 x)^{1/3} \cdot \operatorname{cosec}^2 x + c$$

$$\therefore \lambda = 3 \text{ and } f\left(\frac{\pi}{3}\right) = \left(\frac{4}{3}\right)\left(\frac{-1}{2}\right) \Rightarrow \lambda f(\pi/3) = -2$$

25. Answer (3)

$$\int \frac{dx}{(x-3)^{\frac{6}{7}}(x+4)^{\frac{8}{7}}}$$

$$= \int \frac{dx}{\left(\frac{x-3}{x+4}\right)^{\frac{6}{7}} \cdot (x+4)^2}$$

Let $\frac{x-3}{x+4} = t = 1 - \frac{7}{x+4}$

$$= \frac{1}{7} \int \frac{dt}{t^{\frac{6}{7}}}$$

$$dt = \frac{7}{(x+4)^2} dx$$

$$= t^{\frac{1}{7}} + C$$

$$= \left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$$

26. Answer (2)

Let $I = \int \frac{\sec^2 \theta d\theta}{\sec 2\theta + \tan 2\theta}$

$$= \int \frac{\sec^2 \theta}{1 + \tan^2 \theta + 2 \tan \theta} d\theta$$

$$= \int \frac{\sec^2 \theta (1 - \tan \theta)}{1 + \tan \theta} d\theta$$

Put $\tan \theta = t \quad \therefore \sec^2 \theta d\theta = dt$

$$I = \int \frac{(1-t)dt}{1+t} = \int \left(-1 + \frac{2}{1+t}\right) dt = -t + 2 \log(1+t) + c$$

$$\therefore I = -\tan \theta + 2 \log_e |1 + \tan \theta| + c$$

$$\Rightarrow \lambda = -1, f(\theta) = 1 + \tan \theta.$$

27. Answer (1)

$$I = \int \sin^{-1} \left(\frac{\sqrt{x}}{\sqrt{1+x}} \right) dx = \int \tan^{-1}(\sqrt{x}) dx$$

$$= \int \tan^{-1} \sqrt{x} \cdot 1 dx$$

$$I = x \tan^{-1} \sqrt{x} - \int \frac{x}{1+x} \cdot \frac{1}{2\sqrt{x}} dx$$

$$\begin{aligned}
 &= x \tan^{-1} \sqrt{x} - \frac{1}{2} \int \frac{\sqrt{x}}{1+x} dx \\
 &= x \tan^{-1} \sqrt{x} - \int \frac{t^2}{1+t^2} dt \quad \left[\begin{array}{l} \text{Put } x = t^2 \\ dx = 2t dt \end{array} \right] \\
 &= x \tan^{-1} \sqrt{x} - \int 1 dt + \int \frac{1}{1+t^2} dt \\
 &= x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} + c \\
 &= (x+1) \tan^{-1} \sqrt{x} - \sqrt{x} + c \\
 \Rightarrow A(x) &= (x+1), B(x) = -\sqrt{x}
 \end{aligned}$$

28. Answer (1)

$$\begin{aligned}
 &\int \frac{x^2}{(x \sin x + \cos x)^2} dx \\
 \therefore \frac{d}{dx} (x \sin x + \cos x) &= x \cos x \\
 &= \int \frac{x \cos x}{(x \sin x + \cos x)^2} \cdot \left(\frac{x}{\cos x} \right) dx \\
 &= \frac{x}{\cos x} \left[\frac{-1}{x \sin x + \cos x} \right] - \int \frac{x \sin x + \cos x}{\cos^2 x} \cdot \left[\frac{-1}{x \sin x + \cos x} \right] dx \\
 &= \frac{-x \sec x}{x \sin x + \cos x} + \tan x + C
 \end{aligned}$$

29. Answer (1)

$$\begin{aligned}
 &\int \frac{\sqrt{x}}{(1+x)^2} dx \quad (x > 0) \\
 \text{Put } x &= \tan^2 \theta \Rightarrow 2x dx = 2 \tan \theta \sec^2 \theta d\theta \\
 I &= \int \frac{2 \tan^2 \theta \sec^2 \theta}{\sec^4 \theta} d\theta = \int 2 \sin^2 \theta d\theta = \int (1 - \cos 2\theta) d\theta \\
 &= \theta - \frac{\sin 2\theta}{2} + c \\
 \Rightarrow f(x) &= \theta - \frac{1}{2} \times \frac{2 \tan \theta}{1 + \tan^2 \theta} + c \\
 f(x) &= \theta - \frac{\tan \theta}{1 + \tan^2 \theta} + c = \tan^{-1} \sqrt{x} - \frac{\sqrt{x}}{1+x} + c \\
 \text{Now } f(3) - f(1) &= \tan^{-1}(\sqrt{3}) - \frac{\sqrt{3}}{1+3} - \tan^{-1}(1) + \frac{1}{2} \\
 &= \frac{\pi}{3} - \frac{\sqrt{3}}{4} - \frac{\pi}{4} + \frac{1}{2} \\
 &= \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}
 \end{aligned}$$

