Chapter 7

Determinants

1. Let A be a 2 × 2 matrix

   Statement-1 : adj (adj A) = A

   Statement-2 : |adj A| = |A| **[AIEEE-2009]**

   (1) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

   (2) Statement-1 is true, Statement-2 is false

   (3) Statement-1 is false, Statement-2 is true

   (4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

2. Let a, b, c be such that $b(a + c) \neq 0$. If

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0,$$

   then the value of n is **[AIEEE-2009]**

   (1) Any even integer (2) Any odd integer

   (3) Any integer (4) Zero

3. Consider the system of linear equations:

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + 3x_2 + x_3 = 3$$
$$3x_1 + 5x_2 + 2x_3 = 1$$

   The system has **[AIEEE-2010]**

   (1) Infinite number of solutions

   (2) Exactly 3 solutions

   (3) A unique solutions

   (4) No solution

4. If the trivial solution is the only solution of the system of equations

$$x - ky + z = 0$$
$$kx + 3y - kz = 0$$
$$3x + y - z = 0$$

   then the set of all values of k is **[AIEEE-2011]**

   (1) $R - \{-3\}$ (2) $\{2, -3\}$

   (3) $R - \{2, -3\}$ (4) $R - \{2\}$

5. Let P and Q be 3 × 3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to **[AIEEE-2012]**

   (1) 1 (2) 0

   (3) –1 (4) –2

6. The number of values of k, for which the system of equations

$$(k + 1)x + 8y = 4k$$
$$kx + (k + 3)y = 3k - 1$$

   has no solution, is **[JEE (Main)-2013]**

   (1) Infinite (2) 1

   (3) 2 (4) 3

7. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and $|A| = 4$, then α is equal to **[JEE (Main)-2013]**

   (1) 4 (2) 11

   (3) 5 (4) 0

8. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2\ (1-\beta)^2\ (\alpha - \beta)^2,$$

   then K is equal to **[JEE (Main)-2014]**

   (1) 1 (2) –1

   (3) $\alpha\beta$ (4) $\dfrac{1}{\alpha\beta}$

9. The set of all values of λ for which the system of linear equations

$$2x_1 - 2x_2 + x_3 = \lambda x_1$$
$$2x_1 - 3x_2 + 2x_3 = \lambda x_2$$
$$-x_1 + 2x_2 = \lambda x_3$$

   has a non-trivial solution **[JEE (Main)-2015]**

   (1) Is an empty set

   (2) Is a singleton

   (3) Contains two elements

   (4) Contains more than two elements

10. The system of linear equations

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for **[JEE (Main)-2016]**

(1) Exactly one value of λ

(2) Exactly two values of λ

(3) Exactly three values of λ

(4) Infinitely many values of λ

11. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k,$$ then k is equal to

[JEE (Main)-2017]

(1) z (2) -1

(3) 1 (4) $-z$

12. If S is the set of distinct values of b for which the following system of linear equations

$x + y + z = 1$

$x + ay + z = 1$

$ax + by + z = 0$

has no solution, then S is **[JEE (Main)-2017]**

(1) An infinite set

(2) A finite set containing two or more elements

(3) A singleton

(4) An empty set

13. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to **[JEE (Main)-2018]**

(1) $(-4, -5)$ (2) $(-4, 3)$

(3) $(-4, 5)$ (4) $(4, 5)$

14. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\dfrac{xz}{y^2}$ is equal to **[JEE (Main)-2018]**

(1) -10 (2) 10

(3) -30 (4) 30

15. The system of linear equations

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$ **[JEE (Main)-2019]**

(1) has infinitely many solutions for $a = 4$

(2) is inconsistent when $|a| = \sqrt{3}$

(3) has a unique solution for $|a| = \sqrt{3}$

(4) is inconsistent when $a = 4$

16. If the system of linear equations

$x - 4y + 7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

is consistent, then **[JEE (Main)-2019]**

(1) $g + h + k = 0$ (2) $g + 2h + k = 0$

(3) $g + h + 2k = 0$ (4) $2g + h + k = 0$

17. Let $d \in R$, and

$$A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix},$$

$\theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is **[JEE (Main)-2019]**

(1) -5 (2) $2(\sqrt{2}+1)$

(3) -7 (4) $2(\sqrt{2}+2)$

18. If the system of equations

$x + y + z = 5$

$x + 2y + 3z = 9$

$x + 3y + \alpha z = \beta$

has infinitely many solutions, then $\beta - \alpha$ equals

[JEE (Main)-2019]

(1) 18 (2) 21

(3) 8 (4) 5

19. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$

has a non-trivial solution, is **[JEE (Main)-2019]**

(1) Four (2) One

(3) Three (4) Two

20. If the system of linear equations

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

where a, b, c are non-zero real numbers, has more than one solution, then **[JEE (Main)-2019]**

(1) $b - c + a = 0$

(2) $b + c - a = 0$

(3) $a + b + c = 0$

(4) $b - c - a = 0$

21. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a + b + c)(x + a + b + c)^2$, $x \neq 0$ and $a + b + c \neq 0$, then x is equal to **[JEE (Main)-2019]**

(1) $2(a + b + c)$ (2) $-(a + b + c)$

(3) abc (4) $-2(a + b + c)$

22. An ordered pair (α, β) for which the system of linear equations

$(1 + \alpha) x + \beta y + z = 2$

$\alpha x + (1 + \beta)y + z = 3$

$\alpha x + \beta y + 2z = 2$

has a unique solution, is **[JEE (Main)-2019]**

(1) $(1, -3)$ (2) $(2, 4)$

(3) $(-3, 1)$ (4) $(-4, 2)$

23. If $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$; then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, det (A) lies in the interval

[JEE (Main)-2019]

(1) $\left(1, \frac{5}{2}\right]$ (2) $\left(0, \frac{3}{2}\right]$

(3) $\left[\frac{5}{2}, 4\right)$ (4) $\left(\frac{3}{2}, 3\right]$

24. The set of all values of λ for which the system of linear equations

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$ **[JEE (Main)-2019]**

(has a non-trivial solution)

(1) Contains exactly two elements

(2) Contains more than two elements

(3) Is a singleton

(4) Is an empty set

25. The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$

$cx - y + cz = 0$

$cx + cy - z = 0$

has a non-trivial solution, is **[JEE (Main)-2019]**

(1) -1 (2) 0

(3) 2 (4) $\frac{1}{2}$

26. If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is

[JEE (Main)-2019]

(1) $3x - 4y - 4 = 0$ (2) $3x - 4y - 1 = 0$

(3) $4x - 3y - 1 = 0$ (4) $4x - 3y - 4 = 0$

27. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to

(1) $y(y^2 - 1)$ (2) $y^3 - 1$

(3) $y(y^2 - 3)$ (4) y^3

28. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trivial solution (x, y, z), then $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{2}$ (2) -4

(3) $\frac{3}{4}$ (4) $-\frac{1}{4}$

29. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$; then

for all $\theta \in \left(0, \frac{\pi}{2}\right)$ **[JEE (Main)-2019]**

(1) $\Delta_1 + \Delta_2 = -2x^3$

(2) $\Delta_1 - \Delta_2 = -2x^3$

(3) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$

(4) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$

30. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is

[JEE (Main)-2019]

(1) 10 (2) 12

(3) 7 (4) 9

31. The sum of the real roots of the equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0,$$ is equal to

[JEE (Main)-2019]

(1) 6 (2) 0

(3) –4 (4) 1

32. Let λ be a real number for which the system of linear equations

$x + y + z = 6$

$4x + \lambda y - \lambda z = \lambda - 2$

$3x + 2y - 4z = -5$

has infinitely many solutions. Then λ is a root of the quadratic equation **[JEE (Main)-2019]**

(1) $\lambda^2 + 3\lambda - 4 = 0$ (2) $\lambda^2 - \lambda - 6 = 0$

(3) $\lambda^2 + \lambda - 6 = 0$ (4) $\lambda^2 - 3\lambda - 4 = 0$

33. If $[x]$ denotes the greatest integer $\le x$, then the system of linear equations $[\sin\theta]x + [-\cos\theta]y = 0$

$[\cot\theta]x + y = 0$ **[JEE (Main)-2019]**

(1) Has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(2) Has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

(3) Have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(4) Have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

34. A value of $\theta \in (0, \pi/3)$, for which

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0,$$ is

[JEE (Main)-2019]

(1) $\frac{\pi}{18}$ (2) $\frac{7\pi}{36}$

(3) $\frac{7\pi}{24}$ (4) $\frac{\pi}{9}$

35. If the system of linear equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0$,

where $a, b, c \in R$ are non-zero and distinct; has a non-zero solution, then

[JEE (Main)-2020]

(1) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

(2) a, b, c are in A.P.

(3) $a + b + c = 0$

(4) a, b, c are in G.P.

36. For which of the following ordered pairs (μ, δ), the system of linear equations

$x + 2y + 3z = 1$

$3x + 4y + 5z = \mu$

$4x + 4y + 4z = \delta$

is inconsistent? **[JEE (Main)-2020]**

(1) (4, 3)

(2) (4, 6)

(3) (3, 4)

(4) (1, 0)

37. The system of linear equations

$\lambda x + 2y + 2z = 5$

$2\lambda x + 3y + 5z = 8$

$4x + \lambda y + 6z = 10$ has **[JEE (Main)-2020]**

(1) Infinitely many solutions when $\lambda = 2$

(2) No solution when $\lambda = 8$

(3) A unique solution when $\lambda = -8$

(4) No solution when $\lambda = 2$

38. If for some α and β in R, the intersection of the following three planes

$x + 4y - 2z = 1$

$x + 7y - 5z = \beta$

$x + 5y + \alpha z = 5$

is a line in R^3, then $\alpha + \beta$ is equal to

[JEE (Main)-2020]

(1) -10 (2) 0

(3) 2 (4) 10

39. The following system of linear equations

$7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$, has **[JEE (Main)-2020]**

(1) Infinitely many solutions, (x, y, z) satisfying $x = 2z$

(2) No solution

(3) Only the trivial solution

(4) Infinitely many solutions, (x, y, z) satisfying $y = 2z$

40. Let S be the set of all $\lambda \in R$ for which the system of linear equations

$2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

has no solution. Then the set S

[JEE (Main)-2020]

(1) Is a singleton.

