

Chapter 25

Vector Algebra

- The projections of a vector on the three coordinate axis are 6, -3, 2 respectively. The direction cosines of the vector are **[AIEEE-2009]**
 - $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$
 - $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$
 - $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$
 - 6, -3, 2
- If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{u}, p\vec{v}, p\vec{w}] - [p\vec{v}, \vec{w}, q\vec{u}] - [2\vec{w}, q\vec{v}, q\vec{u}] = 0$ holds for **[AIEEE-2009]**
 - Exactly two values of (p, q)
 - More than two but not all values of (p, q)
 - All values of (p, q)
 - Exactly one value of (p, q)
- Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$ is **[AIEEE-2010]**
 - $-\hat{i} + \hat{j} - 2\hat{k}$
 - $2\hat{i} - \hat{j} + 2\hat{k}$
 - $\hat{i} - \hat{j} - 2\hat{k}$
 - $\hat{i} + \hat{j} - 2\hat{k}$
- If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$ **[AIEEE-2010]**
 - (-3, 2)
 - (2, -3)
 - (-2, 3)
 - (3, -2)
- Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a} + 3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a} + 3\vec{b} + 6\vec{c}$ is **[AIEEE-2011]**
 - $\vec{0}$
 - $\vec{a} + \vec{c}$
 - $\vec{a}$
 - $\vec{c}$
- If the vectors $p\hat{i} + \hat{j} + \hat{k}, \hat{i} + q\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + r\hat{k}$ ($p \neq q \neq r \neq 1$) are coplanar, then the value of $pqr - (p + q + r)$ is **[AIEEE-2011]**
 - 1
 - 2
 - 2
 - 0
- Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is **[AIEEE-2012]**
 - $\frac{\pi}{4}$
 - $\frac{\pi}{6}$
 - $\frac{\pi}{2}$
 - $\frac{\pi}{3}$
- Let $ABCD$ be a parallelogram such that $\vec{AB} = \vec{q}, \vec{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD , then $\vec{r}$ is given by **[AIEEE-2012]**
 - $\vec{r} = -\vec{q} + \left(\frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}} \right) \vec{p}$
 - $\vec{r} = \vec{q} - \left(\frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}} \right) \vec{p}$
 - $\vec{r} = -3\vec{q} + \frac{3(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})} \vec{p}$
 - $\vec{r} = 3\vec{q} - \frac{3(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})} \vec{p}$
- If the vectors $\vec{AB} = 3\hat{i} + 4\hat{k}$ and $\vec{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC , then the length of the median through A is **[JEE (Main)-2013]**
 - $\sqrt{18}$
 - $\sqrt{72}$
 - $\sqrt{33}$
 - $\sqrt{45}$

10. The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is
[JEE (Main)-2014]

- (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$
(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

11. If $[\vec{a} \times \vec{b} \vec{b} \times \vec{c} \vec{c} \times \vec{a}] = \lambda [\vec{a} \vec{b} \vec{c}]^2$ then λ is equal to
[JEE (Main)-2014]

- (1) 0 (2) 1
(3) 2 (4) 3

12. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3} |\vec{b}| |\vec{c}| \vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin \theta$ is
[JEE (Main)-2015]

- (1) $\frac{2\sqrt{2}}{3}$ (2) $\frac{-\sqrt{2}}{3}$
(3) $\frac{2}{3}$ (4) $\frac{-2\sqrt{3}}{3}$

13. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2} (\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is
[JEE (Main)-2016]

- (1) $\frac{\pi}{2}$ (2) $\frac{2\pi}{3}$
(3) $\frac{5\pi}{6}$ (4) $\frac{3\pi}{4}$

14. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30° . Then $\vec{a} \cdot \vec{c}$ is equal to
[JEE (Main)-2017]

- (1) 2 (2) 5
(3) $\frac{1}{8}$ (4) $\frac{25}{8}$

15. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u} \cdot \vec{b} = 24$, then $|\vec{u}|^2$ is equal to
[JEE (Main)-2018]

- (1) 336 (2) 315
(3) 256 (4) 84

16. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{c} = 4$, then $|\vec{c}|^2$ is equal to
[JEE (Main)-2019]

- (1) $\frac{17}{2}$ (2) $\frac{19}{2}$
(3) 9 (4) 8

17. Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$, then $\vec{b}$ is equal to
[JEE (Main)-2019]

- (1) $\sqrt{22}$
(2) $\sqrt{32}$
(3) 4
(4) 6

18. Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}$ and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is
[JEE (Main)-2019]

- (1) (1, 3, 1) (2) (1, 5, 1)
(3) $\left(\frac{1}{2}, 4, -2\right)$ (4) $\left(-\frac{1}{2}, 4, 0\right)$

19. Let $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is
[JEE (Main)-2019]

- (1) -3 (2) -4
(3) 3 (4) 4

20. Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is [JEE (Main)-2019]
- (1) $-14\hat{i} + 5\hat{j}$ (2) $-10\hat{i} - 5\hat{j}$
 (3) $-14\hat{i} - 5\hat{j}$ (4) $-10\hat{i} + 5\hat{j}$
21. Let $\sqrt{3}\hat{i} + \hat{j}$, $\hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is [JEE (Main)-2019]
- (1) 3 (2) 1
 (3) 4 (4) 2
22. The sum of the distinct real values of μ , for which the vectors, $\mu\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + \mu\hat{j} + \hat{k}$, $\hat{i} + \hat{j} + \mu\hat{k}$ are co-planar, is [JEE (Main)-2019]
- (1) 2 (2) 1
 (3) -1 (4) 0
23. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$, the $|\alpha - \beta|$ is equal to [JEE (Main)-2019]
- (1) 90° (2) 45°
 (3) 30° (4) 60°
24. The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is: [JEE (Main)-2019]
- (1) $\sqrt{\frac{3}{2}}$ (2) $3\sqrt{6}$
 (3) $\frac{\sqrt{3}}{2}$ (4) $\sqrt{6}$
25. The vector equation of plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$ is [JEE (Main)-2019]
- (1) $\vec{r} \cdot (\hat{i} - \hat{k}) + 2 = 0$ (2) $\vec{r} \cdot (\hat{i} - \hat{k}) - 2 = 0$
 (3) $\vec{r} \times (\hat{i} - \hat{k}) + 2 = 0$ (4) $\vec{r} \times (\hat{i} + \hat{k}) + 2 = 0$
26. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a} \times \vec{b}| = r$ is possible if [JEE (Main)-2019]
- (1) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$ (2) $\sqrt{\frac{3}{2}} < r \leq 3\sqrt{\frac{3}{2}}$
 (3) $0 < r \leq \sqrt{\frac{3}{2}}$ (4) $r \geq 5\sqrt{\frac{3}{2}}$
27. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to [JEE (Main)-2019]
- (1) $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$ (2) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$
 (3) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (4) $3\hat{i} - 9\hat{j} - 5\hat{k}$
28. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is [JEE (Main)-2019]
- (1) $\frac{5\pi}{12}$ (2) $\frac{2\pi}{3}$
 (3) $\frac{\pi}{4}$ (4) $\frac{5\pi}{6}$
29. The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point (2, 3, -4) and parallel to the vector, $6\hat{i} + 3\hat{j} - 4\hat{k}$ is [JEE (Main)-2019]
- (1) 7 (2) $4\sqrt{3}$
 (3) 6 (4) $2\sqrt{13}$

30. If the volume of parallelopiped (non-zero) formed by the vectors $\hat{i} + \lambda\hat{j} + \hat{k}$, $\hat{j} + \lambda\hat{k}$ and $\lambda\hat{i} + \hat{k}$ is minimum, then λ is equal to **[JEE (Main)-2019]**

- (1) $\frac{1}{\sqrt{3}}$ (2) $-\sqrt{3}$
(3) $\sqrt{3}$ (4) $-\frac{1}{\sqrt{3}}$

31. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is **[JEE (Main)-2019]**

- (1) $4(-2\hat{i} - 2\hat{j} + \hat{k})$ (2) $4(2\hat{i} + 2\hat{j} - \hat{k})$
(3) $4(2\hat{i} + 2\hat{j} + \hat{k})$ (4) $4(2\hat{i} - 2\hat{j} - \hat{k})$

32. Let $\alpha \in R$ and the three vectors

$$\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k} \text{ and}$$

$$\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}. \text{ Then the set}$$

$$S = \{\alpha : \vec{a}, \vec{b} \text{ and } \vec{c} \text{ are coplanar}\}$$

[JEE (Main)-2019]

- (1) Contains exactly two numbers only one of which is positive
(2) Is singleton
(3) Contains exactly two positive numbers
(4) Is empty

33. A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ ($\alpha, \beta \in R$) lies in the plane of the vectors, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then

[JEE (Main)-2020]

- (1) $\vec{a} \cdot \hat{i} + 3 = 0$ (2) $\vec{a} \cdot \hat{i} + 1 = 0$
(3) $\vec{a} \cdot \hat{k} + 2 = 0$ (4) $\vec{a} \cdot \hat{k} + 4 = 0$

34. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. If $\lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ and $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ then the ordered pair, $(\lambda, \vec{d})$ is equal to **[JEE (Main)-2020]**

- (1) $\left(\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$ (2) $\left(-\frac{3}{2}, 3\vec{c} \times \vec{b}\right)$
(3) $\left(-\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$ (4) $\left(\frac{3}{2}, 3\vec{b} \times \vec{c}\right)$

35. Let the volume of a parallelopiped whose coterminal edges are given by $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ be 1 cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then $\cos\theta$ can be **[JEE (Main)-2020]**

- (1) $\frac{5}{7}$ (2) $\frac{5}{3\sqrt{3}}$
(3) $\frac{7}{6\sqrt{6}}$ (4) $\frac{7}{6\sqrt{3}}$

36. Let $a, b, c \in R$ be such that $a^2 + b^2 + c^2 = 1$.

$$\text{If } a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right), \text{ where}$$

$$\theta = \frac{\pi}{9}, \text{ then the angle between the vectors}$$

$$a\hat{i} + b\hat{j} + c\hat{k} \text{ and } b\hat{i} + c\hat{j} + a\hat{k} \text{ is}$$

[JEE (Main)-2020]

- (1) 0 (2) $\frac{\pi}{9}$
(3) $\frac{2\pi}{3}$ (4) $\frac{\pi}{2}$

37. Let x_0 be the point of local maxima of

$$f(x) = \vec{a} \cdot (\vec{b} \times \vec{c}), \text{ where}$$

$$\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$$

$$\text{and } \vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}. \text{ Then the value}$$

$$\text{of } \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} \text{ at } x = x_0 \text{ is}$$

[JEE (Main)-2020]

- (1) 14 (2) -30
(3) -4 (4) -22

38. If the volume of a parallelopiped, whose coterminal edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ ($n \geq 0$), is 158 cu.units, then

[JEE (Main)-2020]

- (1) $n = 7$ (2) $\vec{b} \cdot \vec{c} = 10$
(3) $n = 9$ (4) $\vec{a} \cdot \vec{c} = 17$

39. Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c} \cdot \vec{a} = 0$, then $\vec{c} \cdot \vec{b}$ is equal to
- [JEE (Main)-2020]
- (1) $-\frac{1}{2}$ (2) $-\frac{3}{2}$
(3) -1 (4) $\frac{1}{2}$
40. If the vectors, $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$, $\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$, and $\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$ ($a \in \mathbb{R}$) are coplanar and $3(\vec{p} \cdot \vec{q})^2 - \lambda |\vec{r} \times \vec{q}|^2 = 0$, then the value of λ is
- [JEE (Main)-2020]
41. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}, |\vec{b}| = 5, \vec{b} \cdot \vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to the vector $\vec{b} \times \vec{c}$, then $|\vec{a} \times (\vec{b} \times \vec{c})|$ is equal to
- [JEE (Main)-2020]
42. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$. Then $|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$ is equal to
- [JEE (Main)-2020]
43. Let the position vectors of points 'A' and 'B' be $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda : 1$ ($\lambda > 0$). If O is the origin and $|\vec{OB} \cdot \vec{OP} - 3|\vec{OA} \times \vec{OP}|^2| = 6$, then λ is equal to
- [JEE (Main)-2020]
44. If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $|\hat{i} \times (\vec{a} \times \hat{i})|^2 + |\hat{j} \times (\vec{a} \times \hat{j})|^2 + |\hat{k} \times (\vec{a} \times \hat{k})|^2$ is equal to
- [JEE (Main)-2020]
45. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}| = 2, |\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a} + \vec{b} - \vec{c}|$ is
- [JEE (Main)-2020]
46. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$ is
- [JEE (Main)-2020]
47. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x} + \vec{y}| = |\vec{x}|$ and $2\vec{x} + \lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is
- [JEE (Main)-2020]
48. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors, if $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $\vec{r} \cdot \vec{a}$ is equal to
- [JEE (Main)-2021]
49. A plane passes through the points A(1,2,3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, -1, 1), then the projection of $\vec{OP}$ on this plane is of length:
- [JEE (Main)-2021]
- (1) $\sqrt{\frac{2}{5}}$ (2) $\sqrt{\frac{2}{7}}$
(3) $\sqrt{\frac{2}{3}}$ (4) $\sqrt{\frac{2}{11}}$
50. Let $\vec{a} = \hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - a\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a} \cdot \vec{b}$ is equal to
- [JEE (Main)-2021]
51. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a} \times (\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b})))$ is equal to :
- [JEE (Main)-2021]
- (1) $\vec{a} \times \vec{b}$ (2) $\vec{0}$
(3) $\frac{1}{2}|\vec{a}|^4 \vec{b}$ (4) $|\vec{a}|^4 \vec{b}$
52. If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is :
- [JEE (Main)-2021]
- (1) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ (2) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$
(3) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$ (4) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

53. Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β) , $(0, \beta)$ and $(0, 0)$ is equal to : **[JEE (Main)-2021]**

- (1) $\frac{1}{2}$ (2) $2\sqrt{2}$
(3) 1 (4) $\frac{1}{\sqrt{2}}$

54. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$. If $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}$, $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = 3$ and $\alpha \in \mathbb{R}$, then the value of $\alpha + |\vec{r}|^2$ is equal to : **[JEE (Main)-2021]**

- (1) 15 (2) 13
(3) 9 (4) 11

55. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. If $\vec{c} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 8$, then the value of $\vec{c} \cdot (\vec{a} \times \vec{b})$ is equal to : **[JEE (Main)-2021]**

[JEE (Main)-2021]

56. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$. If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}$, $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$, then $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$ is equal to : **[JEE (Main)-2021]**

- (1) 10 (2) 13
(3) 8 (4) 12

57. If $\vec{a} = \alpha\hat{i} + \beta\hat{j} + 3\hat{k}$,
 $\vec{b} = -\beta\hat{i} - \alpha\hat{j} - \hat{k}$ and
 $\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$

such that $\vec{a} \cdot \vec{b} = 1$ and $\vec{b} \cdot \vec{c} = -3$, then $\frac{1}{3}((\vec{a} \times \vec{b}) \cdot \vec{c})$ is equal to : **[JEE (Main)-2021]**

[JEE (Main)-2021]

58. Let O be the origin. Let $\vec{OP} = x\hat{i} + y\hat{j} - \hat{k}$ and $\vec{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}$, $x, y \in \mathbb{R}$, $x > 0$, be such that $|\vec{PQ}| = \sqrt{20}$ and the vector $\vec{OP}$ is perpendicular to $\vec{OQ}$. If $\vec{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}$, $z \in \mathbb{R}$, is coplanar with $\vec{OP}$ and $\vec{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to : **[JEE (Main)-2021]**

- (1) 7 (2) 9
(3) 2 (4) 1

59. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $(3\hat{i} + 2\hat{j} - \hat{k})$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to : **[JEE (Main)-2021]**

[JEE (Main)-2021]

60. A vector $\vec{a}$ has components $3p$ and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components $p + 1$ and $\sqrt{10}$, then a value of p is equal to : **[JEE (Main)-2021]**

[JEE (Main)-2021]

- (1) -1 (2) 1
(3) $\frac{4}{5}$ (4) $-\frac{5}{4}$

61. In a triangle ABC, if $|\vec{BC}| = 8$, $|\vec{CA}| = 7$, $|\vec{AB}| = 10$, then the projection of the vector $\vec{AB}$ on $\vec{AC}$ is equal to : **[JEE (Main)-2021]**

[JEE (Main)-2021]

- (1) $\frac{115}{16}$ (2) $\frac{25}{4}$
(3) $\frac{127}{20}$ (4) $\frac{85}{14}$

62. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))$ and $\vec{a}$ is equal to : **[JEE (Main)-2021]**

[JEE (Main)-2021]

- (1) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (2) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(3) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$ (4) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

63. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $|\vec{c}|$ is : **[JEE (Main)-2021]**

[JEE (Main)-2021]

- (1) 4 (2) $\frac{2}{3}$
(3) $\frac{3}{2}$ (4) 3

64. Let $\vec{a}, \vec{b}, \vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ , with the vector $\vec{a} + \vec{b} + \vec{c}$. Then $36 \cos^2 \theta$ is equal to _____. [JEE (Main)-2021]

65. In a triangle ABC, if $|\vec{BC}| = 3, |\vec{CA}| = 5$ and $|\vec{BA}| = 7$, then the projection of the vector $\vec{BA}$ on $\vec{BC}$ is equal to : [JEE (Main)-2021]

- (1) $\frac{13}{2}$ (2) $\frac{19}{2}$
(3) $\frac{15}{2}$ (4) $\frac{11}{2}$

66. For $p > 0$, a vector $\vec{v}_2 = 2\hat{i} + (p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1 = \sqrt{3}p\hat{i} + \hat{j}$ by an angle θ about origin in counter clockwise direction. If

$$\tan \theta = \frac{(\alpha\sqrt{3} - 2)}{(4\sqrt{3} + 3)}, \text{ then the value of } \alpha \text{ is equal to}$$

_____. [JEE (Main)-2021]

67. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, and $|\vec{a}| = \sqrt{10}$.