30. Answer (3)

$$\begin{aligned}
 &\int (e^{2x} + 2e^x - e^{-x} - 1) \cdot e^{(e^x + e^{-x})} dx \\
 \text{Let } e^x &= t, dx = \frac{dt}{t} \\
 &= \int \left(t^2 + 2t - \frac{1}{t} - 1 \right) e^{\left(t + \frac{1}{t} \right)} \cdot \frac{dt}{t} \\
 &= \int \left[\frac{(t^2 - 1)(t + 1)}{t^2} + 1 \right] e^{\left(t + \frac{1}{t} \right)} dt \\
 &= \int \underbrace{(t+1)}_I \underbrace{\left(1 - \frac{1}{t^2} \right)}_{II} e^{\left(t + \frac{1}{t} \right)} dt + \int e^{\left(t + \frac{1}{t} \right)} dt \\
 &= (t+1) e^{\left(t + \frac{1}{t} \right)} - \int e^{\left(t + \frac{1}{t} \right)} dt + \int e^{\left(t + \frac{1}{t} \right)} dt \\
 &= (e^x + 1) \cdot e^{(e^x + e^{-x})} + c
 \end{aligned}$$

So, $g(x) = 1 + e^x$ and $g(0) = 2$

31. Answer (3)

$$\begin{aligned}
 \text{Let } \sin \theta &= t \\
 &\Rightarrow \int \frac{dt}{5 + 7t - 2 + 2t^2} \\
 &\Rightarrow \frac{1}{2} \int \frac{dt}{\left(t + \frac{7}{4} \right)^2 - \left(\frac{5}{4} \right)^2} \\
 &= \frac{1}{5} \ln \left| \frac{t + \frac{1}{2}}{t + 3} \right| + c \\
 &= \frac{1}{5} \ln \left| \frac{2t + 1}{t + 3} \right| + c
 \end{aligned}$$

$$\therefore B(\theta) = \frac{2 \sin \theta + 1}{2(\sin \theta + 3)} \text{ and } A = \frac{1}{5}$$

$$\Rightarrow \frac{B(\theta)}{A} = \frac{5(2 \sin \theta + 1)}{(\sin \theta + 3)}$$

32. Answer (4)

$$\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = \int \frac{\cos x - \sin x}{\sqrt{9 - (\sin x + \cos x)^2}} dx$$

$$\text{let } \sin x + \cos x = t$$

$$(\cos x - \sin x)dx = dt$$

$$= \int \frac{dt}{\sqrt{9 - t^2}}$$

$$= \sin^{-1}\left(\frac{t}{3}\right) + c$$

$$= \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c$$

$$\text{Hence } (a, b) = (1, 3)$$

33. Answer (2)

$$\int \frac{\sin \theta (2 \sin \theta) \cos \theta (\sin^6 \theta + \sin^4 \theta + \sin^2 \theta)}{\sqrt{2 \sin^4 \theta + 3 \sin^2 \theta + 6}} \cdot \frac{1}{1 - (1 - 2 \sin^2 \theta)} d\theta$$

$$\text{Put } \sin \theta = t$$

$$\Rightarrow \cos \theta d\theta = dt$$

$$\Rightarrow \int \frac{t^2 (t^6 + t^4 + t^2) \sqrt{2t^4 + 3t^2 + 6}}{t^2} dt$$

$$\int (t^5 + t^3 + t) \sqrt{2t^6 + 3t^4 + 6t^2} dt$$

$$\text{Put } 2t^6 + 3t^4 + 6t^2 = k$$

$$\Rightarrow 12(t^5 + t^3 + t) dt = dk$$

$$\Rightarrow \frac{1}{12} \int \sqrt{k} dk$$

$$\Rightarrow \frac{2k^{\frac{3}{2}}}{12 \cdot 3}$$

$$\Rightarrow \frac{1}{18} (2 \sin^6 \theta + 3 \sin^4 \theta + 6 \sin^2 \theta)^{\frac{3}{2}} + C$$

$$= \frac{1}{18} (11 - 18 \cos^2 \theta + 9 \cos^4 \theta - 2 \sin^6 \theta)^{\frac{3}{2}} + C$$