(2) Contains more than two elements.

(3) Is an empty set.

(4) Contains exactly two elements.

41. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} =$

$Ax^3 + Bx^2 + Cx + D$, then $B + C$ is equal to

[JEE (Main)-2020]

(1) 9 (2) -1

(3) 1 (4) -3

42. If the system of equations

$x + y + z = 2$

$2x + 4y - z = 6$

$3x + 2y + \lambda z = \mu$

has infinitely many solutions, then

[JEE (Main)-2020]

(1) $2\lambda - \mu = 5$

(2) $\lambda - 2\mu = -5$

(3) $\lambda + 2\mu = 14$

(4) $2\lambda + \mu = 14$

43. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, $Ax = b$ when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$ and

$b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$, then the determinant of A is equal to

[JEE (Main)-2020]

(1) 4 (2) $\frac{1}{2}$

(3) 2 (4) $\frac{3}{2}$

44. Let $\lambda \in R$. The system of linear equations

$2x_1 - 4x_2 + \lambda x_3 = 1$

$x_1 - 6x_2 + x_3 = 2$

$\lambda x_1 - 10x_2 + 4x_3 = 3$

is inconsistent for **[JEE (Main)-2020]**

(1) exactly two values of λ

(2) exactly one positive value of λ

(3) every value of λ

(4) exactly one negative value of λ

45. If $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero distinct real numbers, then

$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to **[JEE (Main)-2020]**

(1) $y(b - a)$ (2) $y(a - b)$

(3) $y(a - c)$ (4) 0

46. If the system of linear equations

$x + y + 3z = 0$

$x + 3y + k^2z = 0$

$3x + y + 3z = 0$

has a non-zero solution (x, y, z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to **[JEE (Main)-2020]**

(1) 9 (2) 3

(3) –9 (4) –3

47. The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively **[JEE (Main)-2020]**

(1) 5 and 7 (2) 6 and 8

(3) 4 and 9 (4) 5 and 8

48. If the system of linear equations,

$x + y + z = 6$

$x + 2y + 3z = 10$

$3x + 2y + \lambda z = \mu$

has more than two solutions, then $\mu - \lambda^2$ is equal to ____________. **[JEE (Main)-2020]**

49. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to ______ **[JEE (Main)-2020]**

50. If the system of equations

$x - 2y + 3z = 9$

$2x + y + z = b$

$x - 7y + az = 24$, has infinitely many solutions, then $a - b$ is equal to ______. **[JEE (Main)-2020]**

51. The sum of distinct values of λ for which the system of equations

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0,$

has non-zero solutions, is ________ .

[JEE (Main)-2020]

52. The system of linear equations

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

is inconsistent if :

[JEE (Main)-2021]

(1) $k \ne 3, m \ne \frac{4}{5}$ (2) $k \ne 3, m \in R$

(3) $k = 3, m = \frac{4}{5}$ (4) $k = 3, m \ne \frac{4}{5}$

53. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to _____________.

[JEE (Main)-2021]

54. For the system of linear equations :

$x - 2y = 1$, $x - y + kz = -2$, $ky + 4z = 6$, $k \in R$,

consider the following statements :

[JEE (Main)-2021]

(A) The system has unique solution if $k \ne 2, k \ne -2$.

(B) The system has unique solution if $k = -2$.

(C) The system has unique solution if $k = 2$

(D) The system has no solution if $k = 2$.

(E) The system has infinite number of solutions if $k \ne -2$.

Which of the following statements are correct?

(1) (A) and (E) only (2) (A) and (D) only

(3) (B) and (E) only (4) (C) and (D) only

55. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2, + A)(I_2 - A)^{-1}$

$= \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to ________.

[JEE (Main)-2021]

56. If the system of equations

$kx + y + 2z = 1$

$3x - y - 2z = 2$

$-2x - 2y - 4z = 3$

has infinitely many solutions, then k is equal to ___________.

[JEE (Main)-2021]

57. The following system of linear equations **[JEE (Main)-2021]**

$2x + 3y + 2z = 9$

$3x + 2y + 2z = 9$

$x - y + 4z = 8$

(1) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

(2) has a unique solution

(3) does not have any solution

(4) has infinitely many solutions

58. The value of $\begin{vmatrix} (a+1) & (a+2) & a+2 & 1 \\ (a+2) & (a+3) & a+3 & 1 \\ (a+3) & (a+4) & a+4 & 1 \end{vmatrix}$ is :

[JEE (Main)-2021]

(1) 0 (2) $(a+2)(a+3)(a+4)$

(3) –2 (4) $(a+1)(a+2)(a+3)$

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59. Consider the following system of equations : **[JEE (Main)-2021]**

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c,$

where a, b and c are real constants. Then the system of equations :

(1) has infinite number of solutions when $5a = 2b + c$

(2) has no solution for all a, b and c

(3) has a unique solution when $5a = 2b + c$

(4) has a unique solution for all a, b and c

60. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}, i = \sqrt{-1}$. Then, the system of linear equations $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has

[JEE (Main)-2021]

(1) Exactly two solutions

(2) No solution

(3) A unique solution

(4) Infinitely many solutions

61. Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ and

$A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$ where $\omega = \frac{-1+i\sqrt{3}}{3}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP-I_3)^2$ is $\alpha\omega^2$, then the value α is equal to _____. **[JEE (Main)-2021]**

62. The maximum value of

$f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R$ is :

[JEE (Main)-2021]

(1) $\frac{3}{4}$ (2) 5

(3) $\sqrt{7}$ (4) $\sqrt{5}$

63. The system of equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + zk = k^2$ has no solution equal to : **[JEE (Main)-2021]**

(1) 0 (2) 1

(3) –1 (4) –2

64. If $A - \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}I\right) = 0$, then possible value of α is : **[JEE (Main)-2021]**

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

65. If $A=\begin{bmatrix}2 & 3\\ 0 & -1\end{bmatrix}$, then the value of $\det(A^4)$ + $\det\left(A^{10}-(\text{Adj}(2A))^{10}\right)$ is equal to_____.

[JEE (Main)-2021]

66. If x, y, z are in arithmetic progression with common difference d, $x \neq 3d$, and the determinant of the matrix $\begin{bmatrix}3 & 4\sqrt{2} & x\\ 4 & 5\sqrt{2} & y\\ 5 & k & z\end{bmatrix}$ is zero, then the value of k^2 is :

[JEE (Main)-2021]

(1) 12 (2) 36

(3) 72 (4) 6

67. If 1, $\log_{10}(4^x-2)$ and $\log_{10}\left(4^x+\frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant $\begin{vmatrix}2\left(x-\frac{1}{2}\right) & x-1 & x^2\\ 1 & 0 & x\\ x & 1 & 0\end{vmatrix}$ is equal to :

[JEE (Main)-2021]

68. The solutions of the equation

$$\begin{vmatrix}1+\sin^2 x & \sin^2 x & \sin^2 x\\ \cos^2 x & 1+\cos^2 x & \cos^2 x\\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x\end{vmatrix}=0,(0<x<\pi), \text{ are :}$$

[JEE (Main)-2021]

(1) $\frac{\pi}{6},\frac{5\pi}{6}$ (2) $\frac{\pi}{12},\frac{\pi}{6}$

(3) $\frac{5\pi}{12},\frac{7\pi}{12}$ (4) $\frac{7\pi}{12},\frac{11\pi}{12}$

69. Let α, β, γ be real roots of the equation, $x^3 + ax^2 + bx + c = 0$, ($a, b, c \in R$ and $a, b \neq 0$). If the system of equations (in u, v, w) given by $\alpha u + \beta v + \gamma w = 0$; $\beta u + \gamma v + \alpha w = 0$; $\gamma u + \alpha v + \beta w = 0$ has non-trivial solution, then the value of $\frac{a^2}{b}$ is :

[JEE (Main)-2021]

(1) 5 (2) 1

(3) 3 (4) 0

70. Let the system of linear equations

$4x + \lambda y + 2z = 0$

$2x - y + z = 0$

$\mu x + 2y + 3z = 0$, $\lambda, \mu \in R$.

has a non-trivial solution. Then which of the following is true?

[JEE (Main)-2021]

(1) $\mu = -6, \lambda \in R$ (2) $\lambda = 3, \mu \in R$

(3) $\mu = 6, \lambda \in R$ (4) $\lambda = 2, \mu \in R$

71. Let I be an identity matrix of order 2×2 and $P=\begin{bmatrix}2 & -1\\ 5 & -3\end{bmatrix}$. Then the value of $n \in N$ for which $P_n = 5I - 8P$ is equal to _______.

[JEE (Main)-2021]

72. Let a, b, c, d be in arithmetic progression with common difference λ. If $\begin{vmatrix}x+a-c & x+b & x+a\\ x-1 & x+c & x+b\\ x-b+d & x+d & x+c\end{vmatrix}=2$, then value of λ^2 is equal to ______.

[JEE (Main)-2021]

73. The value of $k \in R$, for which the following system of linear equations

$3x - y + 4z = 3$,

$x + 2y - 3z = -2$,

$6x + 5y + kz = -3$,

has infinitely many solutions, is

(1) –3 (2) –5

(3) 5 (4) 3

[JEE (Main)-2021]

74. The values of λ and μ such that the system of equations $x + y + z = 6$, $3x + 5y + 5z = 26$, $x + 2y + \lambda z = \mu$ has no solution, are

[JEE (Main)-2021]

(1) $\lambda \neq 2, \mu = 10$ (2) $\lambda = 3, \mu \neq 10$

(3) $\lambda = 3, \mu = 5$ (4) $\lambda = 2, \mu \neq 10$

75. The values of a and b, for which the system of equations

$2x + 3y + 6z = 8$

$x + 2y + az = 5$

$3x + 5y + 9z = b$

has no solution, are

(1) $a \neq 3, b \neq 13$ (2) $a = 3, b = 13$

(3) $a \neq 3, b = 3$ (4) $a = 3, b \neq 13$

[JEE (Main)-2021]

76. The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$ in the interval

$-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is **[JEE (Main)-2021]**

(1) 2 (2) 1

(3) 4 (4) 3

77. For real numbers α and β consider the following system of linear equations :

$x + y - z = 2$, $x + 2y + \alpha z = 1$, $2x - y + z = \beta$.