Then a possible value of $[\vec{a} \vec{b} \vec{c}] + [\vec{a} \vec{b} \vec{d}] + [\vec{a} \vec{c} \vec{d}]$ is equal to [JEE (Main)-2021]

- (1) -42 (2) -40
(3) -38 (4) -29

68. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}$ and $|\vec{a}| = 2$. Then which one of the following is not true? [JEE (Main)-2021]

- (1) $\vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} - \vec{c})) = \vec{0}$
(2) Projection of $\vec{a}$ on $(\vec{b} \times \vec{c})$ is 2
(3) $[\vec{a} \vec{b} \vec{c}] + [\vec{c} \vec{a} \vec{b}] = 8$
(4) $|3\vec{a} + \vec{b} - 2\vec{c}|^2 = 51$

69. Let the vectors

$$(2+a+b)\hat{i} + (a+2b+c)\hat{j} - (b+c)\hat{k},$$

$$(1+b)\hat{i} + 2b\hat{j} - b\hat{k} \text{ and}$$

$$(2+b)\hat{i} + 2b\hat{j} + (1-b)\hat{k}, a, b, c \in \mathbb{R}$$

be co-planar. Then which of the following is true?

[JEE (Main)-2021]

- (1) $3c = a + b$ (2) $2b = a + c$
(3) $2a = b + c$ (4) $a = b + 2c$

70. Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors.

If a vector $\vec{r} = (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p} + \vec{q})$ and $(\vec{p} - \vec{q})$, and $|\vec{r}| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to _____.

[JEE (Main)-2021]

71. Let a, b and c be distinct positive numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are co-planar, then c is equal to [JEE (Main)-2021]

- (1) $\frac{2}{\frac{1}{a} + \frac{1}{b}}$ (2) $\sqrt{ab}$
(3) $\frac{a+b}{2}$ (4) $\frac{1}{a} + \frac{1}{b}$

72. If $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then $|\vec{a} \cdot \vec{b}|$ is equal to [JEE (Main)-2021]

- (1) 3 (2) 6
(3) 4 (4) 5

73. If $(\vec{a} + 3\vec{b})$ is perpendicular to $(7\vec{a} - 5\vec{b})$ and $(\vec{a} - 4\vec{b})$ is perpendicular to $(7\vec{a} - 2\vec{b})$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is _____.

[JEE (Main)-2021]

74. Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Then the vector product $(\vec{a} + \vec{b}) \times ((\vec{a} - \vec{b}) \times \vec{b})$ is equal to [JEE (Main)-2021]

- (1) $7(34\hat{i} - 5\hat{j} + 3\hat{k})$ (2) $7(30\hat{i} - 5\hat{j} + 7\hat{k})$
(3) $5(30\hat{i} - 5\hat{j} + 7\hat{k})$ (4) $5(34\hat{i} - 5\hat{j} + 3\hat{k})$

75. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b}$ and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is l , then the value of $3l^2$ is equal to _____.

[JEE (Main)-2021]

76. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$. If magnitudes of the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are $\sqrt{2}, 1$ and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta \left(0 < \theta < \frac{\pi}{2}\right)$, then the value of $1 + \tan \theta$ is equal to

[JEE (Main)-2021]

(1) 2 (2) $\frac{\sqrt{3}+1}{\sqrt{3}}$

(3) 1 (4) $\sqrt{3}+1$

77. Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i} - 2\hat{j} + \hat{k}$, where α and β are integers. If $\vec{a} \cdot \vec{b} = -1$ and $\vec{b} \cdot \vec{c} = 10$, $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is equal to _____.

[JEE (Main)-2021]

78. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$, then $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal to

[JEE (Main)-2021]

(1) 6 (2) -2
(3) -6 (4) 2

79. If the projection of the vector $\hat{i} + 2\hat{j} + \hat{k}$ on the sum of the two vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $-\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is 1, then λ is equal to _____.

[JEE (Main)-2021]

80. Let $\vec{a} = \hat{i} + 5\hat{j} + \alpha\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that, $|\vec{b} \times \vec{c}| = 5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is _____.

[JEE (Main)-2021]

81. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a} + 3\vec{b}| = |3\vec{a} + 3\vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60° . If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to :

[JEE (Main)-2021]

(1) 8 (2) 4
(3) 5 (4) 6

82. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies

$$\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = \vec{0},$$

then $\vec{r}$ is equal to

[JEE (Main)-2021]

(1) $\frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$ (2) $\frac{1}{2}(\vec{a} + \vec{b} + 2\vec{c})$

(3) $\frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$ (4) $\frac{1}{3}(2\vec{a} + \vec{b} - \vec{c})$

83. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i} + 2\hat{j} - \hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to _____.

[JEE (Main)-2021]

84. Let $\hat{a}, \hat{b}$ be unit vectors. If $\vec{c}$ be a vector such that the angle between $\hat{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $|6\vec{c}|^2$ is equal to:

[JEE (Main)-2022]

(1) $6(3 - \sqrt{3})$ (2) $3 + \sqrt{3}$

(3) $6(3 + \sqrt{3})$ (4) $6(\sqrt{3} + 1)$

85. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that $|(\hat{a} + \hat{b}) + 2(\hat{a} \times \hat{b})| = 2$. If $\theta \in (0, \pi)$ is the angle between $\hat{a}$ and $\hat{b}$, then among the statements:

(S1) : $2|\hat{a} \times \hat{b}| = |\hat{a} - \hat{b}|$

(S2) : The projection of $\hat{a}$ on $(\hat{a} + \hat{b})$ is $\frac{1}{2}$

[JEE (Main)-2022]

- (1) Only (S1) is true
(2) Only (S2) is true
(3) Both (S1) and (S2) are true
(4) Both (S1) and (S2) are false

86. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $a_i > 0, i = 1, 2, 3$ be a vector which makes equal angles with the coordinate axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{i} + 4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90° . If $\vec{a}, \vec{b}$ and x-axis are coplanar, then projection of a vector $\vec{b}$ on $3\hat{i} + 4\hat{j}$ is equal to: **[JEE (Main)-2022]**
- (1) $\sqrt{7}$ (2) $\sqrt{2}$
 (3) 2 (4) 7
87. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}| = 4, |\vec{b}| = 3$ and $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$. Then $\left|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})\right|^2 + 4(\vec{a} \cdot \vec{b})^2$ is equal to _____. **[JEE (Main)-2022]**
88. Let $\vec{b} = \hat{i} + \hat{j} + \lambda\hat{k}, \lambda \in \mathbb{R}$. If $\vec{a}$ is a vector such that $\vec{a} \times \vec{b} = 13\hat{i} - \hat{j} - 4\hat{k}$ and $\vec{a} \cdot \vec{b} + 21 = 0$, then $(\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{j}) + (\vec{b} + \vec{a}) \cdot (\hat{i} - \hat{k})$ is equal to _____. **[JEE (Main)-2022]**
89. If $\vec{a} \cdot \vec{b} = 1, \vec{b} \cdot \vec{c} = 2$ and $\vec{c} \cdot \vec{a} = 3$, then the value of $\left[\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{b} \times \vec{a})\right]$ is : **[JEE (Main)-2022]**
- (1) 0 (2) $-6\vec{a} \cdot (\vec{b} \times \vec{c})$
 (3) $12\vec{c} \cdot (\vec{a} \times \vec{b})$ (4) $-12\vec{b} \cdot (\vec{c} \times \vec{a})$
90. Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ be three given vectors. Let $\vec{v}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{2}{\sqrt{3}}$. If $\vec{v} \cdot \hat{j} = 7$, then $\vec{v} \cdot (\hat{i} + \hat{k})$ is equal to : **[JEE (Main)-2022]**
- (1) 6 (2) 7
 (3) 8 (4) 9
91. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1, 2, \dots, 10\}$ is : **[JEE (Main)-2022]**
- (1) 0 (2) 1
 (3) 2 (4) 3
92. Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute, $|\vec{a}| = 1$, and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$. If $\vec{c} = 2\sqrt{2}(\vec{a} \times \vec{b}) - 2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is **[JEE (Main)-2022]**
- (1) $\frac{\pi}{4}$ (2) $-\frac{\pi}{4}$
 (3) $\frac{5\pi}{6}$ (4) $\frac{3\pi}{4}$
93. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar vectors and $\vec{a} \cdot \vec{c} = 5, \vec{b} \perp \vec{c}$, then $122(c_1^2 + c_2^2 + c_3^2)$ is equal to _____. **[JEE (Main)-2022]**
94. Let, $\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$, where $\alpha \in \mathbb{R}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2 + 4)}$, then the value of $2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$ is equal to : **[JEE (Main)-2022]**
- (1) 10 (2) 7
 (3) 9 (4) 14
95. Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is : **[JEE (Main)-2022]**
- (1) $\frac{1}{3}$ (2) 1
 (3) $\frac{5}{3}$ (4) $\frac{7}{3}$

96. Let $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ where $\alpha, \beta \in \mathbf{R}$, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $\alpha + \beta$ is equal to : **[JEE (Main)-2022]**

- (1) 3 (2) 4
(3) 5 (4) 6

97. Let A, B, C be three points whose position vectors respectively are

$$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$$

$$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in \mathbf{R}$$

$$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$$

If α is the smallest positive integer for which $\vec{a}, \vec{b}, \vec{c}$ are non collinear, then the length of the median, in $\triangle ABC$, through A is: **[JEE (Main)-2022]**

- (1) $\frac{\sqrt{82}}{2}$ (2) $\frac{\sqrt{62}}{2}$
(3) $\frac{\sqrt{69}}{2}$ (4) $\frac{\sqrt{66}}{2}$

98. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} + (\vec{b} \times \vec{c}) = \vec{0}$ and $\vec{b} \cdot \vec{c} = 5$. Then the value of $3(\vec{c} \cdot \vec{a})$ is equal to _____.

[JEE (Main)-2022]

99. Let ABC be a triangle such that $\vec{BC} = \vec{a}$, $\vec{CA} = \vec{b}$, $\vec{AB} = \vec{c}$, $|\vec{a}| = 6\sqrt{2}$, $|\vec{b}| = 2\sqrt{3}$ and $\vec{b} \cdot \vec{c} = 12$. Consider the statements :

$$(S1) : |(\vec{a} \times \vec{b}) + (\vec{c} \times \vec{b})| - |\vec{c}| = 6(2\sqrt{2} - 1)$$

$$(S2) : \angle ACB = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

Then

[JEE (Main)-2022]

- (1) Both (S1) and (S2) are true
(2) Only (S1) is true
(3) Only (S2) is true
(4) Both (S1) and (S2) are false

100. Let $\vec{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$, $\alpha > 0$. If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{i} + 2\hat{j} - 2\hat{k}$ is 30, then α is equal to **[JEE (Main)-2022]**

- (1) $\frac{15}{2}$ (2) 8
(3) $\frac{13}{2}$ (4) 7

101. If the maximum value of a, for which the function $f_a(x) = \tan^{-1} 2x - 3ax + 7$ is non-decreasing in $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$, is $\bar{a}$, then $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ is equal to

[JEE (Main)-2022]

- (1) $8 - \frac{9\pi}{4(9 + \pi^2)}$ (2) $8 - \frac{4\pi}{9(4 + \pi^2)}$
(3) $8\left(\frac{1 + \pi^2}{9 + \pi^2}\right)$ (4) $8 - \frac{\pi}{4}$

102. Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} + 4\hat{k}$ be two vectors, such that $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to

- (1) 2 (2) $\frac{39}{5}$
(3) 9 (4) $\frac{46}{5}$

103. Let $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$. If $((\vec{a} \times \vec{b}) \times \hat{i}) \cdot \hat{k} = \frac{23}{2}$, then $|\vec{b} \times 2\hat{j}|$ is equal to

[JEE (Main)-2022]

- (1) 4 (2) 5
(3) $\sqrt{21}$ (4) $\sqrt{17}$

104. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors such that $\vec{a} \times \vec{b} = 4\vec{c}$, $\vec{b} \times \vec{c} = 9\vec{a}$ and $\vec{c} \times \vec{a} = \alpha\vec{b}$, $\alpha > 0$. If $|\vec{a}| + |\vec{b}| + |\vec{c}| = \frac{1}{36}$, then α is equal to _____.