34. Answer (1)

$$I = \int \frac{e^{3 \ln 2x} + 5e^{2 \ln 2x}}{e^{4 \ln x} + 5e^{3 \ln x} - 7e^{2 \ln x}} dx$$

$$I = \int \frac{(2x)^3 + 5(2x)^2}{x^4 + 5x^3 - 7x^2} dx = \int \frac{8x + 20}{x^2 + 5x - 7} dx$$

$$\text{Let } x^2 + 5x - 7 = t$$

$$(2x + 5)dx = dt$$

$$I = 4 \int \frac{dt}{t} = 4 \ln |t| + c$$

$$I = 4 \ln |x^2 + 5x - 7| + c$$

35. Answer (6)

$$I = \int \frac{x^2 - 1}{(x^4 + 3x^2 + 1) \tan^{-1}\left(\frac{x^2 + 1}{x}\right)} dx + \int \frac{1}{x^4 + 3x^2 + 1} dx$$

$$I = I_1 + I_2 \quad \dots(i)$$

$$\text{For } I_1, \text{ Let } \tan^{-1} \frac{x^2 + 1}{x} = t$$

$$I_1 = \int \frac{1}{t} dt = \ln t = \ln \left| \tan^{-1} \left(\frac{x^2 + 1}{x} \right) \right| + C_1$$

$$I_2 = \frac{1}{2} \int \frac{(x^2 + 1) - (x^2 - 1)}{x^4 + 3x^2 + 1} dx$$

$$= \frac{1}{2} \int \frac{x^2 + 1}{x^4 + 3x^2 + 1} - \frac{1}{2} \int \frac{x^2 - 1}{x^4 + 3x^2 + 1} dx$$

Divide Nr and Dr by x^2

$$= \frac{1}{2} \int \frac{1 + \frac{1}{x^2}}{\left(x - \frac{1}{x}\right)^2 + 5} - \frac{1}{2} \int \frac{1 - \frac{1}{x^2}}{\left(x + \frac{1}{x}\right)^2 + 1} dx$$

$$= \frac{1}{2\sqrt{5}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{5}x} \right) - \frac{1}{2} \tan^{-1} \left(\frac{x^2 + 1}{x} \right)$$

$$\alpha = 1, \beta = \frac{1}{2\sqrt{5}}, \gamma = \frac{1}{\sqrt{5}}, \delta = -\frac{1}{2}$$

$$\text{Required value} = 10 \left(1 + \frac{1}{10} - \frac{1}{2} \right)$$

$$= 6$$

36. Answer (2)

$$\text{Put } (2x - 1)^2 + 5 = t^2$$

$$4(2x - 1)dx = 2t dt$$

$$\Rightarrow \int \frac{t \cos t}{t} \cdot dt$$

$$\Rightarrow \frac{1}{2} \sin t + c$$

$$\Rightarrow \frac{1}{2} \sin(\sqrt{(2x-1)^2 + 5}) + c$$

37. Answer (04)

$$\int \frac{5x^8 + 7x^6}{x^{14} \left(\frac{1}{x^5} + \frac{1}{x^7} + 2 \right)} dx$$

$$\Rightarrow \int \frac{5x^{-6} + 7x^{-8}}{\left(2 + \frac{1}{x^5} + \frac{1}{x^7} \right)^2} dx$$

$$\text{Put } 2 + \frac{1}{x^5} + \frac{1}{x^7} = t$$

$$(-5x^{-6} - 7x^{-8})dx = dt$$

$$\Rightarrow \int \frac{-dt}{t^2} = \frac{1}{t} + c$$

$$f(x) = \frac{x^7}{2x^7 + x^2 + 1} + c$$

$$f(0) = 0 \Rightarrow c = 0 \Rightarrow f(1) = \frac{1}{4} \Rightarrow k = 4$$

38. Answer (15)

$$\int \frac{dx}{\left(\left(x + \frac{1}{2} \right)^2 + \frac{3}{4} \right)^2} \quad \text{Let } x + \frac{1}{2} = \frac{\sqrt{3}}{2} \tan \theta$$

$$\int \frac{\frac{\sqrt{3}}{2} \sec^2 \theta}{\frac{9}{16} \sec^4 \theta} d\theta = \frac{8}{3\sqrt{3}} \int \cos^2 \theta d\theta = \frac{4}{3\sqrt{3}} \int 2 \cos^2 \theta d\theta$$