If the system has infinite solutions, then $\alpha + \beta$ is equal to _______. **[JEE (Main)-2021]**

78. Let

$$f(x) = \begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}, x \in [0, \pi].$$

Then the maximum value of $f(x)$ is equal to ______.

[JEE (Main)-2021]

79. If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}, i = \sqrt{-1}$ and Q = A^TBA, then the inverse of the matrix $AQ^{2021}A^T$ is equal to **[JEE (Main)-2021]**

(1) $\begin{pmatrix} 1 & 0 \\ 2021\,i & 1 \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 0 \\ -2021\,i & 1 \end{pmatrix}$

(3) $\begin{pmatrix} 1 & -2021\,i \\ 0 & 1 \end{pmatrix}$ (4) $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$

80. Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations.

$(1 + \cos^2\theta)\, x + \sin^2\theta\, y + 4 \sin 3\theta\, z = 0$

$\cos^2\theta\, x + (1 + \sin^2\theta)\, y + 4 \sin 3\theta\, z = 0$

$\cos^2\theta\, x + \sin^2\theta\, y + (1 + 4 \sin 3\theta)\, z = 0$

has a non-trivial solution, then the value of θ is

[JEE (Main)-2021]

(1) $\frac{7\pi}{18}$ (2) $\frac{\pi}{18}$

(3) $\frac{4\pi}{9}$ (4) $\frac{5\pi}{18}$

81. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to

[JEE (Main)-2021]

(1) $A^6 - A$ (2) A^5

(3) $A^5 - A$ (4) A^6

82. Let A be a 3 × 3 real matrix. If det(2 Adj(2 Adj(Adj (2A)))) = 2^{41}, then the value of det (A^2) equals ______. **[JEE (Main)-2021]**

83. If the system of linear equations

$2x + y - z = 3$

$x - y - z = \alpha$

$3x + 3y + \beta z = 3$

has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to ______. **[JEE (Main)-2021]**

84. Let $[\lambda]$ be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations $x + y + z = 4$, $3x + 2y + 5z = 3$, $9x + 4y + (28 + [\lambda])z = [\lambda]$ has a solution is **[JEE (Main)-2021]**

(1) $(-\infty, -9) \cup [-8, \infty)$ (2) $[-9, -8)$

(3) R (4) $(-\infty, -9) \cup (-9, \infty)$

85. Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where [t] denotes the greatest integer less than or equal to t. If det(A) = 192, then the set of values of x is the interval

[JEE (Main)-2021]

(1) [60, 61) (2) [65, 66)

(3) [62, 63) (4) [68, 69)

86. If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :

[JEE (Main)-2021]

(1) $a = -\frac{1}{3}, b \ne \frac{7}{3}$ (2) $a \ne -\frac{1}{3}, b = \frac{7}{3}$

(3) $a \ne \frac{1}{3}, b = \frac{7}{3}$ (4) $a = \frac{1}{3}, b \ne \frac{7}{3}$

87. If $a_r = \cos\frac{2r\pi}{9} + i\sin\frac{2r\pi}{9}, r = 1, 2, 3...., i = \sqrt{-1},$

then the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to :

[JEE (Main)-2021]

(1) $a_1a_9 - a_3a_7$ (2) $a_2a_6 - a_4a_8$

(3) a_9 (4) a_5

88. If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$x + (\cos\gamma)y + (\cos\beta)z = 0$

$(\cos\gamma)x + y + (\cos\alpha)z = 0$

$(\cos\beta)x + (\cos\alpha)y + z = 0$

has :

(1) a unique solution

(2) no solution

(3) infinitely many solutions

(4) exactly two solutions

[JEE (Main)-2021]

89. Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S_1 be the set of all $a \in \boldsymbol{R}$ for which system is inconsistent and S_2 be the set of all $a \in \boldsymbol{R}$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then **[JEE (Main)-2021]**

(1) $n(S_1) = 2, n(S_2) = 0$ (2) $n(S_1) = 1, n(S_2) = 0$

(3) $n(S_1) = 2, n(S_2) = 2$ (4) $n(S_1) = 0, n(S_2) = 2$

90. Let $a_1, a_2, a_3 ... , a_{10}$ in G.P with $a_i > 0$ for $i = 1, 2, ... , 10$ and S be the set of pairs (r, k), $r, k \in N$ (the set of natural numbers for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

[JEE (Main)-2021]

(1) 2 (2) 10

(3) 4 (4) Infinitely many

91. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ji}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is **[JEE (Main)-2021]**

(1) 1/9 (2) 1/81

(3) 3 (4) 1/3

92. Let $a - 2b + c = 1$.

If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then

[JEE (Main)-2021]

(1) $f(50) = 1$ (2) $f(-50) = 501$

(3) $f(-50) = -1$ (4) $f(50) = -501$

93. The number of values of α for which the system of equations :

$x + y + z = \alpha$

$\alpha x + 2\alpha y + 3z = -1$

$x + 3\alpha y + 5z = 4$

is inconsistent, is

[JEE (Main)-2022]

(1) 0 (2) 1

(3) 2 (4) 3

94. Let $S = \left\{\sqrt{n} : 1 \le n \le 50 \text{ and } n \text{ is odd}\right\}$.

Let $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$

If $\sum_{a \in S} \det(\text{adj } A) = 100\lambda$, then λ is equal to :

[JEE (Main)-2022]

(1) 218 (2) 221

(3) 663 (4) 1717

95. Let the system of linear equations

$x + y + az = 2$

$3x + y + z = 4$

$x + 2z = 1$

have a unique solution (x^*, y^*, z^*). If (α, x^*), (y^*, α) and $(x^*, -y^*)$ are collinear points, then the sum of absolute values of all possible values of α is

[JEE (Main)-2022]

(1) 4 (2) 3

(3) 2 (4) 1

96. Let A be a 3 × 3 real matrix such that

$$A\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix};\ A\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix}\text{ and }A\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}.$$

If $X = (x_1, x_2, x_3)^T$ and I is an identity matrix of order 3, then the system $(A-2I)X=\begin{pmatrix}4\\1\\1\end{pmatrix}$ has :

[JEE (Main)-2022]

(1) No solution

(2) Infinitely many solutions

(3) Unique solution

(4) Exactly two solutions

97. The system of equations

$-kx + 3y - 14z = 25$

$-15x + 4y - kz = 3$

$-4x + y + 3z = 4$

is consistent for all k in the set

[JEE (Main)-2022]

(1) R (2) $R - \{-11, 13\}$

(3) $R - \{13\}$ (4) $R - \{-11, 11\}$

98. Let A be a 3 × 3 invertible matrix. If $|\text{adj}\ (24A)| = |\text{adj}\ (3\ \text{adj}\ (2A))|$, then $|A|^2$ is equal to :

[JEE (Main)-2022]

(1) 6^6 (2) 2^{12}

(3) 2^6 (4) 1

99. The ordered pair (a, b), for which the system of linear equations

$3x - 2y + z = b$

$5x - 8y + 9z = 3$

$2x + y + az = -1$

has no solution, is :

[JEE (Main)-2022]

(1) $\left(3, \frac{1}{3}\right)$ (2) $\left(-3, \frac{1}{3}\right)$

(3) $\left(-3, -\frac{1}{3}\right)$ (4) $\left(3, -\frac{1}{3}\right)$

100. If the system of equations

$\alpha x + y + z = 5$, $x + 2y + 3z = 4$, $x + 3y + 5z = \beta$

has infinitely many solutions, then the ordered pair (α, β) is equal to:

[JEE (Main)-2022]

(1) $(1, -3)$ (2) $(-1, 3)$

(3) $(1, 3)$ (4) $(-1, -3)$

101. Let the system of linear equations $x + 2y + z = 2$, $\alpha x + 3y - z = \alpha$, $-\alpha x + y + 2z = -\alpha$ be inconsistent. Then α is equal to :

[JEE (Main)-2022]

(1) $\frac{5}{2}$ (2) $-\frac{5}{2}$

(3) $\frac{7}{2}$ (4) $-\frac{7}{2}$

102. The positive value of the determinant of the matrix A, whose

$$\text{Adj}(\text{Adj}(A))=\begin{pmatrix}14 & 28 & -14\\-14 & 14 & 28\\28 & -14 & 14\end{pmatrix},$$ is __________.

[JEE (Main)-2022]

103. Let $f(x) = \begin{vmatrix} a & -1 & 0\\ ax & a & -1\\ ax^2 & ax & a\end{vmatrix}$, $a \in R$. Then the sum of the square of all the values of a, for which $2f'(10) - f'(5) + 100 = 0$, is

[JEE (Main)-2022]

(1) 117 (2) 106

(3) 125 (4) 136

104. Let A and B be two 3 × 3 matrices such that $AB = I$ and $|A| = \frac{1}{8}$. Then $|adj\ (B\ adj(2A))|$ is equal to

[JEE (Main)-2022]

(1) 16 (2) 32

(3) 64 (4) 128

105. If the system of linear equations

$2x + 3y - z = -2$

$x + y + z = 4$

$x - y + |\lambda|z = 4\lambda - 4$

where $\lambda \in$ R, has no solution, then

[JEE (Main)-2022]

(1) $\lambda = 7$ (2) $\lambda = -7$

(3) $\lambda = 8$ (4) $\lambda^2 = 1$

106. Let A be a matrix of order 3 × 3 and det (A) = 2. Then det (det (A) adj (5 adj (A^3))) is equal to ______.

[JEE (Main)-2022]

(1) 512×10^6 (2) 256×10^6

(3) 1024×10^6 (4) 256×10^{11}

107. If the system of linear equations

$2x - 3y = \gamma + 5,$

$\alpha x + 5y = \beta + 1$, where $\alpha, \beta, \gamma \in R$ has infinitely many solutions, then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to _______.