[JEE (Main)-2022]

105. Let the vectors $\vec{a} = (1+t)\hat{i} + (1-t)\hat{j} + \hat{k}$, $\vec{b} = (1-t)\hat{i} + (1+t)\hat{j} + 2\hat{k}$ and $\vec{c} = t\hat{i} - t\hat{j} + \hat{k}$, $t \in \mathbf{R}$ be such that for $\alpha, \beta, \gamma \in \mathbf{R}$, $\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = \vec{0} \Rightarrow \alpha = \beta = \gamma = 0$. Then, the set of all values of t is

[JEE (Main)-2022]

- (1) A non-empty finite set
- (2) Equal to N
- (3) Equal to $\mathbf{R} - \{0\}$
- (4) Equal to $\mathbf{R}$

106. Let a vector $\vec{a}$ has magnitude 9. Let a vector $\vec{b}$ be such that for every $(x, y) \in \mathbf{R} \times \mathbf{R} - \{(0, 0)\}$, the vector $(x\vec{a} + y\vec{b})$ is perpendicular to the vector $(6y\vec{a} - 18x\vec{b})$. Then the value of $|\vec{a} \times \vec{b}|$ is equal to

[JEE (Main)-2022]

- (1) $9\sqrt{3}$
- (2) $27\sqrt{3}$
- (3) 9
- (4) 81

107. Let S be the set of all $a \in \mathbf{R}$ for which the angle between the vectors $\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a(\log_e b)\hat{k}$, ($b > 1$) is acute. Then S is equal to :

[JEE (Main)-2022]

- (1) $\left(-\infty, -\frac{4}{3}\right)$
- (2) ϕ
- (3) $\left(-\frac{4}{3}, 0\right)$
- (4) $\left(\frac{12}{7}, \infty\right)$

108. Let $\vec{a}, \vec{b}, \vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) + (\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) + (\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b}) = 168$, then $|\vec{a}| + |\vec{b}| + |\vec{c}|$ is equal to :

[JEE (Main)-2022]

- (1) 10
- (2) 14
- (3) 16
- (4) 18

109. Let $\vec{a}$ and $\vec{b}$ be two vectors such that

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2, \vec{a} \cdot \vec{b} = 3 \text{ and } |\vec{a} \times \vec{b}|^2 = 75.$$

Then $|\vec{a}|^2$ is equal to _____. [JEE (Main)-2022]

110. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that the angle

between them is $\frac{\pi}{4}$. If θ is the angle between the

vectors $(\hat{a} + \hat{b})$ and $(\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b}))$, then the value of $164 \cos^2 \theta$ is equal to :

- (1) $90 + 27\sqrt{2}$
- (2) $45 + 18\sqrt{2}$
- (3) $90 + 3\sqrt{2}$
- (4) $54 + 90\sqrt{2}$

[JEE (Main)-2022]

111. Let $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and let $\vec{b}$ be a vector such that $\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$ and $\vec{a} \cdot \vec{b} = 3$. Then the projection of $\vec{b}$ on the vector $\vec{a} - \vec{b}$ is :

[JEE (Main)-2022]

- (1) $\frac{2}{\sqrt{21}}$
- (2) $2\sqrt{\frac{3}{7}}$
- (3) $\frac{2}{3}\sqrt{\frac{7}{3}}$
- (4) $\frac{2}{3}$

112. Let $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. Let $\vec{c}$ be a vector satisfying $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \lambda\vec{c}$. If $\vec{b}$ and $\vec{c}$ are non-parallel, then the value of λ is [JEE (Main)-2022]

- (1) -5
- (2) 5
- (3) 1
- (4) -1

Chapter 25

Vector Algebra

1. Answer (2)

Direction ratios are $a = 6$, $b = -3$ and $c = 2$

Then direction cosines are

$$\frac{6}{\sqrt{36+9+4}}, \frac{-3}{\sqrt{36+9+4}}, \frac{2}{\sqrt{36+9+4}}$$

$$= \frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$$

2. Answer (4)

$$[3\vec{u} \quad p\vec{v} \quad p\vec{w}] - [p\vec{v} \quad \vec{w} \quad q\vec{u}] - [2\vec{w} \quad q\vec{v} \quad q\vec{u}]$$

$$= 3p^2[\vec{u} \cdot (\vec{v} \times \vec{w})] - pq[\vec{v} \cdot (\vec{w} \times \vec{u})] - 2q^2[\vec{w} \cdot (\vec{v} \times \vec{u})]$$

$$\Rightarrow (3p^2 - pq + 2q^2)[\vec{u} \cdot (\vec{v} \times \vec{w})] = 0$$

But $\vec{u} \cdot (\vec{v} \times \vec{w}) \neq 0$

$$\Rightarrow 3p^2 - pq + 2q^2 = 0$$

$$\Rightarrow p = q = 0$$

3. Answer (1)

We have

$$\vec{a} \times \vec{b} + \vec{c} = \vec{0}$$

$$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) + \vec{a} \times \vec{c} = \vec{0}$$

$$\Rightarrow (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} + \vec{a} \times \vec{c} = \vec{0}$$

$$\Rightarrow 3\vec{a} - 2\vec{b} + \vec{a} \times \vec{c} = \vec{0}$$

$$\Rightarrow 2\vec{b} = 3\vec{a} + \vec{a} \times \vec{c}; \vec{a} \times \vec{c} = -2\hat{i} - \hat{j} - \hat{k}$$

$$= 3\hat{j} - 3\hat{k} - 2\hat{i} - \hat{j} - \hat{k}$$

$$= -2\hat{i} + 2\hat{j} - 4\hat{k}$$

$$\Rightarrow \vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$$

4. Answer (1)

We have

$$\vec{a} \cdot \vec{b} = 2 - 4 + 2 = 0$$

$$\vec{a} \cdot \vec{c} = \lambda - 1 + 2\mu = 0$$

$$\vec{b} \cdot \vec{c} = 2\lambda + 4 + \mu = 0$$

$$\text{Thus } \lambda = 1 - 2\mu$$

$$\text{and } 2 - 4\mu + 4 + \mu = 0$$

$$\Rightarrow 3\mu = 6, \Rightarrow \mu = 2$$

$$\lambda = -3$$

$$(\lambda, \mu) = (-3, 2)$$

5. Answer (1)

$$\vec{c} = \mu(\vec{a} + 3\vec{b})$$

$$\vec{b} + 2\vec{c} = \lambda\vec{a}$$

$$\vec{b} + 2\mu(\vec{a} + 3\vec{b}) = \lambda\vec{a}$$

$$(1 + 6\mu)\vec{b} + (2\mu - \lambda)\vec{a} = \vec{0}$$

$$6\mu + 1 = 0, 2\mu = \lambda$$

$$\mu = -\frac{1}{6}, \lambda = -\frac{1}{3}$$

$$\text{Now, } \vec{c} = -\frac{1}{6}(\vec{a} + 3\vec{b}) = -\frac{1}{6}\vec{a} - \frac{1}{2}\vec{b}$$

6. Answer (2)

Given vectors $p\hat{i} + \hat{j} + \hat{k}, \hat{i} + q\hat{j} + \hat{k}, \hat{i} + \hat{j} + r\hat{k}$ to be coplanar

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$

$$p(qr - 1) - 1(r - 1) + (1 - q) = 0$$

$$pqr - p - r + 1 - q + 1 = 0$$

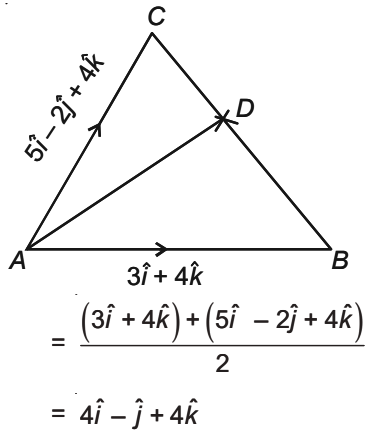
$$pqr - p - q - r = -2$$

7. Answer (4)

8. Answer (1)

9. Answer (3)

$$\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$



$$\therefore |\overrightarrow{AD}| = \sqrt{16+1+16} = \sqrt{33}$$

10. Answer (3)

$$l + m + n = 0$$

$$l^2 = m^2 + n^2$$

$$\text{Now, } (-m - n)^2 = m^2 + n^2$$

$$\Rightarrow mn = 0$$

$$m = 0 \text{ or } n = 0$$

$$\text{If } m = 0$$

$$\text{then } l = -n$$

$$l^2 + m^2 + n^2 = 1$$

Gives

$$\Rightarrow n = \pm \frac{1}{\sqrt{2}}$$

$$\text{i.e. } (l_1, m_1, n_1)$$

$$= \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right)$$

$$\therefore \cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

11. Answer (2)

L.H.S.

$$= (\vec{a} \times \vec{b}) \cdot [(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})]$$

$$= (\vec{a} \times \vec{b}) \cdot [(\vec{b} \times \vec{c} \cdot \vec{a})\vec{c} - (\vec{b} \times \vec{c} \cdot \vec{c})\vec{a}]$$

$$= (\vec{a} \times \vec{b}) \cdot [(\vec{b} \cdot \vec{c} \cdot \vec{a})\vec{c}] \quad [\because \vec{b} \times \vec{c} \cdot \vec{c} = 0]$$

$$= [\vec{a} \cdot \vec{b} \cdot \vec{c}] \cdot (\vec{a} \times \vec{b} \cdot \vec{c}) = [\vec{a} \cdot \vec{b} \cdot \vec{c}]^2$$

$$[\vec{a} \times \vec{b} \cdot \vec{b} \times \vec{c} \cdot \vec{c} \times \vec{a}] = [\vec{a} \cdot \vec{b} \cdot \vec{c}]^2$$

So $\lambda = 1$

12. Answer (1)

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a} = \frac{1}{3} |\vec{b}| |\vec{c}| \vec{a}$$

$$\therefore -(\vec{b} \cdot \vec{c}) = \frac{1}{3} |\vec{b}| |\vec{c}|$$

$$\therefore \cos \theta = -\frac{1}{3}$$

$$\therefore \sin \theta = \frac{2\sqrt{2}}{3}$$

13. Answer (3)

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c}) \text{ and}$$

after comparing

$$\vec{a} \cdot \vec{b} = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos \theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \frac{5\pi}{6}$$

14. Answer (1)

$$|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$$

$$\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| |\vec{c}| \sin 30^\circ = 3 \quad |\vec{a}| = 3 = |\vec{a} \times \vec{b}|$$

$$\Rightarrow |\vec{c}| = 2$$

$$|\vec{c} - \vec{a}| = 3$$

$$\Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2(\vec{a} \cdot \vec{c}) = 9$$

$$\vec{a} \cdot \vec{c} = \frac{9-3-2}{2} = 2$$

15. Answer (1)

Clearly, $\vec{u} = \lambda(\vec{a} \times (\vec{a} \times \vec{b}))$

$$\Rightarrow \vec{u} = \lambda((\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2 \vec{b})$$

$$\Rightarrow \vec{u} = \lambda(2\vec{a} - 14\vec{b}) = 2\lambda\{(2\hat{i} + 3\hat{j} - \hat{k}) - 7(\hat{j} + \hat{k})\}$$

$$\Rightarrow \vec{u} = 2\lambda(2\hat{i} - 4\hat{j} - 8\hat{k})$$

$$\text{as, } \vec{u} \cdot \vec{b} = 24$$

$$\Rightarrow 4\lambda(\hat{i} - 2\hat{j} - 4\hat{k}) \cdot (\hat{j} + \hat{k}) = 24$$

$$\Rightarrow \lambda = -1$$

$$\text{So, } \vec{u} = -4(\hat{i} - 2\hat{j} - 4\hat{k})$$

$$\Rightarrow |\vec{u}|^2 = 336$$

16. Answer (2)

$$|\vec{a} \times \vec{c}|^2 = |\vec{a}|^2 |\vec{c}|^2 - (\vec{a} \cdot \vec{c})^2$$

$$\Rightarrow |\vec{b}|^2 = 2|\vec{c}|^2 - 16$$

$$\Rightarrow 3 = 2|\vec{c}|^2 - 16$$

$$\Rightarrow |\vec{c}|^2 = \frac{19}{2}$$

17. Answer (4)

$$\text{Projection of } \vec{b} \text{ on } \vec{a} = \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} = \frac{b_1 + b_2 + 2}{4}$$

$$\text{According to question } \frac{b_1 + b_2 + 2}{2} = \sqrt{1+1+2} = 2$$

$$\Rightarrow b_1 + b_2 = 2 \quad \dots(1)$$

$$\text{Also } \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} = 0$$

$$\Rightarrow 8 + 5b_1 + b_2 + 2 = 0 \quad \dots(2)$$

From (1) and (2),

$$b_1 = -3, b_2 = 5$$

$$\Rightarrow \vec{b} = -3\hat{i} + 5\hat{j} + \sqrt{2}\hat{k}$$

$$|\vec{b}| = \sqrt{9+25+2} = 6$$

18. Answer (4)

$$\therefore \vec{b} = 2\vec{a}$$

$$\therefore 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k} = 4\hat{i} + 2\lambda_1\hat{j} + 6\hat{k}$$

$$\therefore 3 - \lambda_2 = 2\lambda_1 \quad \dots(1)$$

$\therefore \vec{a}$ is perpendicular to $\vec{c}$

$$\therefore 6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$$

$$2 + 2\lambda_1 + \lambda_3 - 1 = 0$$

$$2\lambda_1 + \lambda_3 + 1 = 0$$

$$\therefore \lambda_3 = -2\lambda_1 - 1 \quad \dots(2)$$

from equations (1) and (2) one of possible value of

$$\lambda_1 = -\frac{1}{2}, \lambda_2 = 4 \text{ and } \lambda_3 = 0$$

19. Answer (2)

$\vec{\alpha}$ and $\vec{\beta}$ are collinear

$$\Rightarrow \vec{\alpha} = t\vec{\beta}$$

$$(\lambda - 2)\vec{a} + \vec{b} = t((4\lambda - 2)\vec{a} + 3\vec{b})$$

$$(\lambda - 2 - t(4\lambda - 2))\vec{a} + \vec{b}(1 - 3t) = \vec{0}$$

$\vec{a}$ and $\vec{b}$ are non-collinear

$$\Rightarrow \lambda - 2 - t(4\lambda - 2) = 0, 1 - 3t = 0$$

$$\Rightarrow t = \frac{1}{3} \text{ and } \lambda - 2 - \frac{1}{3}(4\lambda - 2) = 0$$

$$3\lambda - 6 - 4\lambda + 2 = 0$$

$$\boxed{\lambda = -4}$$

Option (2) is correct

20. Answer (4)

For coplanar vectors,

$$\begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & (\lambda^2 - 1) \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - \lambda - 16 + 2(8 - \lambda^2 + 1) + 4(4 - 2\lambda) = 0$$

$$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$$

$$\text{i.e., } (\lambda - 2)(\lambda - 3)(\lambda + 3) = 0$$

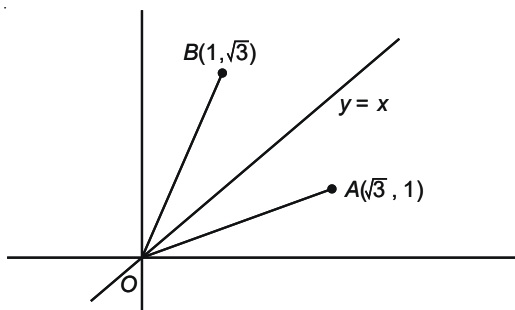
$$\text{For } \lambda = 2, \vec{c} = 2\hat{i} + 4\hat{j} + 3\hat{k}$$

$$\therefore \vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = -10\hat{i} + 5\hat{j}$$

For $\lambda = 3$ or -3 , $\vec{a} \times \vec{c} = \vec{0}$ (Rejected)

21. Answer (2)

By observing point A, B angle bisector of acute angle, OA and OB would be $y = x$



Now, according to question

$$\left| \frac{\beta - (1 - \beta)}{\sqrt{2}} \right| = \frac{3}{\sqrt{2}}$$

$$\Rightarrow 2\beta = \pm 3 + 1$$

$$\beta = 2 \text{ or } \beta = -1$$

22. Answer (3)

For coplanar vectors,

$$\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$$

$$\Rightarrow \mu(\mu^2 - 1) + 1 - \mu + 1 - \mu = 0$$

$$\Rightarrow (1 - \mu)[2 - \mu(\mu + 1)] = 0$$

$$\Rightarrow (1 - \mu)[\mu^2 + \mu - 2] = 0$$

$$\Rightarrow \mu = 1, -2$$

$$\text{Sum of all real values} = 1 - 2 = -1$$

23. Answer (3)

$$\therefore |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$\text{Now } \vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2} \vec{b}$$

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{1}{2} \vec{b}$$

$$\therefore \vec{a} \cdot \vec{c} = \frac{1}{2} \text{ and } \vec{a} \cdot \vec{b} = 0$$

$$|\vec{a}| |\vec{c}| \cos \beta = \frac{1}{2} \text{ and } \alpha = 90^\circ$$

$$\beta = 60^\circ$$

$$\therefore |\alpha - \beta| = |90^\circ - 60^\circ| = 30^\circ$$

24. Answer (1)

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ vector perpendicular to $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$$

Projection of vector $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ on $\vec{a} \times \vec{b}$ is

$$= \frac{|\vec{c} \cdot (\vec{a} \times \vec{b})|}{|\vec{a} \times \vec{b}|} = \frac{|2 - 6 + 1|}{\sqrt{6}} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