$$= \frac{4}{\sqrt{3}} \left(\theta + \frac{\sin 2\theta}{2} \right)$$

$$\tan \theta = \frac{2x+1}{\sqrt{3}} \Rightarrow \sin 2\theta = \frac{2x+1}{2(x^2+x+1)} \cdot \sqrt{3}$$

$$\text{So } \int \frac{dx}{(x^2+x+1)^2}$$

$$= \frac{4}{3\sqrt{3}} \left(\tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) + \frac{\sqrt{3}}{4} \frac{2x+1}{x^2+x+1} \right)$$

$$a = \frac{4}{3\sqrt{3}}, b = \frac{1}{3}$$

$$9(a+b) = 15$$

39. Answer (07)

$$\text{Write } 2e^x + 3e^{-x} = A(4e^x + 7e^{-x}) + B(4e^x - 7e^{-x})$$

Comparing both sides

$$4A + 4B = 2 \quad \dots(i)$$

$$7A - 7B = 3 \quad \dots(ii)$$

$$\text{on solving } A = \frac{13}{28} \text{ and } B = \frac{1}{28}$$

$$I = \int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}} dx = \int \left(\frac{\frac{13}{28}(4e^x + 7e^{-x}) + \frac{1}{28}(4e^x - 7e^{-x})}{4e^x + 7e^{-x}} \right) dx$$

$$= \frac{13}{28} x + \frac{1}{18} \ln(4e^x + 7e^{-x}) + C$$

$$\text{Comparing LHS and RHS gives } u = \frac{13}{2} \text{ and } v = \frac{1}{2}$$

$$\Rightarrow u + v = 7$$

40. Answer (4)

$$I = \int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}} dx$$

$$= \int \frac{dx}{(x+2)^2 \left(\frac{x-1}{x+2} \right)^{3/4}}$$

$$\text{Let } \frac{x-1}{x+2} = t \Rightarrow \frac{3dx}{(x+2)^2} = dt$$

$$\therefore I = \int \frac{1}{t^{3/4}} \cdot \frac{1}{3} dt$$

$$= \frac{1}{3} \left(\frac{t^{-3/4+1}}{-\frac{3}{4}+1} \right) + C$$

$$= \frac{4}{3} \left(t^{1/4} \right) + C$$

$$\therefore I = \frac{4}{3} \left(\frac{x-1}{x+2} \right)^{1/4} + C$$

41. Answer (3)

$$I = \int \frac{\sin x}{\cos^3 x + \sin^3 x} dx = \int \frac{\frac{\sin x}{\cos^3 x}}{\frac{\cos^3 x}{\cos^3 x} + \frac{\sin^3 x}{\cos^3 x}} \cdot dx$$

$$I = \int \frac{\tan x \cdot \sec^2 x}{1 + \tan^3 x} dx,$$

Put $\tan x = t$

$$\sec^2 x \cdot dx = dt$$

$$I = \int \frac{t}{1+t^3} dt$$

$$\frac{t}{1+t^3} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2-t}$$

$$t = A(1-t+t^2) + (1+t)(Bt+C)$$

By comparing coeff of x , x^2 and constant term,

$$A = -\frac{1}{3}, B = \frac{1}{3}, C = \frac{1}{3}$$

$$I = -\frac{1}{3} \int \frac{1}{1+t} dt + \frac{1}{3} \int \frac{t+1}{t^2-t+1} dt$$

$$I = -\frac{1}{3} \ln(1+t)$$

$$+ \frac{1}{6} \left[\int \frac{2t-1}{t^2-t+1} dt + 3 \int \frac{1}{t^2-t+1} dt \right]$$

$$I = -\frac{1}{3} \ln(1+t)$$

$$+ \frac{1}{6} \left[\log(t^2-t+1) + 3 \cdot \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2t-1}{\sqrt{3}} \right) \right] + C$$

$$I = -\frac{1}{3} \ln(1+\tan x)$$

$$+ \frac{1}{6} \cdot \log(\tan^2 x - \tan x + 1) + \frac{1}{\sqrt{3}} \cdot \tan^{-1} \left(\frac{2\tan x - 1}{\sqrt{3}} \right) + C$$