[JEE (Main)-2022]

108. If the system of linear equations

$2x + y - z = 7$

$x - 3y + 2z = 1$

$x + 4y + \delta z = k$, where $\delta, k \in \mathbf{R}$

has infinitely many solutions, then $\delta + k$ is equal to:

[JEE (Main)-2022]

(1) –3 (2) 3

(3) 6 (4) 9

109. Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$. If $B = I - {}^5C_1(\text{adj}A) + {}^5C_2(\text{adj}A)^2 - \ldots - {}^5C_5(\text{adj}A)^5$, then the sum of all elements of the matrix B is

[JEE (Main)-2022]

(1) –5 (2) –6

(3) –7 (4) –8

110. The number of q $\in (0, 4\pi)$ for which the system of linear equations

$3(\sin 3\theta)\, x - y + z = 2$

$3(\cos 2\theta)\, x + 4y + 3z = 3$

$6x + 7y + 7z = 9$

has no solution, is

[JEE (Main)-2022]

(1) 6 (2) 7

(3) 8 (4) 9

111. Let $A = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}, a, b \in \mathbb{R}$.

If for some $n \in N, A^n = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$ then $n + a + b$ is equal to __________.

[JEE (Main)-2022]

112. If the system of linear equations.

$8x + y + 4z = -2$

$x + y + z = 0$

$\lambda x - 3y = \mu$

has infinitely many solutions, then the distance of the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane $8x + y + 4z + 2 = 0$ is

[JEE (Main)-2022]

(1) $3\sqrt{5}$ (2) 4

(3) $\frac{26}{9}$ (4) $\frac{10}{3}$

113. Let A be a 2 × 2 matrix with det (A) = –1 and det ((A + I) (Adj (A) + I)) = 4. Then the sum of the diagonal elements of A can be

[JEE (Main)-2022]

(1) –1 (2) 2

(3) 1 (4) $-\sqrt{2}$

114. Consider a matrix $A = \begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{bmatrix}$,

where α, β, γ are three distinct natural numbers.

If $\frac{\det(\text{adj}(\text{adj}(\text{adj}(\text{adj } A))))}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}} = 2^{32} \times 3^{16}$, then the number of such 3-tuples (α, β, γ) is _________.

[JEE (Main)-2022]

115. Let the matrix $A=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$ and the matrix $B_0 = A^{49} + 2A^{98}$. If $B_n = \text{Adj}(B_{n-1})$ for all $n \geq 1$, then $\det(B_4)$ is equal to :

[JEE (Main)-2022]

(1) 3^{28} (2) 3^{30}

(3) 3^{32} (4) 3^{36}

116. If the system of equations

$x + y + z = 6$

$2x + 5y + \alpha z = \beta$

$x + 2y + 3z = 14$

has infinitely many solutions, then $\alpha + \beta$ is equal to

[JEE (Main)-2022]

(1) 8 (2) 36

(3) 44 (4) 48

117. Let A and B be two 3×3 non-zero real matrices such that AB is a zero matrix. Then

[JEE (Main)-2022]

(1) the system of linear equations $AX = 0$ has a unique solution

(2) the system of linear equations $AX = 0$ has infinitely many solutions

(3) B is an invertible matrix

(4) adj(A) is an invertible matrix

118. The number of matrices $A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$, where $a, b, c, d \in \{-1, 0, 1, 2, 3, \ldots, 10\}$, such that $A = A^{-1}$, is ______.

[JEE (Main)-2022]

119. Let p and $p + 2$ be prime numbers and let

$$\Delta=\begin{vmatrix}p! & (p+1)! & (p+2)!\\(p+1)! & (p+2)! & (p+3)!\\(p+2)! & (p+3)! & (p+4)!\end{vmatrix}$$

Then the sum of the maximum values of α and β, such that p^{α} and $(p + 2)^{\beta}$ divide Δ, is _______.

[JEE (Main)-2022]

120. The number of real values of λ, such that the system of linear equations

$2x - 3y + 5z = 9$

$x + 3y - z = -18$

$3x - y + (\lambda^2 - |\lambda|)z = 16$

has no solutions, is **[JEE (Main)-2022]**

(1) 0 (2) 1

(3) 2 (4) 4

❑ ❑ ❑

Chapter 7

Determinants

1. Answer (1)

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then adj $(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\therefore$ $|A| = |\text{adj } A| = ad - bc$

Also adj[adj A] $= \begin{bmatrix} a & b \\ c & d \end{bmatrix} = A$

$\therefore$ Both statements are true but (2) is not correct explanation of (1).

2. Answer (2)

Applying D' = D is first determinant and $R_2 \leftrightarrow R_3$ and $R_1 \leftrightarrow R_2$ in second determinant

$$\begin{vmatrix} a & -b & c \\ a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \end{vmatrix} + \begin{vmatrix} a(-1)^{n+2} & b(-1)^{n+1} & c(-1)^{n} \\ a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \end{vmatrix} = 0$$

Then $\begin{vmatrix} a+(-1)^{n+2}a & -b+(-1)^{n+1}b & c+(-1)^{n}c \\ a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \end{vmatrix} = 0$

if n is an odd integer.

3. Answer (4)

The given system of linear equations can be put in the matrix form as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ -8 \end{bmatrix} \quad \text{by} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 5 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

Clearly the given system of equations has no solution.

Alter

Subtracting the addition of first two equations from third equation, we get,

0 = –5, which is an absurd result.

Hence the given system of equation has no solution.

4. Answer (3)

For non-trivial solution

$$\begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow$ $1(-3 + k) + k(-k + 3k) + 1(k - 9) = 0$

$\Rightarrow$ $k - 3 + 2k^2 + k - 9 = 0$

$\Rightarrow$ $2k^2 + 2k - 12 = 0$

$\Rightarrow$ $k^2 + k - 6 = 0$

$\Rightarrow$ $(k + 3)(k - 2) = 0$

$\Rightarrow$ $k = -3, 2$

Thus, the set of values of k is $R - \{-3, 2\}$ for trivial solution.

5. Answer (2)

$P^2 + Q^2 = 0 \Rightarrow \det(P^2 + Q^2) = 0$

6. Answer (2)

$$\begin{vmatrix} k+1 & 8 \\ k & k+3 \end{vmatrix} = 0$$

$\Rightarrow$ $k^2 + 4k + 3 - 8k = 0$

$\Rightarrow$ $k = 1, 3$

When $k = 1$, equation change to

$2x + 8y = 4 \quad \Rightarrow \quad x + 4y = 2$

and $x + 4y = 2 \quad \Rightarrow \quad x + 4y = 2$

$\Rightarrow$ Infinitely many solutions

When $k = 3$

$4x + 8y = 12 \quad \Rightarrow \quad k + 2y = 3$

and $3x + 6y = 8 \qquad$ and $x + 2y = \dfrac{8}{3}$

$\Rightarrow$ No solution

$\therefore$ One value of k exists for which system of equation has no solution.

7. Answer (2)

$$\begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$$

$|P| = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6)$

$= 2\alpha - 6$

Also, $|P| = |A|^2 = 16$

$2\alpha = 22$

$\alpha = 11$

8. Answer (1)

$$\begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & 1 \\ \alpha^2 & \beta^2 & 1 \end{vmatrix} \times \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$= [(1 - \alpha)(1 - \beta)(1 - \beta)]^2$

So, $\boxed{k = 1}$

9. Answer (3)

$x_1(2-\lambda) - 2x_2 + x_3 = 0$

$2x_1 + x_2(-\lambda - 3) + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

$$\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -\lambda-3 & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$(2-\lambda)(\lambda^2 + 3\lambda - 4) + 2(-2\lambda + 2) + (4 - \lambda - 3) = 0$

$2\lambda^2 + 6\lambda - 8 - \lambda^3 - 3\lambda^2 + 4\lambda - 4\lambda + 4 - \lambda + 1 = 0$

$\Rightarrow \quad -\lambda^3 - \lambda^2 + 5\lambda - 3 = 0$

$\Rightarrow \quad \lambda^3 + \lambda^2 - 5\lambda + 3 = 0$

$\lambda^3 - \lambda^2 + 2\lambda^2 - 2\lambda - 3\lambda + 3 = 0$

$\lambda^2(\lambda - 1) + 2\lambda(\lambda - 1) - 3(\lambda - 1) = 0$

$(\lambda - 1)(\lambda^2 + 2\lambda - 3) = 0$

$(\lambda - 1)(\lambda + 3)(\lambda - 1) = 0$

$\Rightarrow \quad \lambda = 1, 1, -3$

Two elements.

10. Answer (3)

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$1(\lambda + 1) - \lambda(-\lambda^2 + 1) + 1(-\lambda - 1) = 0$

$\lambda^3 - \lambda + \lambda + 1 - \lambda - 1 = 0$

$\lambda^3 - \lambda = 0$

$\lambda(\lambda^2 - 1) = 0$

$\lambda = 0, \lambda = \pm 1$

Exactly three values of λ

11. Answer (4)

$2\omega + 1 = z, \; z = \sqrt{3}\, i$

$\omega = \dfrac{-1+\sqrt{3}i}{2} \rightarrow$ Cube root of unity.

$C_1 \rightarrow C_1 + C_2 + C_3$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} = \begin{vmatrix} 3 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix}$$

$= 3(\omega^2 - \omega^4)$

$= 3\left[\left(\dfrac{-1-\sqrt{3}i}{2}\right) - \left(\dfrac{-1+\sqrt{3}i}{2}\right)\right]$

$= -3\sqrt{3}i$

$= -3z$

$\therefore k = -z$

12. Answer (3)

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $-(1-a)^2 = 0$

$\Rightarrow$ $a = 1$

For $a = 1$

Eq. (1) & (2) are identical *i.e.*, $x + y + z = 1$

To have no solution with $x + by + z = 0$.