25. Answer (1)

Equation of the plane passing through the line of intersection of $x + y + z = 1$ and $2x + 3y + 4z = 5$ is

$$(2x + 3y + 4z - 5) + \lambda(x + y + z - 1) = 0$$

$$(2 + \lambda)x + (3 + \lambda)y + (4 + \lambda)z + (-5 - \lambda) = 0 \dots (i)$$

(i) is perpendicular to $x - y + z = 0$

$$\Rightarrow (2 + \lambda)(1) + (3 + \lambda)(-1) + (4 + \lambda)(1) = 0$$

$$2 + \lambda - 3 - \lambda + 4 + \lambda = 0$$

$$\lambda = -3$$

$\Rightarrow$ Equation of required plane is

$$-x + z - 2 = 0$$

$$\Rightarrow x - z + 2 = 0$$

$$\Rightarrow \vec{r} \cdot (\hat{i} - \hat{k}) + 2 = 0$$

26. Answer (4)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix} = (2 + x)\hat{i} + (x - 3)\hat{j} - 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = r = \sqrt{(2 + x)^2 + (x - 3)^2 + (-5)^2}$$

$$\Rightarrow r = \sqrt{4 + x^2 + 4x + x^2 + 9 - 6x + 25}$$

$$= \sqrt{2x^2 - 2x + 38} = \sqrt{2\left(x^2 - x + \frac{1}{4}\right) + 38 - \frac{1}{2}}$$

$$= \sqrt{2\left(x - \frac{1}{2}\right)^2 + \frac{75}{2}}$$

$$\Rightarrow r \geq \sqrt{\frac{75}{2}} \Rightarrow r \geq 5\sqrt{\frac{3}{2}}$$

27. Answer (2)

$$\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2 \quad \dots(i)$$

$$\therefore \vec{\beta}_2 \cdot \vec{\alpha} = 0$$

$$\text{and Let } \vec{\beta}_1 = \lambda \vec{\alpha}$$

$$\vec{\alpha} \cdot \vec{\beta} = \vec{\alpha} \cdot \vec{\beta}_1 - \vec{\alpha} \cdot \vec{\beta}_2$$

$$\Rightarrow 5 = \lambda \alpha^2$$

$$\Rightarrow 5 = \lambda \times 10$$

$$\Rightarrow \lambda = \frac{1}{2}$$

$$\therefore \boxed{\vec{\beta}_1 = \frac{\vec{\alpha}}{2}}$$

Cross product with $\vec{\beta}_1$ in equation (i),

$$\vec{\beta} \times \vec{\beta}_1 = -\vec{\beta}_2 \times \vec{\beta}_1$$

$$\Rightarrow \boxed{\vec{\beta} \times \vec{\beta}_1 = \vec{\beta}_1 \times \vec{\beta}_2} = \frac{(\vec{\beta} \times \vec{\alpha})}{2}$$

$$\Rightarrow \vec{\beta}_1 \times \vec{\beta}_2 = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 3 & 1 & 0 \end{vmatrix}$$

$$= \frac{1}{2} [-3\hat{i} - \hat{j}(-9) + \hat{k}(5)]$$

$$= \frac{1}{2} [-3\hat{i} + 9\hat{j} + 5\hat{k}]$$

28. Answer (2)

Let $\cos \alpha, \cos \beta, \cos \gamma$ be direction cosines of $\vec{a}$

Hence, by given data

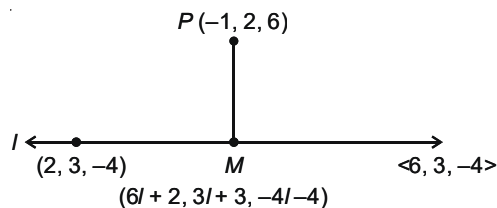
$$\cos \alpha = \cos \frac{\pi}{3}, \cos \beta = \cos \frac{\pi}{4} \text{ \& \ } \cos \gamma = \cos \theta$$

$$\therefore \cos^2 \frac{\pi}{3} + \cos^2 \frac{\pi}{4} + \cos^2 \theta = 1$$

$$\cos^2 \theta = \frac{1}{4} \Rightarrow \cos \theta = \pm \frac{1}{2}, \theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

29. Answer (1)

$$\text{Equation of } l \text{ is } \frac{x-2}{6} = \frac{y-3}{3} = \frac{z+4}{-4}$$



$$\text{Let } M(6\lambda + 2, 3\lambda + 3, -4\lambda - 4)$$

$$\text{DR's of } PM \text{ is } \langle 6\lambda + 3, 3\lambda + 1, -4\lambda - 10 \rangle$$

$$\Rightarrow (6\lambda + 3)(6) + (3\lambda + 1)(3) + (-4\lambda - 10)(-4) = 0$$

$$\Rightarrow \lambda = -1$$

$$\text{i.e. } M \equiv (-4, 0, 0)$$

$$\therefore PM = \sqrt{9 + 4 + 36} = 7$$

30. Answer (1)

Vector are coplanar for $\lambda = \lambda_1$ where $\lambda_1^3 - \lambda_1 + 1 = 0 \Rightarrow$ volume is minimum when $\lambda = \lambda_1$.

$$V = \begin{vmatrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{vmatrix}$$

$$= |1(1) + \lambda(\lambda^2) + 1(-\lambda)|$$

$$= |\lambda^3 - \lambda + 1|$$

$$\text{Let } f(x) = x^3 - x + 1$$

$$f'(x) = 3x^2 - 1$$

For maxima/minima,

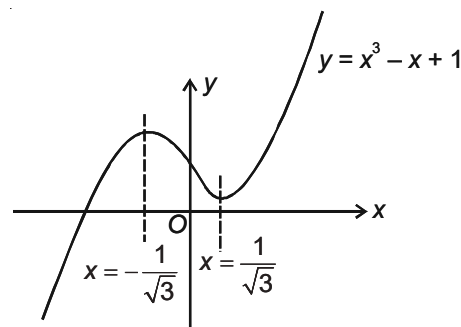
$$f'(x) = 0$$

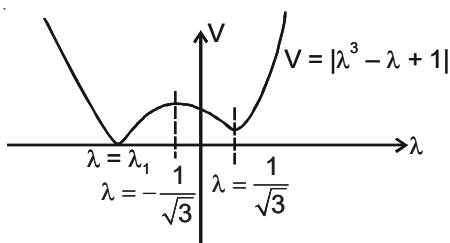
$$x = \pm \frac{1}{\sqrt{3}}$$

$$f''(x) = 6x$$

$$\therefore f''\left(\frac{1}{\sqrt{3}}\right) > 0$$

$$x = \frac{1}{\sqrt{3}} \text{ is point of local minima}$$





When $\lambda = \lambda_1$, volume of parallelopiped is zero (vectors are coplanar)

31. Answer (4)

Let vector be $\lambda [(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})]$

$$\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}$$

$$\vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$$

$$\text{vector} = \lambda [(4\hat{i} + 4\hat{j}) \times (2\hat{i} + 4\hat{k})]$$

$$= \lambda [16\hat{i} - 16\hat{j} - 8\hat{k}]$$

$$= 8\lambda [2\hat{i} - 2\hat{j} - \hat{k}]$$

$$\Rightarrow 12 = 8|\lambda|\sqrt{4 + 4 + 1}$$

$$|\lambda| = \frac{1}{2}$$

Hence required vector is $\pm 4(2\hat{i} - 2\hat{j} - \hat{k})$

32. Answer (4)

If $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then

$$[\vec{a}, \vec{b}, \vec{c}] = 0$$

$$\Rightarrow \begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -\alpha \\ \alpha & -2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \alpha^2 + 6 = 0$$

No value of ' α ' exist

Set S is an empty set.

33. Answer (3)

$$\vec{a} = \lambda_1(\hat{b} + \hat{c})$$

$$= \lambda_1 \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} + \frac{\hat{i} - \hat{j} + 4\hat{k}}{3\sqrt{2}} \right)$$

$$= \frac{\lambda_1}{3\sqrt{2}} (4\hat{i} + 2\hat{j} + 4\hat{k})$$

$$\text{As } \vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$$

$$\therefore \lambda_1 = 3\sqrt{2}, \alpha = 4, \beta = 4$$

$$\vec{a} = 4\hat{i} + 2\hat{j} + 4\hat{k} \text{ no option is satisfied}$$

$$\text{Also } \vec{a} = \lambda_2(\hat{b} - \hat{c})$$

$$= \frac{\lambda_2}{3\sqrt{2}} ((3\hat{i} + 3\hat{j}) - (\hat{i} - \hat{j} + 4\hat{k})) = \frac{\lambda_2}{3\sqrt{2}} (2\hat{i} + 4\hat{j} - 4\hat{k})$$

$$= \frac{2\lambda_2}{3\sqrt{2}} (\hat{i} + 2\hat{j} - 2\hat{k})$$

$$\Rightarrow \alpha = 1, \beta = -2 \text{ and } \frac{2\lambda_2}{3\sqrt{2}} = 1$$

$$\therefore \vec{a} \cdot \hat{k} + 2 = 0$$

34. Answer (3)

$$\therefore |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$\text{and } \vec{a} + \vec{b} + \vec{c} = \vec{0}$$

On squaring both sides

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c}) = 0$$

$$\therefore \lambda = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} = -\frac{3}{2}$$

$$\text{and } \vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$$

$$= \vec{a} \times \vec{b} + \vec{b} \times (-\vec{a} - \vec{b}) + (-\vec{a} - \vec{b}) \times \vec{a}$$

$$= \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - 0 - 0 - \vec{b} \times \vec{a}$$

$$= 3(\vec{a} \times \vec{b})$$

$$\therefore (\lambda, \vec{d}) = \left(-\frac{3}{2}, 3(\vec{a} \times \vec{b}) \right)$$

35. Answer (4)

$$v = [\vec{a} \ \vec{b} \ \vec{c}]$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & \lambda \\ 2 & 1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = \pm 1$$

$$\Rightarrow 2 - 1(5) + \lambda(1) = \pm 1$$

$$\Rightarrow \lambda - 3 = 1 \text{ or } \lambda - 3 = -1$$

$$\Rightarrow \lambda = 4 \text{ or } 2$$

$$\vec{u} = \hat{i} + \hat{j} + 4\hat{k} \text{ or } \hat{i} + \hat{j} + 2\hat{k}$$

$$\begin{aligned} \therefore \cos \theta &= \frac{2+1+4}{\sqrt{18}\sqrt{6}} \text{ or } \frac{2+1+2}{\sqrt{6}\sqrt{6}} \\ &= \frac{7}{6\sqrt{3}} \text{ or } \frac{5}{6} \end{aligned}$$

36. Answer (4)

$$\therefore a \cos \theta = b \cos \left(\theta + \frac{2\pi}{3} \right) = c \cos \left(\theta + \frac{4\pi}{3} \right) = k$$

$$ab + bc + ca = k^2 \left[\frac{1}{\cos \theta \cdot \cos \left(\theta + \frac{2\pi}{3} \right)} + \right.$$

$$\left. \frac{1}{\cos \left(\theta + \frac{2\pi}{3} \right) \cdot \cos \left(\theta + \frac{4\pi}{3} \right)} + \right.$$

$$\left. \frac{1}{\cos \left(\theta + \frac{4\pi}{3} \right) \cdot \cos \theta} \right] \\ = k^2 \left[\frac{\cos \theta + \cos \left(\theta + \frac{2\pi}{3} \right) + \cos \left(\theta + \frac{4\pi}{3} \right)}{\cos \theta \cdot \cos \left(\theta + \frac{2\pi}{3} \right) \cdot \cos \left(\theta + \frac{4\pi}{3} \right)} \right]$$

$$= 0$$

So, angle between the given vectors will be $\frac{\pi}{2}$.

37. Answer (4)

$$\text{Here } \vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix} = x^3 - 27x + 26$$

$$\Rightarrow f(x) = x^3 - 27x + 26$$

$$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = -3, 3$$

$$\begin{array}{c} + \quad - \quad + \\ -3 \quad 3 \\ \text{Max} \quad \text{Min} \end{array}$$

$$\Rightarrow x_0 = -3$$

$$\text{Now } \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -2x - 2x - 3 - 14 - 2x - x + 7x + 4 + 3x = 3x - 13$$

$$\text{So value at } x = x_0 = 3 \times -3 - 13 = -22$$

38. Answer (2)

$$\text{Volume of parallelopiped} = \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = 158$$

$$\Rightarrow (12 + n^2) - 1(6 + n) + n(2n - 4) = 158$$

$$\Rightarrow 3n^2 - 5n - 152 = 0$$

$$\Rightarrow 3n^2 - 24n + 19n - 152 = 0$$

$$\Rightarrow 3n(n - 8) + 19(n - 8) = 0$$

$$\Rightarrow n = 8$$

$\therefore$

$$\vec{a} = \hat{i} + \hat{j} + 8\hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 8\hat{k} \text{ and } \vec{c} = \hat{i} + 8\hat{j} + 3\hat{k}$$

$$\vec{a} \cdot \vec{c} = 1 + 8 + 24 = 33$$

$$\vec{b} \cdot \vec{c} = 2 + 32 - 24 = 10$$

39. Answer (1)

$$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}; \vec{b} = \hat{i} - \hat{j} + \hat{k}$$

$$\therefore \vec{b} \times \vec{c} = \vec{b} \times \vec{a} \Rightarrow \vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{a})$$

$$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$$

$$\therefore \vec{a} \cdot \vec{c} = 0$$

$$\Rightarrow \vec{c} = -\frac{1}{2}(\hat{i} + \hat{j} + \hat{k})$$

$$\therefore \vec{b} \cdot \vec{c} = \frac{-1}{2}$$

40. Answer (1.00)

$$\therefore \vec{P}, \vec{Q}, \vec{R} \text{ are coplanar, so } \begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a+1 & a & a \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -2 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 3a + 1 = 0$$

$$\Rightarrow a = -\frac{1}{3}$$

$$\therefore \vec{P} \cdot \vec{Q} = 3a^2 + 2a = -\frac{1}{3}$$

$$\text{and } \vec{R} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & a & a+1 \\ a & a+1 & a \end{vmatrix}$$

$$= (-2a-1)\hat{i} + a\hat{j} + a\hat{k} = -\frac{\hat{i} + \hat{j} + \hat{k}}{3}$$

$$\Rightarrow |\vec{R} \times \vec{Q}| = \frac{1}{\sqrt{3}}$$

$$\text{Now, } \lambda = \frac{3(\vec{P} \cdot \vec{Q})^2}{|\vec{R} \times \vec{Q}|^2} = \frac{3 \times 1}{9 \times \frac{1}{3}} = 1$$

41. Answer (30)

$$\vec{b} \cdot \vec{c} = |\vec{b}| |\vec{c}| \cos \frac{\pi}{3} = 10$$

$$\Rightarrow 5|\vec{c}| \cdot \frac{1}{2} = 10 \Rightarrow |\vec{c}| = 4$$

$$\begin{aligned} \text{Now } |\vec{a} \times (\vec{b} \times \vec{c})| &= |\vec{a}| |\vec{b} \times \vec{c}| \sin \frac{\pi}{2} \\ &= \sqrt{3} |\vec{b}| |\vec{c}| \sin \frac{\pi}{3} \\ &= \sqrt{3} \cdot 5 \cdot 4 \cdot \frac{\sqrt{3}}{2} \\ &= 30 \end{aligned}$$

42. Answer (2)

$$\text{Given, } |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$\text{Also } |\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$$

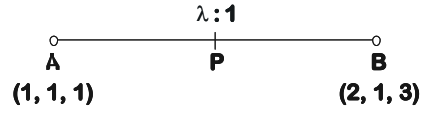
$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} + |\vec{a}|^2 + |\vec{c}|^2 - 2\vec{a} \cdot \vec{c} = 8$$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = -2$$

$$\begin{aligned} \text{Now, } |\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2 &= |\vec{a}|^2 + 4|\vec{b}|^2 + 4\vec{a} \cdot \vec{b} + |\vec{a}|^2 + 4|\vec{c}|^2 + 4\vec{a} \cdot \vec{c} \\ &= 10 + 4(\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}) \\ &= 10 + 4(-2) \\ &= 2 \end{aligned}$$