$$\alpha = -\frac{1}{3}, \beta = \frac{1}{6}, \gamma = \frac{1}{\sqrt{3}}$$

$$18(\alpha + \beta + \gamma^2)$$

$$18 \left(-\frac{1}{3} + \frac{1}{6} + \frac{1}{3} \right) = 3$$

42. Answer (4)

$$\cos x dy - (\sin x) y dx = 6x dx$$

$$\Rightarrow \int d(y \cos x) = \int 6x dx$$

$$\Rightarrow y \cos x = 3x^2 + C$$

$$\text{As } y\left(\frac{\pi}{3}\right) = 0 \Rightarrow (0) \times \left(\frac{1}{2}\right) = \frac{3\pi^2}{9} + C \Rightarrow C = -\frac{\pi^2}{3}$$

$$\Rightarrow y \cos x = 3x^2 - \frac{\pi^2}{3}$$

$$\text{For } y\left(\frac{\pi}{6}\right)$$

$$y \frac{\sqrt{3}}{2} = \frac{3\pi^2}{36} - \frac{\pi^2}{3}$$

$$\frac{\sqrt{3}y}{2} = \frac{-3\pi^2}{12} \Rightarrow y = \frac{-\pi^2}{2\sqrt{3}}$$

43. Answer (4)

$$I = \int \left\{ e^{\left(x+\frac{1}{x}\right)} + x \left(1 - \frac{1}{x^2}\right) e^{\frac{x+1}{x}} \right\} dx$$

$$= x \cdot e^{\frac{x+1}{x}} + c$$

$$\text{As } \int (xf'(x) + f(x)) dx = xf(x) + c$$

44. Answer (1)

$$\therefore \int \frac{1}{x} \sqrt{\frac{1-x}{1+x}} dx = g(x) + c$$

$$\int_1^2 \frac{1}{x} \sqrt{\frac{1-x}{1+x}} dx = g\left(\frac{1}{2}\right) - g(1)$$

$$\therefore g\left(\frac{1}{2}\right) = \int_1^2 \frac{1}{x} \sqrt{\frac{1-x}{1+x}} dx$$

$$\cot x = \cos 2\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{\cos 2\theta} \cdot \frac{\sin \theta}{\cos \theta} (-2 \sin 2\theta) d\theta$$

$$= - \int_0^{\frac{\pi}{6}} \frac{4 \sin^2 \theta}{\cos 2\theta} d\theta$$

$$= 2 \int_0^{\frac{\pi}{6}} \frac{(1 - 2 \sin^2 \theta) - 1}{\cos 2\theta} d\theta$$

$$= 2 \int_0^{\frac{\pi}{6}} (1 - \sec 2\theta) d\theta$$

$$= \frac{\pi}{3} - 2 \cdot \frac{1}{2} [\ln |\sec 2\theta + \tan 2\theta|]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{3} - [\ln |2 + \sqrt{3}| - \ln 1]$$

$$= \frac{\pi}{3} + \ln \left(\frac{1}{2 + \sqrt{3}} \right)$$

$$= \frac{\pi}{3} + \ln \left| \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \right|$$

45. Answer (2)

$$I = \int \frac{e^x (x^2 + 1)}{(x+1)^2} dx = f(x)e^x + c$$

$$= \int \frac{e^x (x^2 - 1 + 1 + 1)}{(x+1)^2} dx$$

$$= \int e^x \left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2} \right] dx$$

$$= e^x \left(\frac{x-1}{x+1} \right) + c$$

$$\therefore f(x) = \frac{x-1}{x+1}$$

$$f(x) = 1 - \frac{2}{x+1}$$

$$f'(x) = 2 \left(\frac{1}{x+1} \right)^2$$

$$f''(x) = -4 \left(\frac{1}{x+1} \right)^3$$

$$f'''(x) = \frac{12}{(x+1)^4}$$

$$\text{for } x = 1$$

$$f'''(1) = \frac{12}{2^4} = \frac{12}{16} = \frac{3}{4}$$

46. Answer (1)

$$I(x) = \int \frac{\sec^2 x - 2022}{\sin^{2022} x} dx$$

$$= \int (\sec^2 x \cdot \sin^{-2022} x - 2022 \sin^{-2022} x) dx$$

$$= \sin^{-2022} x \tan x + \int 2022 \sin^{-2023} x \cos x \cdot \tan x dx$$

$$- \int 2022 \sin^{-2022} x dx + c$$

$$I(x) = \sin^{-2022} x \tan x + c$$

$$\therefore I\left(\frac{\pi}{4}\right) = 2^{1011} \Rightarrow c = 2^{1011} - 2^{1011} = 0$$

$$\therefore I\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)^{2022} \sqrt{3}, I\left(\frac{\pi}{6}\right) = 2^{2022} \frac{1}{\sqrt{3}}$$

$$\text{So, option (1) : } \frac{3^{1010} 2^{2022}}{3^{1011}} \cdot \sqrt{3} - \frac{2^{2022}}{\sqrt{3}} = 0$$