$b = 1$

13. Answer (3)

$$\Delta = \begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$$

$x = -4$ makes all three row identical

hence $(x + 4)^2$ will be factor

Also, $C_1 \to C_1 + C_2 + C_2$

$$\Delta = \begin{vmatrix} 5x-4 & 2x & 2x \\ 5x-4 & x-4 & 2x \\ 5x-4 & 2x & x-4 \end{vmatrix}$$

$\Rightarrow$ $5x - 4$ is a factor

$$\Delta = \lambda(5x-4)(x+4)^2$$

$\therefore$ $B = 5$, $A = -4$

14. Answer (2)

$\because$ System of equation has non-zero solution.

$$\therefore \quad \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$\Rightarrow$ $44 - 4k = 0$

$\therefore$ $k = 11$

Let $z = \lambda$

$\therefore$ $x + 11y = -3\lambda$

and $3x + 11y = 2\lambda$

$$\therefore \quad x = \frac{5\lambda}{2},\ y = -\frac{\lambda}{2},\ z = \lambda$$

$$\therefore \quad \frac{xz}{y^2} = \frac{\frac{5\lambda}{2}\cdot\lambda}{\left(-\frac{\lambda}{2}\right)^2} = 10$$

15. Answer (2)

The equations are

$x + y + z = 2$ ...(1)

$2x + 3y + 2z = 5$ ...(2)

$2x + 3y + (a^2 - 1)z = a + 1$ ...(3)

$$\text{Here, } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2-1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 0 & a^2-3 \end{vmatrix}$$

$= a^2 - 3$

$a^2 - 3 = 0 \quad \Rightarrow \quad |a| = \sqrt{3}$

If $a^2 = 3$, then plane represented by (2) and (3) are parallel.

$\therefore$ Given system of equation is inconsistent.

16. Answer (4)

$\because$ $x - 4y + 7z = g$ ...(i)

$3y - 5z = h$ ...(ii)

$-2x + 5y - 9z = k$ ...(iii)

from 2 (equation (i)) + equation (ii) + equation (iii):

$0 = 2g + h + k$.

$\therefore$ $2g + h + k = 0$

then system of equation is consistent.

17. Answer (1)

$$\det(A) = \begin{vmatrix} -2 & 4+d & \sin\theta-2 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & -\sin\theta+2+2d \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 4+d & \sin\theta-2 \\ 1 & \sin\theta+2 & d \\ 1 & 0 & 0 \end{vmatrix}$$

$= d(4 + d) - (\sin^2\theta - 4)$

$\Rightarrow$ $\det(A) = d^2 + 4d + 4 - \sin^2\theta = (d + 2)^2 - \sin^2\theta$

min det (A) is attained when $\sin^2\theta = 1$

$\therefore$ $(d + 2)^2 - 1 = 8 \Rightarrow (d + 2)^2 = 9 \Rightarrow d + 2 = \pm 3$

$\Rightarrow$ $d = -5$ or 1

18. Answer (3)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = 2\alpha - 9 - \alpha + 3 + 1 = \alpha - 5$$

$$\Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & \alpha \end{vmatrix} = 5(2\alpha - 9) - 1(9\alpha - 3\beta) + (27 - 2\beta)$$

$= 10\alpha - 45 - 9\alpha + 3\beta + 27 - 2\beta$

$= \alpha + \beta - 18$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 9 & 3 \\ 1 & \beta & \alpha \end{vmatrix} = 9\alpha - 3\beta - 5(\alpha - 3) + 1(\beta - 9)$$

$= 9\alpha - 3\beta - 5\alpha + 15 + \beta - 9$

$= 4\alpha - 2\beta + 6$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & \beta \end{vmatrix} = 2\beta - 27 - \beta + 9 + 5 = \beta - 13$$

$\Rightarrow$ for infinitely many solutions

$\Delta_1 = \Delta_2 = \Delta_3 = \Delta = 0$

$\Rightarrow \alpha = 5,\ \beta = 13 \Rightarrow \beta - \alpha = 8$

19. Answer (4)

For non-trivial solution

$\Delta = 0$

$$\begin{vmatrix} \sin 3\theta & \cos 2\theta & 2 \\ 1 & 3 & 7 \\ -1 & 4 & 7 \end{vmatrix} = 0$$

$\sin 3\theta(21 - 28) - \cos 2\theta(7 + 7) + 2(4 + 3) = 0$

$\sin 3\theta + 2\cos 2\theta - 2 = 0$

$3\sin\theta - 4\sin^3\theta + 2 - 4\sin^2\theta - 2 = 0$

$4\sin^3\theta + 4\sin^2\theta - 3\sin\theta = 0$

$\sin\theta\,(4\sin^2\theta + 4\sin\theta - 3) = 0$

$\sin\theta\,(4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3) = 0$

$\sin\theta\,[2\sin\theta\,(2\sin\theta - 1) + 3\,(2\sin\theta - 1)] = 0$

$\sin\theta = 0,\ \sin\theta = \frac{1}{2}$ $\quad\left(\because \quad \sin\theta \neq -\frac{3}{2}\right)$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Option (4) is correct.

20. Answer (4)

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$ for infinite solution

$$\Delta_1 = \begin{vmatrix} a & 2 & 3 \\ b & -1 & 5 \\ c & -3 & 2 \end{vmatrix} = a(13) + 2(5c - 2b) + 3(-3b + c)$$

$= 13a - 13b + 13c = 0$

i.e, $a - b + c = 0$

or $b - c - a = 0$

21. Answer (4)

$$\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_1 \to C_1 - C_3,\ C_2 \to C_2 - C_3$

$$\Delta = (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & -b-c-a & 2b \\ c+a+b & c+a+b & c-a-b \end{vmatrix}$$

$= (a + b + c)(a + b + c)^2$

Option (4) is correct

22. Answer (2)

For unique solution,

$$\Delta = \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$\begin{vmatrix} 1+\alpha+\beta+1 & \beta & 1 \\ \alpha & \beta+1 & 1 \\ \alpha+\beta+2 & \beta & 2 \end{vmatrix} \neq 0$$

$$(\alpha+\beta+2)\begin{vmatrix} 1 & \beta & 1 \\ 1 & \beta+1 & 1 \\ 1 & \beta & 2 \end{vmatrix} \neq 0$$

$$(\alpha+\beta+2)\begin{vmatrix}1 & \beta & 1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix} \neq 0$$

$(\alpha + \beta + 2)\ 1(1) \neq 0$

$\boxed{\alpha+\beta+2 \neq 0}$

23. Answer (4)

$$|A| = \begin{vmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix}$$

$$= \begin{vmatrix}0 & 0 & 2\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix} R_1 \to R_1 + R_3$$

$= 2(\sin^2\theta + 1)$

as $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$

$\Rightarrow \sin^2\theta \in \left(0, \frac{1}{2}\right)$

$\therefore$ $\det(A) \in [2, 3)$

$[2, 3) \subset \left(\frac{3}{2}, 3\right]$

24. Answer (3)

$x(1 - \lambda) - 2y - 2z = 0$

$x + (2 - \lambda)y + z = 0$

$-x - y - \lambda z = 0$

for getting a non-trivial solution

$\Delta = 0$

$$\begin{vmatrix}1-\lambda & -2 & -2\\ 1 & 2-\lambda & 1\\ -1 & -1 & -\lambda\end{vmatrix} = 0$$

$\Rightarrow (1 - \lambda)^3 = 0$

$\lambda = 1$

25. Answer (4)

If the system of equations has non-trivial solutions, then

$$\begin{vmatrix}1 & -c & -c\\ c & -1 & c\\ c & c & -1\end{vmatrix} = 0$$

$\Rightarrow (1-c^2)+c(-c-c^2)-c(c^2+c)=0$

$\Rightarrow (1+c)(1-c)-2c^2(1+c)=0$

$\Rightarrow (1+c)(1-c-2c^2)=0$

$\Rightarrow (1+c)^2(1-2c)=0$

$\Rightarrow c=-1 \text{ or } \frac{1}{2}$

26. Answer (4)

$x - 2y + kz = 1,\ 2x + y + z = 2,\ 3x - y - kz = 3$

$$\Delta = \begin{vmatrix}1 & -2 & k\\ 2 & 1 & 1\\ 3 & -1 & -k\end{vmatrix} = 1(-k + 1) + 2(-2k - 3) + k(-2 - 3)$$

$= -k + 1 - 4k - 6 - 5k$

$= -10k - 5 = -5(2k + 1)$

$$\Delta_1 = \begin{vmatrix}1 & -2 & k\\ 2 & 1 & 1\\ 3 & -1 & -k\end{vmatrix} = -5(2k+1)$$

$$\Delta_2 = \begin{vmatrix}1 & 1 & k\\ 2 & 2 & 1\\ 3 & 3 & -k\end{vmatrix} = 0,\ \Delta_3 = \begin{vmatrix}1 & -2 & 1\\ 2 & 1 & 2\\ 3 & -1 & 3\end{vmatrix} = 0$$

$\because$ $z \neq 0$

$\Rightarrow \Delta = 0$

$\Rightarrow k = -\frac{1}{2}$

$\therefore$ System of equation has infinite many solutions.