43. Answer (0.8)

$$\text{Let position vector of P is } \vec{OP} = \frac{\lambda \vec{b} + \vec{a}}{\lambda + 1}$$



$$\text{Given } \vec{OB} \cdot \vec{OP} - 3|\vec{OA} \times \vec{OP}|^2 = 6$$

$$\Rightarrow \vec{b} \cdot \left(\frac{\lambda \vec{b} + \vec{a}}{\lambda + 1} \right) - 3 \left| \vec{a} \times \frac{\lambda \vec{b} + \vec{a}}{\lambda + 1} \right|^2 = 6$$

$$\Rightarrow \frac{\vec{a} \cdot \vec{b} + \lambda |\vec{b}|^2}{\lambda + 1} - \frac{3\lambda^2}{(\lambda + 1)^2} |\vec{a} \times \vec{b}|^2 = 6$$

$$(\because \vec{a} \times \vec{b} = 2\hat{i} - \hat{j} - \hat{k})$$

$$\Rightarrow \frac{6 + 14\lambda}{\lambda + 1} - \frac{18\lambda^2}{(\lambda + 1)^2} = 6$$

$$\Rightarrow 6 + \frac{8\lambda}{\lambda + 1} - \frac{18\lambda^2}{(\lambda + 1)^2} = 6$$

$$\begin{aligned} \text{Let } \frac{\lambda}{\lambda + 1} &= t \\ 18t^2 - 8t &= 0 \end{aligned}$$

$$t = 0, \frac{4}{9}$$

$$\therefore \frac{\lambda}{\lambda + 1} = \frac{4}{9}$$

$$\therefore \lambda = \frac{4}{5} = 0.8$$

44. Answer (18)

$$\text{Let } \vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\text{Now } \hat{i} \times (\vec{a} \times \hat{i}) = (\hat{i} \cdot \hat{i})\vec{a} - (\hat{i} \cdot \vec{a})\hat{i}$$

$$= y\hat{j} + z\hat{k}$$

$$\text{Similarly } \hat{j} \times (\vec{a} \times \hat{j}) = x\hat{i} + z\hat{k}$$

$$\hat{k} \times (\vec{a} \times \hat{k}) = x\hat{i} + y\hat{j}$$

$$\begin{aligned} \text{Now } |y\hat{j} + z\hat{k}|^2 + |x\hat{i} + z\hat{k}|^2 + |x\hat{i} + y\hat{j}|^2 &= 2(x^2 + y^2 + z^2) = 2(4 + 1 + 4) = 18 \end{aligned}$$

45. Answer (6)

 Projection of $\vec{b}$ on $\vec{a}$ = Projection of $\vec{c}$ on $\vec{a}$

$$\Rightarrow \vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$$

$$\text{Given } \vec{b} \cdot \vec{c} = 0$$

 $\therefore$

$$\begin{aligned} |\vec{a} + \vec{b} - \vec{c}|^2 &= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{c} - 2\vec{a} \cdot \vec{c} \\ &= 4 + 16 + 16 \\ &= 36 \end{aligned}$$

$$\Rightarrow |\vec{a} + \vec{b} - \vec{c}| = 6$$

46. Answer (4)

 Let angle between $\vec{a}$ and $\vec{b}$ be θ

$$|\vec{a} + \vec{b}| = \sqrt{1+1+2\cos\theta} = 2\left|\cos\frac{\theta}{2}\right|$$

$$\text{and } |\vec{a} - \vec{b}| = 2\left|\sin\frac{\theta}{2}\right|$$

$$\text{So, } \sqrt{3}|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}| = 2\left[\sqrt{3}\left|\cos\frac{\theta}{2}\right| + \left|\sin\frac{\theta}{2}\right|\right]$$

$$\begin{aligned} \max\{\sqrt{3}|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|\} &= 2\sqrt{(\sqrt{3})^2 + (1)^2} \\ &= 4 \end{aligned}$$

47. Answer (01.00)

$$|\vec{x} + \vec{y}| = |\vec{x}|$$

Squaring both sides we get

$$\begin{aligned} |\vec{x}|^2 + 2\vec{x} \cdot \vec{y} + |\vec{y}|^2 &= |\vec{x}|^2 \\ \Rightarrow 2\vec{x} \cdot \vec{y} + \vec{y} \cdot \vec{y} &= 0 \quad \dots(1) \end{aligned}$$

 Also $2\vec{x} + \vec{y}$ and $\vec{y}$ are perpendicular

$$\therefore 2\vec{x} \cdot \vec{y} + \lambda \vec{y} \cdot \vec{y} = 0 \quad \dots(2)$$

 Comparing (1) & (2) $\lambda = 1$

48. Answer (12)

$$\therefore \vec{a} \cdot \vec{b} = -1, \vec{b} \cdot \vec{c} = 2, \vec{c} \cdot \vec{a} = 0$$

$$\vec{r} \times \vec{a} = \vec{c} \times \vec{a} \Rightarrow (\vec{r} \times \vec{a}) \times \vec{b} = (\vec{c} \times \vec{a}) \times \vec{b}$$

$$\Rightarrow (\vec{r} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{b})\vec{r} = (\vec{b} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{b})\vec{c}$$

$$\Rightarrow \vec{r} = 2\vec{a} + \vec{c}$$

$$\begin{aligned} \text{then } \vec{r} \cdot \vec{a} &= 2|\vec{a}|^2 + \vec{a} \cdot \vec{c} \\ &= 12 \end{aligned}$$

49. Answer (4)

$$\vec{AB} = \hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{AC} = \hat{i} + 2\hat{j} - \hat{k}$$

 Normal to plane $\vec{n} = \vec{AB} \times \vec{AC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

$$\vec{OP} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\sin\theta = \frac{|\vec{OP} \cdot \vec{n}|}{|\vec{OP}| \times |\vec{n}|} = \frac{6+1+1}{\sqrt{11} \cdot \sqrt{6}} = \frac{8}{\sqrt{66}}$$

$$\cos\theta = \sqrt{1 - \frac{64}{66}} = \frac{1}{\sqrt{33}}$$

$$\text{Projection} = |\vec{OP}| \cos\theta = \sqrt{6} \times \frac{1}{\sqrt{33}} = \sqrt{\frac{2}{11}}$$

50. Answer (2)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \alpha & 3 \\ 3 & -\alpha & 1 \end{vmatrix} = 4\alpha\hat{i} + 8\hat{j} - 4\alpha\hat{k} = 4(\alpha\hat{i} + 2\hat{j} - \alpha\hat{k})$$

$$\therefore 8\sqrt{3} = 4\sqrt{2\alpha^2 + 4} \Rightarrow \alpha = \pm 2$$

$$\vec{a} \cdot \vec{b} = 3 - \alpha^2 + 3 = 2$$

51. Answer (4)

 Let $\hat{c}$ be a unit vector in the direction of $\vec{a} \times \vec{b}$.

$$\Rightarrow \hat{a} \times \hat{b} = \hat{c}, \hat{b} \times \hat{c} = \hat{a} \text{ \& } \hat{c} \times \hat{a} = \hat{b}$$

$$\vec{a} \times \vec{b} = |\vec{a}| \times |\vec{b}| \times \hat{c}$$

$$\vec{a} \times (\vec{a} \times \vec{b}) = -|\vec{a}|^2 |\vec{b}| \hat{b}$$

$$\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b})) = -|\vec{a}|^3 |\vec{b}| \hat{c}$$

$$\begin{aligned} \vec{a} \times (\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b}))) &= |\vec{a}|^4 |\vec{b}| \hat{b} \\ &= |\vec{a}|^4 \vec{b} \end{aligned}$$

52. Answer (1)

$$\vec{a}_2 = \lambda \vec{a}_1$$

$$\hat{i} + y\hat{j} + z\hat{k} = \lambda(x\hat{i} - j + \hat{k})$$

$$1 = \lambda x, y = -\lambda, z = \lambda$$

$$x\hat{i} + y\hat{j} + z\hat{k} = \frac{1}{\lambda} \hat{i} - \lambda \hat{j} + \lambda \hat{k}$$

$$\text{Unit vector} = \frac{\frac{1}{\lambda}\hat{i} - \lambda\hat{j} + \lambda\hat{k}}{\sqrt{\frac{1}{\lambda^2} + \lambda^2 + \lambda^2}}$$

$$= \frac{\hat{i} - \lambda^2\hat{j} + \lambda^2\hat{k}}{\sqrt{1 + 2\lambda^4}}$$

Let $\lambda^2 = 1$, possible unit vector $= \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

53. Answer (1)

$$\therefore |\alpha\hat{i} + \beta\hat{j}| = |\sqrt{3}\hat{i} + \hat{j}| \Rightarrow \alpha^2 + \beta^2 = 4 \quad \dots(i)$$

Also $\frac{\sqrt{3}\alpha + \beta}{2 \cdot 2} = \frac{1}{\sqrt{2}} \Rightarrow \sqrt{3}\alpha + \beta = 2\sqrt{2} \quad \dots(ii)$

$\therefore \alpha, \beta > 0$, then from (i) and (ii)

$$\alpha = \frac{\sqrt{3}-1}{\sqrt{2}} \text{ and } \beta = \frac{\sqrt{3}+1}{\sqrt{2}}$$

Area of required triangle

$$= \frac{1}{2}\alpha\beta = \frac{1}{2}\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)$$

$$= \frac{1}{2}$$

54. Answer (1)

$$\vec{r} \times \vec{a} = -\vec{r} \times \vec{b}$$

$$\vec{r} \times (\vec{a} + \vec{b}) = 0$$

$$\vec{r} = \lambda(\vec{a} + \vec{b}) = \lambda(3\hat{i} - \hat{j} + 2\hat{k})$$

$$\vec{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3 \Rightarrow \alpha\lambda = 1 \quad \dots(i)$$

$$\vec{r} \cdot (2\hat{i} + 5\hat{j} + \alpha\hat{k}) = -1 \Rightarrow \lambda - 2\alpha\lambda = -1$$

$$\Rightarrow \lambda = 1 \text{ and } \alpha = 1 \text{ [using (i)]}$$

$$\alpha + |\vec{r}|^2 = 1 + (9 + 1 + 4) = 15$$

55. Answer (28)

$$\vec{a} \times \vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{C} = \lambda(\vec{a} \times \vec{b}) = \lambda(3\hat{i} - 2\hat{j} + \hat{k})$$

$$\vec{C} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 8 \Rightarrow \lambda = 2$$

$$\vec{C} = 2(\vec{a} \times \vec{b})$$

$$\vec{C} \cdot (\vec{a} \times \vec{b}) = 2(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{b})$$

$$= 2|\vec{a} \times \vec{b}|^2 = 2(9 + 4 + 1) = 28$$

56. Answer (4)

$$\vec{r} \times \vec{a} = \vec{r} \times \vec{b} \Rightarrow \vec{r} \times (\vec{a} - \vec{b}) = 0$$

$$\Rightarrow \vec{r} = \lambda(\vec{a} - \vec{b}), \lambda \in \mathbb{R}.$$

$$\Rightarrow \vec{r} = \lambda(-5\hat{i} - 4\hat{j} + 10\hat{k})$$

$$\therefore \vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$$

$$\Rightarrow \lambda(-5 - 8 + 10) = -3$$

$$\Rightarrow \lambda = 1$$

$$\text{Hence } \vec{r} = -5\hat{i} - 4\hat{j} + 10\hat{k}$$

$$\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k}) = 12$$

57. Answer (2)

$$\vec{a} \cdot \vec{b} = 1 \Rightarrow -\alpha\beta - \alpha\beta - 3 = 1$$

$$\Rightarrow \alpha\beta = -2 \quad \dots(i)$$

$$\vec{b} \cdot \vec{c} = -3 \Rightarrow -\beta + 2\alpha + 1 = -3$$

$$2\alpha - \beta = -4 \quad \dots(ii)$$

Solving (i) & (ii) $\alpha = -1, \beta = 2$,

$$\frac{1}{3}((\vec{a} \times \vec{b}) \cdot \vec{c}) = \frac{1}{3} \begin{vmatrix} -1 & 2 & 3 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= 2$$

58. Answer (2)

$$\therefore \vec{OP} \cdot \vec{OQ} = 0 \Rightarrow -x + 2y - 3x = 0 \Rightarrow 2x = y$$

$$\text{and } |\vec{OQ} - \vec{OP}|^2 = 20 \Rightarrow (x + 1)^2 + (y - 2)^2 + (3x + 1)^2 = 20$$

$$\Rightarrow 14x^2 = 14 \Rightarrow x = 1$$

$$\therefore [\vec{OP} \vec{OQ} \vec{OR}] = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 3 \\ 3 & z & -7 \end{vmatrix} = 0 \Rightarrow z = -2$$

$$\text{So } x^2 + y^2 + z^2 = 9$$

59. Answer (486)

$$\text{Let } \vec{x} = \lambda(\vec{a} \times \vec{b}) \times \vec{c}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix} = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$(\vec{a} \times \vec{b}) \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & 5 \\ 3 & 2 & -1 \end{vmatrix} = -13\hat{i} + 14\hat{j} - 11\hat{k}$$

$$\therefore \frac{\vec{x} \cdot \vec{a}}{|\vec{a}|} = \frac{17\sqrt{6}}{2} \Rightarrow \left| \frac{(-26-14-11)\lambda}{\sqrt{6}} \right| = \frac{17\sqrt{6}}{2}$$

$$\Rightarrow \lambda = \pm 1$$

$$|\vec{x}|^2 = 13^2 + 14^2 + 11^2 = 486$$

60. Answer (1)

Magnitude of vector remains same hence

$$9p^2 + 1 = (p + 1)^2 + 10$$

$$\Rightarrow 8p^2 - 2p - 10 = 0$$

$$\Rightarrow 4p^2 - p - 5 = 0$$

$$\Rightarrow 4p^2 - 5p + 4p - 5 = 0$$

$$\Rightarrow (p + 1)(4p - 5) = 0 \Rightarrow p = -1 \text{ or } \frac{5}{4}$$

61. Answer (4)

$$\text{Projection of } \vec{AB} \text{ on } \vec{AC} = \frac{(\vec{AB}) \cdot (\vec{AC})}{|\vec{AC}|} = p(\text{say})$$

$$= \frac{|\vec{AB}| |\vec{AC}| \cos \theta}{|\vec{AC}|}$$

$$\text{where } \cos \theta = \frac{10^2 + 7^2 - 8^2}{2 \cdot 10 \cdot 7}$$

$$\Rightarrow p = \frac{10 \cdot 85}{2 \cdot 10 \cdot 7} = \frac{85}{14}$$

62. Answer (4)

Angle required is say θ

$$\Rightarrow \cos \theta = \frac{\vec{a} \cdot (\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))}{|\vec{a}| |\vec{a} + \vec{b} + (\vec{a} \times \vec{b})|}$$

$$= \frac{|\vec{a}|^2 + 0 + 0}{|\vec{a}| |\vec{a} + \vec{b} + (\vec{a} \times \vec{b})|}$$

$$= \frac{|\vec{a}|^2}{|\vec{a}| \sqrt{3} |\vec{a}|}$$

(as $\vec{a}, \vec{b}$ and $\vec{a} \times \vec{b}$ are mutually perpendicular to each other)

$$\cos \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

63. Answer (3)

$$\therefore \vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$$

$$\Rightarrow |\vec{a}| = \sqrt{4+1+4} = 3$$

$$\text{Now, } |\vec{c} - \vec{a}| = 2\sqrt{2}$$

$$\Rightarrow (\vec{c} - \vec{a})^2 = 8$$

$$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c} \cdot \vec{a} = 8$$

$$|\vec{c}|^2 + 9 - 2|\vec{c}| = 8 \quad (\because \vec{a} \cdot \vec{c} = |\vec{c}|)$$

$$|\vec{c}|^2 - 2|\vec{c}| + 1 = 0$$

$$\therefore |\vec{c}| = 1$$

and angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is θ then

$$|(\vec{a} \times \vec{b}) \times \vec{c}| = |\vec{a} \times \vec{b}| \cdot |\vec{c}| \cdot \sin \theta$$

$$|2\hat{i} - 2\hat{j} + \hat{k}| \cdot 1 \cdot \sin \frac{\pi}{6}$$

$$= \frac{3}{2}$$

64. Answer (4)

$$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3} |\vec{a}|$$

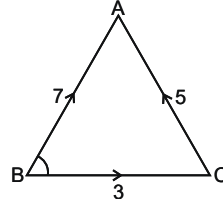
$$\text{Now } \cos \theta = \frac{\vec{a} \cdot (\vec{a} + \vec{b} + \vec{c})}{|\vec{a}| |\vec{a} + \vec{b} + \vec{c}|} = \frac{|\vec{a}|^2}{|\vec{a}| (\sqrt{3} |\vec{a}|)} = \frac{1}{\sqrt{3}}$$

$$\text{So, } 36 \cos^2 2\theta = 36(2 \cos^2 \theta - 1)^2$$

$$= 36 \left(\frac{2}{3} - 1 \right)^2 = 4$$

65. Answer (4)

Projection of $\vec{BA}$ on $\vec{BC}$



$$= \frac{|\vec{BA} \cdot \vec{BC}|}{|\vec{BC}|}$$

$$= \frac{|\vec{BA}| \cdot |\vec{BC}| \cos B}{|\vec{BC}|}$$

$$= 7 \cdot \left(\frac{7^2 + 3^2 - 5^2}{2 \times 7 \times 3} \right)$$

$$= \frac{11}{2} \text{ units}$$

66. Answer (6)

$$\therefore \cos \theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| \cdot |\vec{v}_2|} \text{ and } |\vec{v}_1| = |\vec{v}_2|$$

$$\Rightarrow \cos \theta = \frac{2\sqrt{3}p + p + 1}{|\vec{v}_1|^2} \text{ and } 4 + (p + 1)^2 = 3p^2 + 1$$

$$\Rightarrow p = 2$$

$$\Rightarrow \cos \theta = \frac{4\sqrt{3} + 3}{13} \Rightarrow \tan \theta = \frac{6\sqrt{3} - 2}{4\sqrt{3} + 3}$$