$\therefore$ Option (1) is correct

47. Answer (4)

$$\int \left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2} \right) dx = \frac{xg(x)}{e^x + 1} + c$$

Differentiating on both sides

$$\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2}$$

$$= \frac{(e^x + 1) \left(g(x) + xg'(x) \right) - xg(x)e^x}{(e^x + 1)^2}$$

$$= \frac{g(x)[e^x + 1 - xe^x]}{(e^x + 1)^2} + \frac{xg'(x)(e^x + 1)}{(e^x + 1)^2}$$

$$= \frac{x(\cos x - \sin x)}{e^x + 1} = \frac{xg'(x)}{e^x + 1}$$

$$\Rightarrow g'(x) = \cos x - \sin x > 0 \text{ in } \left(0, \frac{\pi}{4}\right)$$

$$g(x) \text{ is increasing in } \left(0, \frac{\pi}{4}\right) \Rightarrow A \text{ is wrong}$$

$$\text{Now, } g''(x) = -\sin x - \cos x < 0 \text{ in } \left(0, \frac{\pi}{4}\right)$$

$$\Rightarrow g(x) \text{ is increasing in } \left(0, \frac{\pi}{4}\right) \Rightarrow B \text{ is wrong}$$

$$\text{Let } h(x) = g(x) + g'(x)$$

$$\Rightarrow h'(x) = g'(x) + g''(x) = -2\sin x < 0 \text{ in } x \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow g + g' \text{ is decreasing in } \left(0, \frac{\pi}{2}\right) \Rightarrow C \text{ is wrong}$$

$$\text{Let } J(x) = g(x) - g'(x)$$

$$J'(x) = g'(x) - g''(x) = 2\cos x > 0 \text{ in } \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow g - g' \text{ is increasing in } \left(0, \frac{\pi}{2}\right) \Rightarrow \text{correct}$$

48. Answer (A)

$$= \int \frac{\left(1 - \frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(1 + \frac{2}{\sqrt{3}} \sin 2x\right)} dx$$

$$= \int \frac{\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \sqrt{2} \sin\left(\frac{\pi}{4} - x\right)}{\left(\frac{2}{\sqrt{3}}\right) \left(\sin \frac{\pi}{3} + \sin 2x\right)} dx$$

$$= \int \frac{\frac{(\sqrt{3}-1)}{\sqrt{2}} \sin\left(\frac{\pi}{4} - x\right)}{\left(\sin \frac{\pi}{3} + \sin 2x\right)} dx$$

$$= \int \frac{\frac{\sqrt{3}-1}{2\sqrt{2}} \sin\left(\frac{\pi}{4} - x\right)}{\sin\left(\frac{\pi}{6} + x\right) \cos\left(\frac{\pi}{6} - x\right)} dx$$

$$= \frac{1}{2} \int \frac{2 \sin \frac{\pi}{12} \sin\left(\frac{\pi}{4} - x\right)}{\sin\left(\frac{\pi}{6} + x\right) \cos\left(\frac{\pi}{6} - x\right)} dx$$

$$= \frac{1}{2} \int \frac{\cos\left(\frac{\pi}{6} - x\right) - \cos\left(\frac{\pi}{3} - x\right)}{\sin\left(\frac{\pi}{6} + x\right) \cos\left(\frac{\pi}{6} - x\right)} dx$$

$$= \frac{1}{2} \left[\int \operatorname{cosec}\left(\frac{\pi}{6} + x\right) dx - \int \sec\left(\frac{\pi}{6} - x\right) dx \right]$$

$$= \frac{1}{2} \left[\ln \left| \tan\left(\frac{\pi}{12} + \frac{x}{2}\right) \right| - \int \operatorname{cosec}\left(\frac{\pi}{3} - x\right) dx \right]$$

$$= \frac{1}{2} \left[\ln \left| \tan\left(\frac{\pi}{12} + \frac{x}{2}\right) \right| - \ln \left| \frac{\pi}{6} + \frac{x}{2} \right| \right] + C$$

$$= \frac{1}{2} \ln \left| \frac{\tan\left(\frac{\pi}{12} + \frac{x}{2}\right)}{\tan\left(\frac{\pi}{6} + \frac{x}{2}\right)} \right| + C$$