Let $z = \lambda \neq 0$ then $x = \frac{10-3\lambda}{10}$ and $y = -\frac{2\lambda}{5}$

$\therefore$ (x, y) must lie on line $4x - 3y - 4 = 0$

27. Answer (4)

Let $\alpha = \omega$ and $\beta = \omega^2$ are roots of $x^2 + x + 1 = 0$

$$\therefore \begin{vmatrix}y+\omega & 1 & \omega^2\\ 1 & y+\omega^2 & \omega\\ \omega^2 & \omega & 1+y\end{vmatrix} \text{ operate } c_1 \to c_1 + c_2 + c_3$$

$$= y\begin{vmatrix}1 & 1 & \omega^2\\ 1 & y+\omega^2 & \omega\\ 1 & \omega & 1+y\end{vmatrix} \left(\text{By } \begin{matrix}R_2 \to R_2 - R_1\\ R_3 \to R_3 - R_1\end{matrix}\right)$$

$$= y\begin{vmatrix} 1 & 1 & \omega^2 \\ 0 & y+\omega^2-1 & \omega-\omega^2 \\ 0 & \omega-1 & 1+y-\omega^2 \end{vmatrix}$$

$= y\{(y + \omega^2 - 1)(1 + y - \omega^2) - \omega(\omega - 1)(1 - \omega)\}$

$= y(y^2 - (\omega^2 - 1)^2) + y\omega(\omega - 1)^2$

$= y^3 + y(\omega - 1)^2 (\omega - (\omega + 1)^2) = y^3$

28. Answer (1)

$$\Delta = 0 \;\Rightarrow\; \begin{vmatrix} 2 & 3 & -1 \\ 1 & k & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0 \;\Rightarrow\; \boxed{k = \frac{9}{2}}$$

$\therefore$ Equations are $2x + 3y - z = 0$ ...(i)

$2x - y + z = 0$ ...(ii)

$2x + 9y - 4z = 0$ ...(iii)

By (i) – (ii), $\boxed{2y = z}$

$\therefore$ $\boxed{z = -4x}$ and $\boxed{2x + y = 0}$

$$\therefore \frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k = \frac{-1}{2} + \frac{1}{2} - 4 + \frac{9}{2} = \frac{1}{2}$$

29. Answer (1)

$$\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

$= x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta)$

$+ \cos\theta(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \cos\theta\sin\theta$

$+ x\cos^2\theta$

$= -x^3 - x + x$

$= -x^3$

Similarly, $\Delta_1 = -x^3$

$\Delta_1 + \Delta_2 = -2x^3$

30. Answer (1)

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$ have infinite solution

$\Delta = 0, \;\; \Delta x = \Delta y = \Delta z = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$\Rightarrow$ $1(2\lambda - 6) - 1(\lambda - 2) + 1(3 - 2) = 0$

$\Rightarrow$ $2\lambda - 6 - \lambda + 2 + 1 = 0$

$\boxed{\lambda = 3}$

$$\text{Now, } \Delta x = \begin{vmatrix} 5 & 1 & 1 \\ 6 & 2 & 2 \\ \mu & 3 & 3 \end{vmatrix} = 0, \;\; \Delta y = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 6 & 2 \\ 1 & \mu & 3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 5 & 1 \\ 0 & 1 & 1 \\ 0 & \mu-5 & 2 \end{vmatrix} = 0$$

$\Rightarrow$ $1(2 - \mu + 5) = 0$

$\boxed{\mu = 7}$

$$\Delta z = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 6 \\ 1 & 3 & \mu \end{vmatrix} = \begin{vmatrix} 1 & 1 & 5 \\ 0 & -1 & -1 \\ 0 & 2 & \mu-5 \end{vmatrix}$$

$\Rightarrow$ $1(5 - \mu + 2) = 0$

$\Rightarrow$ $\mu = 7$

So, $\lambda + \mu = 10$

31. Answer (2)

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

$\Rightarrow$ $x(-3x^2 - 6x - 2x^2 + 6x) - 6(-3x + 9 - 2x - 4)$

$-(4x - 9xA) = 0$

$\Rightarrow$ $x(-5x^2) - 6(-5x + 5) - 4x + 9x = 0$

$\Rightarrow$ $x^3 - 7x + 6 = 0$

All the roots are real

$\therefore$ Sum of real roots = $\frac{0}{1}$ = 0

32. Answer (2)

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 4-\lambda & 2\lambda & -\lambda \\ 1 & 6 & -4 \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda = 3$$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ \lambda-2 & \lambda & -\lambda \\ -5 & 2 & -4 \end{vmatrix} = 0 \quad \text{for } \lambda = 3$$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 4 & \lambda-2 & -\lambda \\ 3 & -5 & -4 \end{vmatrix} = 0 \quad \text{for } \lambda = 3$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 4 & \lambda & \lambda-2 \\ 3 & 2 & -5 \end{vmatrix} = 0 \quad \text{for } \lambda = 3$$

$\therefore$ For $\lambda = 3$, infinitely many solutions is obtained.

33. Answer (3)

There are two cases.

Case 1 : $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

So ; $[\sin\theta] = 0$, $[-\cos\theta] = 0$, $[\cot\theta] = -1$

The system of equations will be ;

$0 \cdot x + 0 \cdot y = 0$ and $-x + y = 0$

(Infinitely many solutions)

Case 2 : $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

So ; $[\sin\theta] = -1$, $[-\cos\theta] = 0$,

The system of equations will be ;

$-x + 0 \cdot y = 0$ and $[\cot\theta]\, x + y = 0$

Clearly $x = 0$ and $y = 0$ (unique solution)

34. Answer (4)

$C_1 \rightarrow C_1 + C_2$

$$\begin{vmatrix} 2 & \sin^2\theta & 4\cos 6\theta \\ 2 & 1+\sin^2\theta & 4\cos 6\theta \\ 1 & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 - R_2$, $R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & (1+4\cos 6\theta) \end{vmatrix} = 0$$

$\Rightarrow 2 + 4\cos 6\theta = 0$

$$\cos 6\theta = \frac{-1}{2}$$

$\because 6\theta \in (0, 2\pi)$

So, $6\theta = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{9}$ or $\frac{2\pi}{9}$

35. Answer (1)

For non-trivial solution, $\Delta = 0$

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & a & a \\ 1 & 2b & b \\ 1 & 3c & c \end{vmatrix} = 0$$

$\Rightarrow 2bc - 3bc + a(b - c) + a(3c - 2b) = 0$

$\Rightarrow -bc - ab + 2ac = 0$

$ab + bc = 2ac$

$$\frac{1}{c} + \frac{1}{a} = \frac{2}{b}$$

a, b, c are in HP

36. Answer (1)

For inconsistent system we need

$\Delta = 0$ and atleast one of Δx, Δy $\Delta z \neq 0$

$$\therefore \quad \Delta = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 4 & 4 \end{vmatrix} = 0$$

$$\Delta_x = \begin{vmatrix} 1 & 2 & 3 \\ \mu & 4 & 5 \\ \delta & 4 & 4 \end{vmatrix}$$

$= (-4) - 2(4\mu - 5\delta) + 3(4\mu - 4\delta) \neq 0$

$\Rightarrow -4 + 4\mu - 2\delta \neq 0$

$\Rightarrow 2\mu \neq \delta + 2$

$\therefore$ $(\mu, \delta) = (4, 3)$ is only possible in given options

37. Answer (4)

$$\Delta = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix}$$

$= \lambda(18 - 5\lambda) - 2(12\lambda - 20) + 2(2\lambda^2 - 12)$

$\Rightarrow$ $\Delta = -\lambda^2 - 6\lambda + 16$

$\Rightarrow$ $\Delta = (\lambda + 8)(2 - \lambda)$

$\Rightarrow$ $\Delta = 0$ for $\lambda = 2$ or $\lambda = -8$

$\Rightarrow$ for $\lambda = 2$ $\Delta_x = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix}$

$\Rightarrow$ $\Delta_x = 5(8) - 2(-2) + 2(-14)$

$\Delta_x = 40 + 4 - 28 = 16 \neq 0$

$\therefore$ System has no solution.

38. Answer (4)

$\because$ Three equations have infinitely many solutions, so

$$\begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$$

$\Rightarrow$ $\boxed{\alpha = -3}$

Putting the value of α in (3)

$x + 4y - 2z = 1$ and $x + 5y - 3z = 5$

On solving $y = z + 4$ and $x = -2z - 15$

Substituting these values in (2)

$x + 7y - 5z = \beta = -2z - 15 + 7z + 28 - 5z$

$\Rightarrow$ $\boxed{\beta = 13}$

So $\alpha + \beta = 10$

39. Answer (1)

As $\begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 0$

$\Rightarrow$ The system of equations has infinite trivial solutions.

Also adding equations (1) and 3(3) yield

$10x = 20z \quad \Rightarrow \quad x = 2z$

40. Answer (4)

$$\begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $2\lambda^2 - \lambda - 1 = 0$

$\Rightarrow$ $\lambda = 1$ or $-\frac{1}{2}$

When $\lambda = 1$

$2x - y + 2z = 2$ ...(1)

$x - 2y + z = -4$ ...(2)

$x + y + z = 4$ ...(3)

Adding (2) and (3), we get

$2x - y + 2z = 0$ (contradiction) hence no solution.

When $\lambda = -\frac{1}{2}$

$2x - y + 2z = 2$ ...(1)

$x - 2y - \frac{1}{2}z = -4$ ...(2)

$x - \frac{1}{2}y + z = 4$ ...(3)

(1) and (3) contradict each other, hence no solution.

41. Answer (4)

$$\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x-2 & x-1 & x-1 \\ 2x-3 & x-1 & x-1 \\ 3x-5 & 2x-3 & 5x-9 \end{vmatrix} \quad \begin{matrix} C_3 \to C_3 - C_2 \\ C_2 \to C_2 - C_1 \end{matrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x-2 & x-1 & x-1 \\ x-1 & 0 & 0 \\ 3x-5 & 2x-3 & 5x-9 \end{vmatrix} \quad R_2 \to R_2 - R_1$$

$\Rightarrow$ $\Delta = -(x-1)\left[\left(5x^2 - 14x + 9\right) - \left(2x^2 - 5x + 3\right)\right]$

$= -3x^3 + 12x^2 - 15x + 6$

So; $B + C = -3$

42. Answer (4)

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0$$

$\Rightarrow$ $-15 + 6 + 2\lambda = 0$

$\Rightarrow$ $\lambda = \frac{9}{2}$

Substituting the value of λ in equations, we get

$x + y + z = 2$ ...(1)

$2x + 4y - z = 6$ ...(2)

$6x + 4y + 9z = 2\mu$ ...(3)

1 $\times$ 8 $-$ 2 gives, $6x + 4y + 9z = 10$

So for infinitely many solutions, $2\mu = 10$

$\Rightarrow$ $\mu = 5$

43. Answer (3)

A $x = b$ has solutions x_1, x_2, x_3

$\therefore$ $x_1 + y_1 + z_1 = 1$

$2y_1 + z_1 = 2$

$z_1 = 2$

Above system equation has solution

Here $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2$

44. Answer (4)

$\because$ $\begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0 \Rightarrow 3\lambda^2 - 7\lambda - 12 = 0$

$\Rightarrow$ $\lambda = 3$ or $-\frac{2}{3}$

Adding first two equations, we get

$3x_1 - 10x_2 + (\lambda + 1)x_3 = 3$

and the last equation is $\lambda x_1 - 10x_2 + 4x_3 = 3$

So, for $\lambda = 3$ there will be infinitely many solutions

and for $\lambda = -\frac{2}{3}$ there will be no solution (i.e. equations will be inconsistent).