67. Answer (1)

$$[\vec{b} \ \vec{c} \ \vec{d}] = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 3 & 2 & 6 \end{vmatrix} = 2(-6 - 2) - 1(3) + 1(5) = -16 - 3 + 5 = -14$$

$$\text{Let } \vec{a} = \lambda \vec{b} + \mu \vec{c}$$

$$\begin{aligned} \therefore [\vec{a} \ \vec{b} \ \vec{c}] + [\vec{a} \ \vec{b} \ \vec{d}] + [\vec{a} \ \vec{c} \ \vec{d}] \\ = -\mu[\vec{b} \ \vec{c} \ \vec{d}] + \lambda[\vec{b} \ \vec{c} \ \vec{d}] \\ = (\lambda - \mu)[\vec{b} \ \vec{c} \ \vec{d}] \end{aligned}$$

$$\therefore \vec{a} = (2\lambda + \mu)\hat{i} + (\lambda - \mu)\hat{j} + (\lambda + \mu)\hat{k}$$

$$(2\lambda + \mu)^2 + (\lambda - \mu)^2 + (\lambda + \mu)^2 = 10 \quad (\text{as } |\vec{a}| = \sqrt{10})$$

$$\Rightarrow 6\lambda^2 + 3\mu^2 + 4\lambda\mu = 10 \quad \dots(i)$$

$$\& \hat{a} \cdot \hat{b} = 0 \Rightarrow 3(2\lambda + \mu) + 2(\lambda - \mu) + 6(\lambda + \mu) = 0$$

$$14\lambda + 7\mu = 0 \Rightarrow \mu = -2\lambda \quad \dots(ii)$$

$$\text{by (i) \& (ii), } 6\lambda^2 + 12\lambda^2 - 8\lambda^2 = 10$$

$$\Rightarrow \lambda = \pm 1 \Rightarrow \mu = \mp 2$$

$$(\lambda - \mu) = 3 \text{ or } -3$$

$$\therefore \text{Required quantity} = -42$$

68. Answer (4)

$$\therefore \vec{a} \times \vec{b} = \vec{c} \Rightarrow (\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{c} \cdot \vec{c}$$

$$\therefore [\vec{a} \ \vec{b} \ \vec{c}] = |\vec{c}|^2 \quad \dots(i)$$

$$\text{and } (\vec{b} \times \vec{c}) = \vec{a} \Rightarrow \vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot \vec{a}$$

$$\therefore [\vec{a} \ \vec{b} \ \vec{c}] = |\vec{a}|^2 = 4 \quad \dots(ii)$$

$$\therefore |\vec{a}| = |\vec{c}| = 2$$

Option: (1)

$$\vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} - \vec{c})) = \vec{a} \times (-\vec{b} \times \vec{c} + \vec{c} \times \vec{b})$$

$$= 2\vec{a} \times (\vec{c} \times \vec{b}) = 2\vec{a} \times (-\vec{a}) = 0$$

Option: (2)

$$\text{Projection of } \vec{a} \text{ on } (\vec{b} \times \vec{c}) = \frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|} = \frac{|\vec{a}|^2}{|\vec{a}|} = 2$$

Option: (3)

$$[\vec{a} \ \vec{b} \ \vec{c}] + [\vec{c} \ \vec{a} \ \vec{b}] = 2[\vec{a} \ \vec{b} \ \vec{c}] = 8$$

Option: (4)

$$\begin{aligned} |3\vec{a} + \vec{b} - 2\vec{c}|^2 \\ = 9\vec{a}^2 + \vec{b}^2 + 4\vec{c}^2 + 6\vec{a} \cdot \vec{b} - 4\vec{b} \cdot \vec{c} - 12\vec{a} \cdot \vec{c} \\ = 9 \cdot 2^2 + 1^2 + 4 \cdot 2^2 + 0 \\ = 53 \end{aligned}$$

$$\therefore \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

$$\text{and } [\vec{a} \ \vec{b} \ \vec{c}]^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix}$$

$$\therefore 16 = |\vec{a}|^2 |\vec{b}|^2 |\vec{c}|^2$$

$$\therefore |\vec{b}| = 1$$

69. Answer (2)

For coplanarity S.T.P. of given vectors shall vanish

$$\text{i.e. } \begin{vmatrix} 2+a+b & a+2b+c & -(b+c) \\ 1+b & 2b & -b \\ 2+b & 2b & (1-b) \end{vmatrix} = 0$$

$$R_3 \rightarrow R_3 - R_2 \text{ and } R_1 \rightarrow R_1 - R_2$$

$$\begin{vmatrix} 1+a & a+c & -c \\ 1+b & 2b & -b \\ 1 & 0 & 1 \end{vmatrix} = 0$$

Expanding

$$\therefore 1(-b(a+c) + 2bc) + 1(2b(1+a) - (b+1)(a+c)) = 0$$

$$\therefore a + c = 2b$$

70. Answer (3)

$$\vec{r} = \sqrt{3} \frac{(\vec{p} + \vec{q}) \times (\vec{p} - \vec{q})}{|(\vec{p} + \vec{q}) \times (\vec{p} - \vec{q})|}$$

$$= \frac{\sqrt{3} \times \begin{vmatrix} i & j & k \\ 3 & 5 & 2 \\ 1 & 1 & 0 \end{vmatrix}}{|(\vec{p} + \vec{q}) \times (\vec{p} - \vec{q})|} = \frac{\sqrt{3}(-2\hat{i} + 2\hat{j} - 2\hat{k})}{2\sqrt{3}}$$

$$= -\hat{i} + \hat{j} - \hat{k}$$

$$|\alpha| = |\beta| = |\gamma| = 1$$

71. Answer (2)

$\therefore a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar.

$$\therefore \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} a-c & a & c \\ 0 & 0 & 1 \\ c-b & c & b \end{vmatrix} = 0$$

$$\therefore (-1) \{ac - c^2 - ca + ab\} = 0$$

$$\therefore c^2 = ab$$

$$\therefore c = \sqrt{ab}$$

72. Answer (2)

$$|\vec{a}| = 2, |\vec{b}| = 5, |\vec{a} \times \vec{b}| = 8$$

$$\therefore |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

$$\begin{aligned} \therefore (\vec{a} \cdot \vec{b})^2 &= 5^2 \cdot 2^2 - 8^2 \\ &= 100 - 64 \\ &= 36 \end{aligned}$$

$$\therefore \vec{a} \cdot \vec{b} = \pm 6$$

73. Answer (60)

$$\therefore (\vec{a} + 3\vec{b}) \cdot (7\vec{a} - 5\vec{b}) = 0$$

$$\Rightarrow 7|\vec{a}|^2 - 15|\vec{b}|^2 + 16\vec{a} \cdot \vec{b} = 0 \quad \dots(i)$$

$$\text{and } (7\vec{a} - 2\vec{b}) \cdot (\vec{a} - 4\vec{b}) = 0$$

$$\Rightarrow 7|\vec{a}|^2 + 8|\vec{b}|^2 - 30\vec{a} \cdot \vec{b} = 0 \quad \dots(ii)$$

From (i) and (ii)

$$|\vec{b}|^2 = 2\vec{a} \cdot \vec{b} \text{ and } |\vec{a}|^2 = 2\vec{a} \cdot \vec{b}$$

$$\therefore \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \Rightarrow \vec{a} \cdot \vec{b} = 2\vec{a} \cdot \vec{b} \cos \theta$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

74. Answer (1)

$$(\vec{a} - \vec{b}) \times \vec{b} = \vec{a} \times \vec{b}$$

$\therefore$ Given expression is

$$(\vec{a} + \vec{b}) \times (\vec{a} \times (\vec{a} \times \vec{b}) \times \vec{b})$$

$$\Rightarrow (\vec{a} + \vec{b}) \times (\vec{a} \times (\vec{b} \cdot \vec{a})\vec{b} - (\vec{b} \cdot \vec{b})\vec{a})$$

$$\Rightarrow ((\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})) (\vec{b} \cdot \vec{a})$$

$$\Rightarrow (\vec{a} \times (\vec{a} \times \vec{b}) + \vec{b} \times (\vec{a} \times \vec{b})) (\vec{b} \cdot \vec{a})$$

$$\Rightarrow ((\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} + (\vec{b} \cdot \vec{b})\vec{a} - (\vec{b} \cdot \vec{a})\vec{b}) (\vec{b} \cdot \vec{a})$$

Put $\vec{a} \cdot \vec{b} = 7, \vec{a} \cdot \vec{a} = 6, \vec{b} \cdot \vec{b} = 14$ we get

$$\Rightarrow (7\vec{a} - 6\vec{b} + 14\vec{a} - 7\vec{b}) 7$$

$$\Rightarrow 7(21\vec{a} - 13\vec{b})$$

$$\Rightarrow 7(21\hat{i} + 21\hat{j} + 42\hat{k} + 13\hat{i} - 26\hat{j} - 39\hat{k})$$

$$\Rightarrow 7(34\hat{i} - 5\hat{j} + 3\hat{k})$$

75. Answer (2)

$$\therefore \vec{a} \times \vec{b} = \vec{c}$$

$$\Rightarrow (\vec{a} \times \vec{b}) \cdot \vec{c} = |\vec{c}|^2 = 2 = [\vec{a} \vec{b} \vec{c}]$$

$$\therefore I = \frac{|\vec{b} \cdot (\vec{a} \times \vec{c})|}{|\vec{a} \times \vec{c}|} \quad (\because \vec{a} \text{ and } \vec{c} \text{ are perpendicular})$$

$$\Rightarrow I = \frac{|\vec{a} \vec{b} \vec{c}|}{|\vec{a}| |\vec{c}|} = \frac{2}{\sqrt{3}\sqrt{2}} = \sqrt{\frac{2}{3}} \Rightarrow 3I^2 = 2$$

76. Answer (1)

$$\vec{a} = \vec{b} \times (\vec{b} \times \vec{c}) = (\vec{b} \cdot \vec{c})\vec{b} - |\vec{b}|^2 \vec{c}$$

$$= (\vec{b} \cdot \vec{c})\vec{b} - \vec{c} \quad (\because |\vec{b}| = 1)$$

$$|\vec{a}|^2 = (\vec{b} \cdot \vec{c})^2 |\vec{b}|^2 + |\vec{c}|^2 - 2(\vec{b} \cdot \vec{c})(\vec{b} \cdot \vec{c})$$

$$2 = |\vec{c}|^2 - (\vec{b} \cdot \vec{c})^2$$

$$2 = 4 - (2\cos\theta)^2$$

$$(2\cos\theta)^2 = 2$$

$$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \tan\theta = 1$$

77. Answer (9)

$$\vec{a} \cdot \vec{b} = -1 = 3 - 2\alpha\beta \Rightarrow \alpha\beta = 2$$

$$\vec{b} \cdot \vec{c} = 10 = -3\alpha - 2\beta - \alpha \Rightarrow 2\alpha + \beta = -5$$

Clearly $(\alpha, \beta) = (-2, -1)$

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix} = 9$$

78. Answer (2)

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{b} \times \vec{c}) \cdot \vec{a} = \vec{b} \cdot (\vec{c} \times \vec{a})$$

$$= \vec{b} \cdot (-\vec{b}) \quad [\because \vec{a} \times \vec{c} = \vec{b}]$$

$$= -|\vec{b}|^2$$

$$= -(1^2 + (-1)^2) = -2$$

79. Answer (5)

$$\vec{v}_1 = \hat{i} + 2\hat{j} + \hat{k}, \vec{v}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}, \vec{v}_3 = -\lambda\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{v}_2 + \vec{v}_3 = (2 - \lambda)\hat{i} + 6\hat{j} - 2\hat{k} = \vec{v}_4$$

$$\text{Projection of } \vec{v}_1 \text{ on } \vec{v}_4 = \vec{v}_1 \cdot \frac{\vec{v}_4}{|\vec{v}_4|}$$

$$\Rightarrow \frac{1 \times (2 - \lambda) + 2 \times 6 + 1 \times (-2)}{\sqrt{(2 - \lambda)^2 + 6^2 + (-2)^2}} = 1$$

$$\Rightarrow (12 - \lambda)^2 = (2 - \lambda)^2 + 40$$

On solving

$$\lambda = 5$$

80. Answer (90)

$$\vec{b} \times \vec{c} = (-9 - 2\beta)\hat{i} + (3 - \beta)\hat{j} + 5\hat{k}$$

$$|\vec{b} \times \vec{c}| = 5\sqrt{3} \Rightarrow (9 + 2\beta)^2 + (3 - \beta)^2 + 25 = 75$$

$$\Rightarrow \beta^2 + 6\beta + 8 = 0$$

$$\Rightarrow \beta = -2 \text{ or } \beta = -4$$

$\vec{a}$ is perpendicular to $\vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$

$$\Rightarrow (\hat{i} + 5\hat{j} + \alpha\hat{k}) \cdot (\hat{i} + 3\hat{j} + \beta\hat{k}) = 0$$

$$\Rightarrow 1 + 15 + \alpha\beta = 0 \quad \dots(i)$$

$$|\vec{a}|^2 = 1 + 25 + \alpha^2 = 26 + \left(\frac{-16}{\beta}\right)^2$$

from greatest value of $|\vec{a}|^2$ take $\beta = 2$

$$\Rightarrow \text{greatest value of } |\vec{a}|^2 = 90$$

81. Answer (3)

$$\left|\frac{\vec{a}}{8}\right| = 1 \Rightarrow |\vec{a}| = 8$$

$$|2\vec{a} + 3\vec{b}|^2 = |3\vec{a} + 3\vec{b}|^2$$

$$\therefore 4|\vec{a}|^2 + 9|\vec{b}|^2 + 12\vec{a} \cdot \vec{b} = 9|\vec{a}|^2 + |\vec{b}|^2 + 6\vec{a} \cdot \vec{b}$$

$$8|\vec{b}|^2 + 6|\vec{a} \cdot \vec{b}| - 5|\vec{a}|^2 = 0$$

$$8|\vec{b}|^2 + 6|\vec{a}| \cdot |\vec{b}| \cos 60 - 5 \times 64 = 0$$

$$8|\vec{b}|^2 + 24|\vec{b}| - 320 = 0$$

$$|\vec{b}|^2 + 3|\vec{b}| - 40 = 0$$

$$(|\vec{b}| + 8)(|\vec{b}| - 5) = 0$$

$$|\vec{b}| = 5, (|\vec{b}| = -8 \text{ rejected})$$

82. Answer (2)

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = d \text{ (say)}$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

$$\vec{a} \times ((\vec{r} - \vec{b}) \times \vec{a}) + \vec{b} \times ((\vec{r} - \vec{c}) \times \vec{b}) + \vec{c} \times ((\vec{r} - \vec{a}) \times \vec{c}) = 0$$

$$= \sum (a \cdot a)(\vec{r} - \vec{b}) - (\vec{a} \cdot (\vec{r} - \vec{b}))\vec{a} = 0$$

$$= \sum d^2(\vec{r} - \vec{b}) - (\vec{a} - \vec{r})\vec{a} = 0 \quad [\because \vec{a} \cdot \vec{b} = 0]$$

$$= 3d^2\vec{r} - d^2(\vec{a} + \vec{b} + \vec{c}) - \{(\vec{r} \cdot \vec{a})\vec{a} + (\vec{r} \cdot \vec{b})\vec{b} + (\vec{r} \cdot \vec{c})\vec{c}\} = 0$$

$$= 3d^2\vec{r} - d^2(\vec{a} + \vec{b} + \vec{c}) - d^2\vec{r} = 0$$

$[\because \text{Each term is component of } \vec{r}]$

$$2\vec{r} - (\vec{a} + \vec{b} + \vec{c}) = 0$$

$$\vec{r} = \frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$$

83. Answer (1494)

Normal of plane containing $\hat{a}$ and $\hat{b}$ is

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 1 & 2 & -1 \end{vmatrix} = -3\hat{i} + 4\hat{j} + 5\hat{k}$$

$\vec{v}$ is perpendicular to $(3\hat{i} + 2\hat{j} - \hat{k})$ and also $\vec{n}$

$$\vec{v} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -1 \\ -3 & 4 & 5 \end{vmatrix} = \lambda[14\hat{i} - 12\hat{j} + 18\hat{k}]$$

Given

$$\frac{\vec{a} \cdot \vec{v}}{|\vec{a}|} = 19 \Rightarrow \frac{\lambda((2)(14) + (-12)(-1) + (18)(2))}{3} = 19$$

$$\lambda = \frac{3}{4}$$

$$\vec{v} = \frac{3}{4}(14\hat{i} - 12\hat{j} + 18\hat{k}) \Rightarrow 2\vec{v} = 3(7\hat{i} - 6\hat{j} + 9\hat{k})$$

$$|2\vec{v}|^2 = 1494$$

84. Answer (3)

$$\therefore \hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$$

$$\Rightarrow \hat{b} \cdot \vec{c} = |\vec{c}|^2$$

$$\therefore \hat{b} - \vec{c} = 2(\vec{c} \times \hat{a})$$

$$\Rightarrow |\hat{b}|^2 + |\vec{c}|^2 - 2\hat{b} \cdot \vec{c} = 4|\vec{c}|^2 |\hat{a}|^2 \sin^2 \frac{\pi}{12}$$

$$\Rightarrow 1 + |\vec{c}|^2 - 2|\vec{c}|^2 = 4|\vec{c}|^2 \left(\frac{\sqrt{3}-1}{2\sqrt{2}} \right)^2$$

$$\Rightarrow 1 = |\vec{c}|^2 (3 - \sqrt{3})$$

$$\Rightarrow 36|\vec{c}|^2 = \frac{36}{3-\sqrt{3}} = 6(3+\sqrt{3})$$

85. Answer (3)

$$\therefore |\hat{a} + \hat{b} + 2(\hat{a} \times \hat{b})| = 2, \theta \in (0, \pi)$$

$$\Rightarrow |\hat{a} + \hat{b} + 2(\hat{a} \times \hat{b})|^2 = 4.$$

$$\Rightarrow |\hat{a}|^2 + |\hat{b}|^2 + 4|\hat{a} \times \hat{b}|^2 + 2\hat{a} \cdot \hat{b} = 4.$$

$$\therefore \cos \theta = \cos 2\theta$$

$$\therefore \theta = \frac{2\pi}{3}$$

where θ is angle between $\hat{a}$ and $\hat{b}$.

$$\therefore 2|\hat{a} \times \hat{b}| = \sqrt{3} = |\hat{a} - \hat{b}|$$

(S1) is correct

$$\text{And projection of } \hat{a} \text{ on } (\hat{a} + \hat{b}) = \frac{\hat{a} \cdot (\hat{a} + \hat{b})}{|\hat{a} + \hat{b}|} = \frac{1}{2}.$$

(S2) is correct.

86. Answer (2)

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \Rightarrow \cos^2 \alpha = \frac{1}{3}$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

$$\vec{a} = \frac{\lambda}{3}(\hat{i} + \hat{j} + \hat{k}), \lambda > 0$$

$$\frac{\lambda}{\sqrt{3}} \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot (3\hat{i} + 4\hat{j})}{\sqrt{3^2 + 4^2}} = 7$$

$$\Rightarrow \frac{\lambda}{\sqrt{3}}(3+4) = 7 \times 5$$

$$\therefore \lambda = 5\sqrt{3}$$

$$\vec{a} = 5(\hat{i} + \hat{j} + \hat{k})$$

$$\text{Let } \vec{b} = p\hat{i} + q\hat{j} + r\hat{k}$$

$$\vec{a} \cdot \vec{b} = 0 \text{ and } [\vec{a} \vec{b} \hat{i}] = 0$$

$$\Rightarrow p + q + r = 0$$

...(i)

$$\& \begin{vmatrix} p & q & r \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0 \Rightarrow \begin{matrix} q=r \\ p=-2r \end{matrix}$$

$$\vec{b} = -2\hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} = r(-2\hat{i} + \hat{j} + \hat{k})$$

$$\text{Now } |\vec{a}| = |\vec{b}|$$

$$5\sqrt{3} = |r|\sqrt{b} \Rightarrow |r| = \frac{5}{\sqrt{2}}$$

$$\Rightarrow \text{Projection of } \vec{b} \text{ on } 3\hat{i} + 4\hat{j} = \left| \frac{\vec{b} \cdot (3\hat{i} + 4\hat{j})}{\sqrt{3^2 + 4^2}} \right|$$

$$= |r| \frac{(-6+4)}{5} = \left| \frac{-2r}{5} \right|$$

$$\text{Projection} = \frac{2}{5} \times \frac{5}{\sqrt{2}} = \sqrt{2}$$

$\therefore$ 2 is correct

87. Answer (576)

$$|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})|^2 + 4(\vec{a} \cdot \vec{b})^2$$

$$\Rightarrow |\vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}|^2 + 4(\vec{a} \cdot \vec{b})^2$$

$$\Rightarrow |2(\vec{a} \times \vec{b})|^2 + 4(\vec{a} \cdot \vec{b})^2$$

$$\Rightarrow 4(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2$$

$$\Rightarrow 4|\vec{a}|^2|\vec{b}|^2 = 4.16.9 = 576$$

88. Answer (14)

$$\text{Let } \vec{a} = x\hat{i} = y\hat{j} + z\hat{k}$$

$$\text{So, } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 1 & \lambda \end{vmatrix} = \hat{i}(\lambda y - z) + \hat{j}(z - \lambda x) + \hat{k}(x - y)$$

$$\Rightarrow \lambda y - z = 13, z - \lambda x = -1, x - y = -4$$

$$\text{and } x + y + \lambda z = -21$$

$$\Rightarrow \text{clearly, } \lambda = 3, x = -2, y = 2 \text{ and } z = -7$$

$$\text{So, } \vec{b} - \vec{a} = 3\hat{i} - \hat{j} + 10\hat{k}$$

$$\text{and } \vec{b} + \vec{a} = -\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\Rightarrow (\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{j}) + (\vec{b} + \vec{a}) \cdot (\hat{i} - \hat{k}) = 11 + 3 = 14$$

89. Answer (1)

$$\therefore \vec{a} \times (\vec{b} \times \vec{c}) = 3\vec{b} - \vec{c} = \vec{u}$$

$$\vec{b} \times (\vec{c} \times \vec{a}) = \vec{c} - 2\vec{a} = \vec{v}$$

$$\vec{c} \times (\vec{b} \times \vec{a}) = 3\vec{b} - 2\vec{a} = \vec{w}$$

$$\therefore \vec{u} + \vec{v} = \vec{w}$$

So vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ are coplanar, hence their Scalar triple product will be zero.

90. Answer (4)

$$\text{Let } \vec{v} = \lambda_1 \vec{a} + \lambda_2 \vec{b}, \text{ where } \lambda_1, \lambda_2 \in \mathbb{R}.$$

$$= (\lambda_1 + 2\lambda_2)\hat{i} + (\lambda_1 - 3\lambda_2)\hat{j} + (2\lambda_1 + \lambda_2)\hat{k}$$

$$\therefore \text{Projection of } \vec{v} \text{ on } \vec{c} \text{ is } \frac{2}{\sqrt{3}}.$$

$$\therefore \frac{\lambda_1 + 2\lambda_2 - \lambda_1 + 3\lambda_2 + 2\lambda_1 + \lambda_2}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\therefore \lambda_1 + 3\lambda_2 = 1 \quad \dots(i)$$

$$\text{and } \vec{v} \cdot \hat{j} = 7 \Rightarrow \lambda_1 - 3\lambda_2 = 7 \quad \dots(ii)$$

from equation (i) and (ii)

$$\lambda_1 = 4, \lambda_2 = -1$$

$$\therefore \vec{v} = 2\hat{i} + 7\hat{j} + 7\hat{k}$$

$$\therefore \vec{v} \cdot (\hat{i} + \hat{k}) = 2 + 7$$

$$= 9$$

91. Answer (1)

$$\vec{a} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\text{Now, } \vec{b} \times \vec{c} = \vec{a}$$

$$\vec{c} \cdot (\vec{b} \times \vec{c}) = \vec{c} \cdot \vec{a}$$

$$\vec{c} \cdot \vec{a} = 0$$

$$\Rightarrow (\hat{i} + \hat{j} - \hat{k})(2\hat{i} - 3\hat{j} + 2\hat{k}) = 0$$

$$= 2 - 3 - 2 = 0$$

$$\Rightarrow -3 = 0 \text{ (Not possible)}$$

$$\Rightarrow \text{No possible value of } \vec{b} \text{ is possible.}$$

92. Answer (4)

$\therefore \vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$.

$$\therefore \frac{1}{2}|\vec{a} \times \vec{b}| = 2\sqrt{2}$$

$$|\vec{a}| |\vec{b}| \sin \theta = 4\sqrt{2}$$

$$\Rightarrow |\vec{b}| \sin \theta = 4\sqrt{2}$$

$$\dots(i) \quad 94. \text{ Answer (4)}$$

$$\text{and } |\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$$

$$|\vec{a}| |\vec{b}| \cos \theta = |\vec{a}| |\vec{b}| \sin \theta$$

$$\Rightarrow \boxed{\tan \theta = 1} \therefore \theta = \frac{\pi}{4}$$

$$\text{By (i) } |\vec{b}| = 8$$

$$\text{Now } \vec{c} = 2\sqrt{2}(\vec{a} \times \vec{b}) - 2\vec{b}$$

$$\Rightarrow \vec{c} \cdot \vec{b} = -2|\vec{b}|^2 = -128$$

$$\dots(ii)$$

$$\text{and } \vec{c} \cdot \vec{c} = 8|\vec{a} \times \vec{b}|^2 + 4|\vec{b}|^2$$

$$\Rightarrow |\vec{c}|^2 = 8.32 + 4.64$$

$$\Rightarrow |\vec{c}| = 16\sqrt{2}$$

$$\text{From (ii) and (iii)}$$

$$|\vec{c}| |\vec{b}| \cos \alpha = -128$$

$$\Rightarrow \cos \alpha = \frac{-1}{\sqrt{2}}$$

$$\alpha = \frac{3\pi}{4}$$

$$93. \text{ Answer (150)}$$

$$2C_1 + C_2 + 3C_3 = 5$$

$$\dots(i)$$

$$3C_1 + 3C_2 + C_3 = 0$$

$$\dots(ii)$$

$$[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 1 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

$$= 2(3C_3 - C_2) - 1(3C_3 - C_1) + 3(3C_2 - 3C_1)$$

$$= 3C_3 + 7C_2 - 8C_1$$

$$\Rightarrow 8C_1 - 7C_2 - 3C_3 = 0$$

$$\dots(iii)$$

$$C_1 = \frac{10}{122}, C_2 = \frac{-85}{122}, C_3 = \frac{225}{122}$$

$$\text{So } 122(C_1 + C_2 + C_3) = 150$$

$$\vec{a} = \alpha \hat{i} + 2\hat{j} - \hat{k} \text{ and } \vec{b} = -2\hat{i} + \alpha \hat{j} + \hat{k}$$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -1 \\ -2 & \alpha & 1 \end{vmatrix} = (2+\alpha)\hat{i} - (\alpha-2)\hat{j} + (\alpha^2+4)\hat{k}$$

$$\text{Now } |\vec{a} \times \vec{b}| = \sqrt{15(\alpha^2+4)}$$

$$\Rightarrow (2+\alpha)^2 + (\alpha-2)^2 + (\alpha^2+4)^2 = 15(\alpha^2+4)$$

$$\Rightarrow \alpha^4 - 5\alpha^2 - 36 = 0$$

$$\therefore \alpha = \pm 3$$

$$\text{Now, } 2|\vec{a}|^2 + (\vec{a} - \vec{b})|\vec{b}|^{-2} = 2.14 - 14 = 14$$

$$95. \text{ Answer (3)}$$

$$\text{Let } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\text{and } \vec{a} \cdot \left(3\hat{i} - \frac{1}{2}\hat{j} + 2\hat{k}\right) = 0 \Rightarrow 3a_1 + \frac{a_2}{2} + 2a_3 = 0 \dots(i)$$

$$\text{and } \vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$$

$$\Rightarrow a_2\hat{i} + (2a_3 - a_1)\hat{j} - 2a_2\hat{k} = 2\hat{i} - 13\hat{j} - 4\hat{k}$$

$$\therefore a_2 = 2 \dots(ii)$$

$$\text{and } a_1 - 2a_3 = 13 \dots(iii)$$

$$\text{From eq. (i) and (iii) : } a_1 = 3 \text{ and } a_3 = -5$$

$$\therefore \vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$$

$$\therefore \text{projection of } \vec{a} \text{ on } 2\hat{i} + 2\hat{j} + \hat{k} = \frac{6+4-5}{3} = \frac{5}{3}$$

$$96. \text{ Answer (1)}$$

$$\vec{a} = \alpha \hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{b} = 3\hat{i} - \beta \hat{j} + 4\hat{k}$$

$$\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\text{Projection of } \vec{a} \text{ on } \vec{c} \text{ is}$$

$$\frac{\vec{a} \cdot \vec{c}}{|\vec{b}|} = \frac{10}{3}$$

$$\frac{\alpha + 6 + 2}{\sqrt{1^2 + 2^2 + (-2)^2}} = \frac{\alpha + 8}{3} = \frac{10}{3}$$

$$\therefore \boxed{\alpha = 2}$$

$$\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -\beta & 4 \\ 1 & 2 & -2 \end{vmatrix} = (2\beta - 8)\hat{i} + 10\hat{j} + (6 + \beta)\hat{k} = -6\hat{i} + 10\hat{j} + 7\hat{k}$$