45. Answer (2)

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} + y\begin{vmatrix} x & 1 & x+a \\ y & 1 & y+b \\ z & 1 & z+c \end{vmatrix}$$

$$= 0 + y\begin{vmatrix} x & 1 & x+a \\ y-x & 0 & 0 \\ z-x & 0 & -1 \end{vmatrix} \qquad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$= -y(x - y) = -y(b - a)$

$= y(a - b)$

46. Answer (4)

Here $\begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$

$1(9 - k^2) - 1(3 - 3k^2) + 3(1 - 9) = 0$

$9 - k^2 - 3 + 3k^2 - 24 = 0$

$2k^2 = 18 \Rightarrow k^2 = 9$, $k = \pm 3$

So equations are

$x + y + 3z = 0$ ...(i)

$x + 3y + 9z = 0$ ...(ii)

$3x + y + 3z = 0$ ...(iii)

Now (i) – (ii)

$-2y - 6z = 0 \Rightarrow y = -3z \Rightarrow \frac{y}{z} = -3$ ...(iv)

Now,

(i) – (iii)

$-2x = 0 \Rightarrow x = 0$

So $x + \frac{y}{z} = 0 - 3 = -3$

47. Answer (4)

Here $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$

$\Rightarrow$ $\lambda = 5$

and also

$\Delta_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0$

$\Rightarrow$ $\mu = 8$

48. Answer (13)

Given system of equation more than 2 solutions.

Hence system of equation has infinite many solution.

$\therefore$ $D = D_1 = D_2 = D_3 = 0$

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix} = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 3 & \mu & \lambda \end{vmatrix} = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 3 & 2 & \mu \end{vmatrix} = 0$$

$\therefore$ $\lambda = 1$ and $\mu = 14$.

$\therefore$ $\mu - \lambda^2 = 13$

49. Answer (8)

$x - 2y + 5z = 0$ ...(i)

$-2x + 4y + z = 0$ ...(ii)

$-7x + 14y + 9z = 0$ ...(iii)

From (i) and (ii); $z = 0$ and $x = 2y$

Let $x = 2\alpha$, $y = \alpha$, $z = 0$

Now

$15 \le 4\alpha^2 + \alpha^2 \le 150$

$3 \le \alpha^2 \le 30$

$\alpha = \pm 2, \pm 3, \pm 4, \pm 5$

Hence 8 elements are there in set S.

50. Answer (5)

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$ [For infinite solutions]

$$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$\Rightarrow$ $(a + 7) - 2(1 - 2a) + 3(-15) = 0$

$\Rightarrow$ $a = 8$

$$\Delta_3 = \begin{vmatrix} 1 & -2 & 9 \\ 2 & 1 & b \\ 1 & -7 & 24 \end{vmatrix} = 0$$

$\Rightarrow$ $(24 + 7b) - 2(b - 48) + 9(-15) = 0$

$\Rightarrow$ $b = 3$

$\therefore$ $a - b = 5$

51. Answer (03.00)

Here $\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0$

On solving it we get

$6\lambda^3 - 36\lambda^2 + 54\lambda = 0$

$\Rightarrow$ $6\lambda[\lambda^2 - 6\lambda + 9] = 0$

$\Rightarrow$ $\lambda = 0, \lambda = 3$ [Distinct values]

So sum = 0 + 3 = 3

52. Answer (4)

Here $\Delta = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$

$\Rightarrow$ $3(4 + 4) + 2(-2 + 2) - k(4 + 4) = 0$

$\Rightarrow$ $24 + 0 - 8k = 0$ $\Rightarrow$ $\boxed{k = 3}$

Now,

$$\Delta_1 = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix} = 10(4+4) + 2(-6+10m) - 3(12+20m)$$

$= 80 - 12 + 20m - 36 - 60m$

$= 32 - 40m$

$$\Delta_2 = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix} = 3(-6+10m) - 10(-2+2) - 3(10m-6)$$

$= -18 + 30m + 0 - 30m + 18 = 0$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix} = 3(-20m-12) + 2(10m-6) + 10(4+4)$$

$= -60m - 36 + 20m - 12 + 80$

$= -40m + 32$

For inconsistent we have k = 3, &

$32 - 40m \ne 0 \Rightarrow m \ne \dfrac{4}{5}$

53. Answer (17)

$\because Q = k.P^{-1}$ and $|P||Q| = k^3$, $|Q| = \dfrac{k^2}{2}$ then $|P| = 2k$

$\because q_{23} = \dfrac{kC_{32}}{|P|}$ (Where C_{ij} is co-factor of P_{ij} of P)

$$-\frac{k}{8}=-\frac{(3\alpha+4)k}{2k}\Rightarrow 3\alpha+4=\frac{k}{4} \quad \ldots(1)$$

Also $|P| = 2k \Rightarrow 12\alpha + 20 = 2k$

$\Rightarrow k = 6\alpha + 10 \quad \ldots(2)$

From (1) and (2) we get

$k = 4$ and $\alpha = -1$

then $k^2 + \alpha^2 = 17$

54. Answer (2)

Using Cramer's Rule, we have

$$\Delta=\begin{vmatrix}1 & -2 & 0\\ 1 & -1 & k\\ 0 & k & 4\end{vmatrix}=1(-4-k^2)+2(4)=4-k^2$$

$$\Delta_x=\begin{vmatrix}1 & -2 & 0\\ -2 & -1 & k\\ 6 & k & 4\end{vmatrix}=1(-4-k^2)+2(-2k+6)=8-4k-k^2$$

Now, $\Delta = 0$ if $k = \pm 2$

if $k = -2$, $\Delta = 0$ and $\Delta_x \neq 0$

Hence no solution

Also if $k = 2$, $\Delta = 0$ and $\Delta_x = 0$

Now

$$\Delta_y=\begin{vmatrix}1 & 1 & 0\\ 1 & -2 & k\\ 0 & 6 & 4\end{vmatrix}=1(-8-6k)-1(4)=-6k-12\neq 0$$

Hence, the system has no solution if $k = \pm 2$

and unique solution if $k \neq \pm 2$

55. Answer (13)

$$I_2+A=\begin{bmatrix}1 & -\tan\frac{\theta}{2}\\ \tan\frac{\theta}{2} & 1\end{bmatrix} \quad \ldots(1)$$

$$I_2-A=\begin{bmatrix}1 & \tan\frac{\theta}{2}\\ -\tan\frac{\theta}{2} & 1\end{bmatrix}$$

$$\Rightarrow (I_2-A)^{-1}=\frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix}1 & -\tan\frac{\theta}{2}\\ \tan\frac{\theta}{2} & 1\end{bmatrix} \quad \ldots(2)$$

$(I_2 + A)(I_2 - A)^{-1}$

$$=\frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix}1-\tan^2\frac{\theta}{2} & -2\tan\frac{\theta}{2}\\ 2\tan\frac{\theta}{2} & 1-\tan^2\frac{\theta}{2}\end{bmatrix}$$

$$=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

Clearly $a = \cos\theta$ and $b = \sin\theta$, then $13(a^2 + b^2) = 13$

56. Answer (21)

$kx + y + 2z = 1 \quad \ldots(1)$

$-3x + y + 2z = -2 \quad \ldots(2)$

$x + y + 2z = \frac{-3}{2} \quad \ldots(3)$

from (2) and (3) we get

$x=\frac{1}{8}$ and $y+2z=-\frac{13}{8}$

Substituting these values in (1) we get

$k = 21$

57. Answer (2)

Determinant of coefficients of given equations is

$$\begin{vmatrix}2 & 3 & 2\\ 3 & 2 & 2\\ 1 & -1 & 4\end{vmatrix}=2\,(8+2)-3\,(12-2)+2\,(-3-2)$$

$= 20 - 30 - 10 = -20 \neq 0$

$\therefore$ Hence the system of equation have unique solution

58. Answer (3)

Given determinant is

$$D=\begin{vmatrix}a^2+3a+2 & a+2 & 1\\ a+5a+6 & a+3 & 1\\ a^2+7a+12 & a+4 & 1\end{vmatrix}$$

$R_3 \to R_3 - R_2;\ R_2 \to R_2 - R_1$

$$=\begin{vmatrix}a^2+3a+2 & a+2 & 1\\ 2a+4 & 1 & 0\\ 2a+6 & 1 & 0\end{vmatrix}$$

Expanding by C_3

$D = (2a + 4) - (2a + 6) = -2$

59. Answer (1)

$$0 = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix} = (20) - 2(25) - 3(-10) = 0$$

$x + 2y - 3z = a$ ...(1)

$2x + 6y - 11z = b$ ...(2)

$x - 2y + 7z = c$ ...(3)

5eq (1) = 2eq (2) + eq (3)

it $5a = 2b + c \Rightarrow$ infinite solution

i.e., it will represent family of planes having a line (of intersection) as a solution

60. Answer (2)

$$A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}\begin{bmatrix} i & -i \\ -i & i \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}$$

Given $A^8 = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$$\begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$\Rightarrow$ $128(x - y) = 8$ and $-128(x - y) = 64$

$\Rightarrow$ $x - y = \frac{1}{16}$ and $x - y = -\frac{1}{2}$

Which cannot be equal on same time

Hence no solution.