$$2\beta - 8 = -6 \text{ \& \; } 6 + \beta = 7$$

$$\therefore \boxed{\beta = 1}$$

$$\alpha + \beta = 2 + 1 = 3$$

97. Answer (1)

$$\vec{AB} \parallel \vec{AC} \text{ if}$$

$$\frac{1}{2} = \frac{\alpha - 4}{-6} = \frac{1}{2}$$

$$\Rightarrow \alpha = 1$$

$\vec{a}, \vec{b}, \vec{c}$ are non-collinear for $\alpha = 2$ (smallest positive integer)

$$\text{Mid point of } BC = M\left(\frac{5}{2}, 0, \frac{9}{2}\right)$$

$$AM = \sqrt{\frac{9}{4} + 16 + \frac{9}{4}} = \frac{\sqrt{82}}{2}$$

98. Answer (10*)

Data not correct

$$\therefore \vec{a} \cdot \vec{b} = 1 - 2 + 3 = 2 \quad \{\text{according to } \vec{a} \text{ and } \vec{b}\}$$

but given that

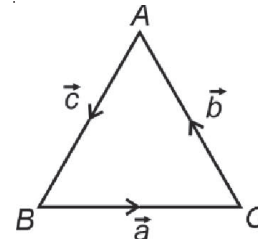
$$\vec{a} + (\vec{b} \times \vec{c}) = \vec{0}$$

$$\vec{a} = -(\vec{b} \times \vec{c})$$

$$\Rightarrow \vec{a} \perp \vec{b} \text{ and } \vec{a} \perp \vec{c}$$

$$\therefore \vec{a} \cdot \vec{b} = 0 \text{ \{Contradicts\}}$$

99. Answer (3*)



$$\therefore \vec{a} + \vec{b} + \vec{c} = \vec{0} \quad \dots(i)$$

$$\text{then } \vec{a} + \vec{c} = -\vec{b}$$

$$\text{then } (\vec{a} + \vec{c}) \times \vec{b} = -\vec{b} \times \vec{b}$$

$$\therefore \vec{a} \times \vec{b} + \vec{c} \times \vec{b} = \vec{0} \quad \dots(ii)$$

$$\text{For (S1): } |\vec{a} \times \vec{b} + \vec{c} \times \vec{b}| - |\vec{c}| = 6(2\sqrt{2} - 1)$$

$$|(\vec{a} + \vec{c}) \times \vec{b}| - |\vec{c}| = 6(2\sqrt{2} - 1)$$

$$|\vec{c}| = 6 - 12\sqrt{2} \quad (\text{not possible})$$

Hence (S1) is not correct

$$\text{For (S2): from (i) } \vec{b} + \vec{c} = -\vec{a}$$

$$\Rightarrow \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{b} = -\vec{a} \cdot \vec{b}$$

$$\Rightarrow 12 + 12 = -6\sqrt{2} \cdot 2\sqrt{3} \cos(\pi - \angle ACB)$$

$$\therefore \cos(\angle ACB) = \sqrt{\frac{2}{3}}$$

$$\therefore \angle ACB = \cos^{-1} \sqrt{\frac{2}{3}}$$

$$\therefore \text{S(2) is correct.}$$

100. Answer (4)

$$\text{Given: } \vec{a} = (\alpha, 1, -1) \text{ and } \vec{b} = (2, 1, -\alpha)$$

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & -1 \\ 2 & 1 & -\alpha \end{vmatrix}$$

$$= (-\alpha + 1)\hat{i} + (\alpha^2 - 2)\hat{j} + (\alpha - 2)\hat{k}$$

$$\text{Projection of } \vec{c} \text{ on } \vec{d} = -\hat{i} + 2\hat{j} - 2\hat{k}$$

$$= \left| \vec{c} \cdot \frac{\vec{d}}{|\vec{d}|} \right| = 30 \text{ \{Given\}}$$

$$\Rightarrow \left| \frac{\alpha - 1 - 4 + 2\alpha^2 - 2\alpha + 4}{\sqrt{1+4+4}} \right| = 30$$

On solving $\alpha = \frac{-13}{2}$ (Rejected as $\alpha > 0$)

and $\alpha = 7$

101. Answer (*)

$$f_a(x) = \tan^{-1} 2x - 3ax + 7$$

$$\therefore f_a(x) \text{ is non-decreasing in } \left(-\frac{\pi}{6}, \frac{\pi}{6} \right)$$

$$\therefore f'_a(x) \geq 0 \Rightarrow \frac{2}{1+4x^2} - 3a \geq 0$$

$$\Rightarrow 3a \leq \frac{2}{1+4x^2}$$

$$\text{So, } a_{\max} = \frac{2}{3} \left(\frac{1}{1+4 \times \frac{\pi^2}{36}} \right) = \frac{6}{9+\pi^2} = \bar{a}$$

$$\therefore f_{\bar{a}}\left(\frac{\pi}{8}\right) = \tan^{-1} \frac{\pi}{4} - 3 \cdot \frac{\pi}{8} \cdot \frac{6}{9+\pi^2} + 7$$

102. Answer (4)

$$\vec{a} = \alpha \hat{i} + \hat{j} + \beta \hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$$

$$\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & \beta \\ 3 & -5 & 4 \end{vmatrix} = -\hat{i} + 9\hat{j} + 12\hat{k}$$

$$4 + 5\beta = -1 \Rightarrow \beta = -1$$

$$-5\alpha - 3 = 12 \Rightarrow \alpha = -3$$

$$\vec{b} - 2\vec{a} = 3\hat{i} - 5\hat{j} + 4\hat{k} - 2(-3\hat{i} + \hat{j} - \hat{k})$$

$$\vec{b} - 2\vec{a} = 9\hat{i} - 7\hat{j} + 6\hat{k}$$

$$\vec{b} + \vec{a} = (3\hat{i} - 5\hat{j} + 4\hat{k}) + (-3\hat{i} + \hat{j} - \hat{k})$$

$$\vec{b} + \vec{a} = -4\hat{j} + 3\hat{k}$$

$$\begin{aligned} \text{Projection of } \vec{b} - 2\vec{a} \text{ on } \vec{b} + \vec{a} &= \frac{(\vec{b} - 2\vec{a}) \cdot (\vec{b} + \vec{a})}{|\vec{b} + \vec{a}|} \\ &= \frac{28 + 18}{5} = \frac{46}{5} \end{aligned}$$

103. Answer (2)

$$\text{Given, } \vec{a} = 2\hat{i} - \hat{j} + 5\hat{k} \text{ and } \vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$$

$$\text{Also, } ((\vec{a} \times \vec{b}) \times \hat{i}) \cdot \hat{k} = \frac{23}{2}$$

$$\Rightarrow ((\vec{a} \cdot \hat{i})\vec{b} - (\vec{b} \cdot \hat{i})\vec{a}) \cdot \hat{k} = \frac{23}{2}$$

$$\Rightarrow (2 \cdot \vec{b} - \alpha \cdot \vec{a}) \cdot \hat{k} = \frac{23}{2}$$

$$\Rightarrow 2 \cdot 2 - 5\alpha = \frac{23}{2} \Rightarrow \alpha = \frac{-3}{2}$$

$$\text{Now, } |\vec{b} \times 2\hat{j}| = |(\alpha\hat{i} + \beta\hat{j} + 2\hat{k}) \times 2\hat{j}|$$

$$= |2\alpha\hat{k} + 0 - 4\hat{i}|$$

$$= \sqrt{4\alpha^2 + 16}$$

$$= \sqrt{4\left(\frac{-3}{2}\right)^2 + 16}$$

$$= 5$$

104. Answer (*)

$$\text{Given } \vec{a} \times \vec{b} = 4 \cdot \vec{c} \quad \dots(i)$$

$$\vec{b} \times \vec{c} = 9 \cdot \vec{a} \quad \dots(ii)$$

$$\vec{c} \times \vec{a} = \alpha \cdot \vec{b} \quad \dots(iii)$$

Taking dot products with $\vec{c}, \vec{a}, \vec{b}$ we get

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

$$\text{Hence (i)} \Rightarrow |\vec{a}| \cdot |\vec{b}| = 4 \cdot |\vec{c}| \quad \dots(iv)$$

$$(ii) \Rightarrow |\vec{b}| \cdot |\vec{c}| = 9 \cdot |\vec{a}| \quad \dots(v)$$

$$(iii) \Rightarrow |\vec{c}| \cdot |\vec{a}| = \alpha \cdot |\vec{b}| \quad \dots(vi)$$

Multiplying (iv), (v) and (vi)

$$\Rightarrow |\vec{a}| \cdot |\vec{b}| \cdot |\vec{c}| = 36\alpha \quad \dots(vii)$$

$$\text{Dividing (vii) by (iv)} \Rightarrow |\vec{c}|^2 = 9\alpha \Rightarrow |\vec{c}| = 3\sqrt{\alpha} \quad \dots(viii)$$

$$\text{Dividing (vii) by (v)} \Rightarrow |\vec{a}|^2 = 4\alpha \Rightarrow |\vec{a}| = 2\sqrt{\alpha}$$

$$\text{Dividing (viii) by (vi)} \Rightarrow |\vec{b}|^2 = 36 \Rightarrow |\vec{b}| = 6$$

$$\text{Now, as given, } 3\sqrt{\alpha} + 2\sqrt{\alpha} + 6 = \frac{1}{36} \Rightarrow \sqrt{\alpha} = \frac{-43}{36}$$

105. Answer (3)

Clearly $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar

$$\begin{vmatrix} 1+t & 1-t & 1 \\ 1-t & 1+t & 2 \\ t & -t & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow (1+t)(1+t+2t) - (1-t)(1-t-2t) + 1(t^2-t-t^2) \neq 0$$

$$\Rightarrow (3t^2+4t+1) - (1-t)(1-3t) - 2t \neq 0$$

$$\Rightarrow (3t^2+4t+1) - (3t^2-4t+1) - 2t \neq 0$$

$$\Rightarrow t \neq 0$$

106. Answer (2)

$$(x\vec{a} + y\vec{b}) \cdot (6y\vec{a} - 18x\vec{b}) = 0$$

$$\Rightarrow (6xy|\vec{a}|^2 - 18xy|\vec{b}|^2) + (6y^2 - 18x^2)\vec{a} \cdot \vec{b} = 0$$

As given equation is identity

Coefficient of x^2 = coefficient of y^2 = coefficient of xy = 0

$$\Rightarrow |\vec{a}|^2 = 3|\vec{b}|^2 \Rightarrow |\vec{b}| = 3\sqrt{3}$$

$$\text{and } \vec{a} \cdot \vec{b} = 0$$

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta$$

$$= 9.3\sqrt{3}.1 = 27\sqrt{3}$$

107. Answer (2)

$$\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$$

$$\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a(\log_e b)\hat{k}$$

For acute angle $\vec{u} \cdot \vec{v} > 0$

$$\Rightarrow a(\log_e b)^2 - 12 + 6a(\log_e b) > 0$$

$$\therefore b > 1$$

$$\text{Let } \log_e b = t \Rightarrow t > 0 \text{ as } b > 1$$

$$at^2 + 6at - 12 > 0 \quad \forall t > 0$$

$$\Rightarrow a \in \phi$$

108. Answer (3)

$$|\vec{a}||\vec{b}||\vec{c}| = 14$$

$$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \wedge \vec{a} = \theta = \frac{2\pi}{3}$$

$$\vec{a} \cdot \vec{b} = -\frac{1}{2}|\vec{a}||\vec{b}|$$

$$\vec{b} \cdot \vec{c} = -\frac{1}{2}|\vec{b}||\vec{c}|$$

$$\vec{c} \cdot \vec{a} = -\frac{1}{2}|\vec{c}||\vec{a}|$$

Now,

$$(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) + (\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) + (\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b})$$

$$= 168$$

...(i)

$$(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{b})(\vec{b} \cdot \vec{c}) - (\vec{a} \cdot \vec{c})|\vec{b}|^2$$

$$= \frac{1}{4}|\vec{b}|^2|\vec{a}||\vec{c}| + \frac{1}{2}|\vec{a}||\vec{b}|^2|\vec{c}|$$

$$= \frac{3}{4}|\vec{a}||\vec{b}|^2|\vec{c}| \quad \dots(ii)$$

$$\text{Similarly } (\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) = \frac{3}{4}|\vec{a}||\vec{b}||\vec{c}|^2 \quad \dots(iii)$$

$$(\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b}) = \frac{3}{4}|\vec{a}|^2|\vec{b}||\vec{c}| \quad \dots(iv)$$

Substitute (ii), (iii), (iv) in (i)

$$\frac{3}{4} |\vec{a}| |\vec{b}| |\vec{c}| [|\vec{a}| + |\vec{b}| + |\vec{c}|] = 168$$

$$\frac{3}{4} \times 14 [|\vec{a}| + |\vec{b}| + |\vec{c}|] = 168$$

$$|\vec{a}| + |\vec{b}| + |\vec{c}| = 16$$

109. Answer (14)

$$\therefore |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$$

$$\text{or } |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + 2|\vec{b}|^2$$

$$\therefore |\vec{b}|^2 = 6 \quad \dots(i)$$

$$\text{Now } |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

$$75 = |\vec{a}|^2 \cdot 6 - 9$$

$$\therefore |\vec{a}|^2 = 14$$

110. Answer (1)

$$\hat{a} \cdot \hat{b} = \frac{1}{\sqrt{2}} \text{ and } |\vec{a} \times \vec{b}| = \frac{1}{\sqrt{2}}$$

$$\frac{(\hat{a} + \hat{b}) \cdot (\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b}))}{|\hat{a} + \hat{b}| |\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b})|} = \cos \theta$$

$$\Rightarrow \cos \theta = \frac{1 + 3\hat{a}\hat{b} + 2}{|\hat{a} + \hat{b}| |\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b})|}$$

$$|\hat{a} + \hat{b}|^2 = 2 + \sqrt{2}$$

$$|\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b})|^2 = 1 + 4 + 4|\hat{a} \times \hat{b}|^2 + 4\hat{a}\hat{b}$$

$$= 5 + 4 \cdot \frac{1}{2} + \frac{4}{\sqrt{2}} = 7 + 2\sqrt{2}$$

$$\text{So, } \cos^2 \theta = \frac{\left(3 + \frac{3}{\sqrt{2}}\right)^2}{(2 + \sqrt{2})(7 + 2\sqrt{2})} = \frac{9\sqrt{2}(5\sqrt{2} + 3)}{164}$$

$$\Rightarrow 164 \cos^2 \theta = 90 + 27\sqrt{2}$$

111. Answer (1)

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$$

$$\vec{a} \cdot \vec{b} = 3$$

$$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 \cdot |\vec{b}|^2$$

$$\Rightarrow 5 + 9 = 6|\vec{b}|^2$$

$$\Rightarrow |\vec{b}|^2 = \frac{7}{3}$$

$$|\vec{a} - \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}} = \sqrt{\frac{7}{3}}$$

$$\text{projection of } \vec{b} \text{ on } \vec{a} - \vec{b} = \frac{\vec{b} \cdot (\vec{a} - \vec{b})}{|\vec{a} - \vec{b}|}$$

$$= \frac{\vec{b} \cdot \vec{a} - |\vec{b}|^2}{|\vec{a} - \vec{b}|} = \frac{3 - \frac{7}{3}}{\sqrt{\frac{7}{3}}}$$

$$= \frac{2}{\sqrt{21}}$$

112. Answer (1)

$$\vec{a} = 3\hat{i} + \hat{j} \text{ \& } \vec{b} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \vec{b} + \lambda \vec{c}$$

If $\vec{b}$ & $\vec{c}$ are non-parallel

$$\text{then } \vec{a} \cdot \vec{c} = 1 \text{ \& } \vec{a} \cdot \vec{b} = -\lambda$$

$$\text{but } \vec{a} \cdot \vec{b} = 5 \Rightarrow \lambda = -5$$