61. Answer (36)

$\because$ $P^{-1}AP - I_3 = P^{-1}AP - P^{-1}P = P^{-1}(A - I)P$

$\Rightarrow$ $|P^{-1}AP - I_3| = |P^{-1}||A - I|\,|P| = |A - I|$

$$\because \quad A - I = \begin{vmatrix} 1 & 7 & \omega^2 \\ -1 & \omega^2 & 1 \\ 0 & -\omega & -\omega \end{vmatrix}$$

$\Rightarrow$ $|A - I| = -6\omega$

$\Rightarrow$ $|P^{-1}AP - I_3|^2 = (-6\omega)^2 = 36\omega^2$

62. Answer (4)

$C_1 \to C_1 + C_2$

$$f(x) = \begin{vmatrix} 2 & 1+\cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_1 \to R_1 - 2R_3$

$$f(x) = \begin{vmatrix} 0 & \sin^2 x & \cos 2x - 2\sin 2x \\ 0 & -1 & 0 \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$f(x) = \cos 2x - 2\sin 2x$

$\text{Max} = \sqrt{5}$

63. Answer (4)

$$\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0$$

$\Rightarrow$ $(k - 1)^2 (k + 2) = 0$

$k = 1$ makes the equation identical hence the system will have infinite solution

System will have no solution for $k = -2$.

64. Answer (4)

$$A = \begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \sin^2\alpha & 0 \\ 0 & \sin^2\alpha \end{bmatrix}$$

$$\det\left(A^2 - \frac{1}{2}I\right) = \begin{vmatrix} \sin^2\alpha - \frac{1}{2} & 0 \\ 0 & \sin^2\alpha - \frac{1}{2} \end{vmatrix} = 0$$

$$\Rightarrow \left(\sin^2\alpha - \frac{1}{2}\right)^2 = 0$$

$$\sin\alpha = \pm\frac{1}{\sqrt{2}}$$

$\alpha = \frac{\pi}{4}$ is one possibility

65. Answer (16)

$$\because \quad A=\begin{bmatrix}2 & 3\\ 0 & -1\end{bmatrix}, A^2=\begin{bmatrix}4 & 3\\ 0 & 1\end{bmatrix}, A^3=\begin{bmatrix}8 & 9\\ 0 & -1\end{bmatrix}, \ldots$$

So by mathematical induction we can conclude that

$$A^n=\begin{bmatrix}2^n & 2^n-(-1)^n\\ 0 & (-1)^n\end{bmatrix}$$

Also 2A · (adj (2A)) = |2A|I

$\Rightarrow$ A · adj(2A) = –4I

Now, $|A^{10}-(\text{adj}2A)^{10}| = \dfrac{\left|A^{20}-A^{10}(\text{adj}(2A))^{10}\right|}{|A|^{10}}$

$$= \frac{\left|A^{20}-2^{20}I\right|}{\left|A^{10}\right|} \quad \ldots(i)$$

$$A^{20}-A^{20}\cdot I=\begin{bmatrix}2^{20} & 2^{20}-1\\ 0 & 1\end{bmatrix}-2^{20}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 2^{20}-1\\ 0 & 1-2^{20}\end{bmatrix}$$

$\Rightarrow$ $|A^{20}-2^{20}I| = 0$

From (i) $|A^{10}-(\text{adj}(2A))^{10}| = 0$

Hence, $\det(A^4) + \det(A^{10} - (\text{adj}(2A)^{10})$

$= |A|^4 + 0$

$= (-2)^4 = 16$

66. Answer (3)

$$\because \quad \begin{vmatrix}3 & 4\sqrt{2} & x\\ 4 & 5\sqrt{2} & y\\ 5 & k & z\end{vmatrix}=0 \Rightarrow \begin{vmatrix}3 & 4\sqrt{2} & x\\ 1 & \sqrt{2} & d\\ 1 & k-5\sqrt{2} & d\end{vmatrix}=0$$

$$\begin{vmatrix}3 & 4\sqrt{2} & x\\ 1 & \sqrt{2} & d\\ 0 & k-6\sqrt{2} & 0\end{vmatrix}=0$$

$$\Rightarrow \quad \left(k-6\sqrt{2}\right)(3d-x)=0$$

$$\Rightarrow \quad k=6\sqrt{2} \Rightarrow k^2=72$$

67. Answer (2)

$$\because \quad \left(4^x-2\right)^2=10\left(4^x+\frac{18}{5}\right)$$

$\Rightarrow$ $4^{2x} - 14\cdot4^x - 32 = 0$

$\Rightarrow$ $4^x = 16 \Rightarrow x = 2$

$$\begin{vmatrix}2x-1 & x-1 & x^2\\ 1 & 0 & x\\ x & 1 & 0\end{vmatrix}=x\left(x^2-x\right)-\left(x^2-x\right)=(x-1)\left(x^2-x\right)$$

$= 2$

68. Answer (4)

By using $C_1 \to C_1 - C_2$ and $C_3 \to C_3 - C_2$ we get

$$\begin{vmatrix}1 & \sin^2 x & 0\\ -1 & 1+\cos^2 x & -1\\ 0 & 4\sin 2x & 1\end{vmatrix}=0$$

Expanding by R_1 we get

$1(1 + \cos^2x + 4\sin2x) - \sin^2x(-1) = 0$

$\Rightarrow$ $2 + 4\sin2x = 0$

$$\Rightarrow \quad \sin 2x=\frac{-1}{2}$$

$$\Rightarrow \quad 2x=n\pi+(-1)^n\left(\frac{-\pi}{6}\right), n\in Z$$

$$\therefore \quad 2x=\frac{7\pi}{6},\frac{11\pi}{6} \Rightarrow x=\frac{7\pi}{12},\frac{11\pi}{12}$$

69. Answer (3)

For non-trivial solutions of given system we have

$$\begin{vmatrix}\alpha & \beta & \gamma\\ \beta & \gamma & \alpha\\ \gamma & \alpha & \beta\end{vmatrix}=0$$

$\therefore$ $-(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha) = 0$

$\Rightarrow$ $-(-a)(a^2 - 3b) = 0$

$$\Rightarrow \quad \frac{a^2}{b}=3 \qquad \text{(as } a\neq 0\text{)}$$

70. Answer (3)

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow$ $\mu\lambda + 2\mu - 6\lambda - 12 = 0$

$\Rightarrow$ $(\lambda + 2)(\mu - 6) = 0$

$\lambda = -2$ or $\mu = 6$

71. Answer (6)

$$P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$$

$$\begin{vmatrix} 2-\lambda & -1 \\ 5 & -3-\lambda \end{vmatrix} = 0$$

$\Rightarrow$ $\lambda^2 + \lambda - 1 = 0$

$\Rightarrow$ $P^2 + P - I = 0$

$\Rightarrow$ $P^2 = I - P$

$\Rightarrow$ $P^4 = I + P^2 - 2P$

$\Rightarrow$ $P^4 = 2I - 3P$

Now, $P^4 \cdot P^2 = (2I - 3P)(I - P) = 2I - 5P + 3P^2$

$\Rightarrow$ $P^6 = 5I - 8P$

so n = 6.

72. Answer (1)

$R_1 \rightarrow R_1 + R_3 - 2R_2$

$$D = \begin{vmatrix} 2 & 0 & 0 \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix}$$

$= 2((x^2 + 2cx + c^2) - (x^2 + (b + d)x + bd))$

$= 2(c^2 - bd) = 2(c^2 - (c - \lambda)(c + \lambda))$

$= 2\lambda^2$

$D = 2 \Rightarrow \lambda^2 = 1$

73. Answer (2)

$$\Delta = \begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & k \end{vmatrix} = 0 \Rightarrow k = -5$$

For $k = -5$, $\Delta_1 = \Delta_2 = \Delta_3 = 0$

74. Answer (4)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 5 \\ 1 & 2 & \lambda \end{vmatrix} = 0$$

$\Rightarrow$ $\lambda = 2$

For $\mu = 10$, $\Delta_1, \Delta_2, \Delta_3 = 0$ which corresponds to the case of infinite solutions

$\therefore$ $\mu \neq 10$

75. Answer (4)

For no solution $\Delta = 0$ (by Cramer's rule)

$$\begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 0$$

$\Rightarrow$ $2(18 - 5a) - 3(9 - 3a) + 6(-1) = 0$

$\Rightarrow$ $-a + 3 = 0 \Rightarrow a = 3$

Let solutions of $2x + 3y + 6z = 8$ & $x + 2y + 3z = 5$ be $z = k$ gives $y = 2$ & $x = 1 - 3k$

for no solution $(1 - 3k, 2, k)$ shall not satisfy $3x + 5y + 9z = b$

$\therefore$ $3(1 - 3k) + 10 + 9k \neq b$

$\Rightarrow$ $b \neq 13$

76. Answer (1)

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$\begin{vmatrix} \sin x + 2\cos x & \cos x & \cos x \\ \sin x + 2\cos x & \sin x & \cos x \\ \sin x + 2\cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$(\sin x + 2\cos x)\begin{vmatrix} 0 & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$$

$\therefore$ $(\sin x + 2\cos x)(\cos x - \sin x)^2 = 0$

$\therefore$ $\tan x = -2$ and $\tan x = 1$

$\therefore$ Number of roots in $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is 1.

77. Answer (5)

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow \alpha = -2$$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 2 & -2 \\ \beta & -1 & 1 \end{vmatrix} = 0 \Rightarrow \beta = 7$$

$\Delta_3 = 0 \Rightarrow \beta = 7$

$\alpha + \beta = 5$

78. Answer (6)

$$f(x) = \begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2 & 2 & 0 \\ 0 & 2 & 1 \end{vmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$\Rightarrow$ $f(x) = -2(-2\cos 2x) + (2\sin^2 x + 4 - 2\cos^2 x)$

$= 4\cos 2x + 4 - 2\cos 2x = 4 + 2\cos 2x$

$\Rightarrow$ $f(x)_{max} = 6$

79. Answer (2)

$\because$

$$A \cdot A^T = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

So, $A \cdot Q^{2021} \cdot A^T = A(A^T BA)^{2021} \cdot A^T = B^{2021}$

$$\because \quad B^2 = \begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2i & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & 0 \\ 2i & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3i & 1 \end{bmatrix}$$

$$\text{Hence, } B^{2021} = \begin{bmatrix} 1 & 0 \\ 2021i & 1 \end{bmatrix}$$

$$\text{So } (B^{2021})^{-1} = \begin{bmatrix} 1 & 0 \\ -2021i & 1 \end{bmatrix}$$

80. Answer (1)

$$\Delta = \begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \quad \Delta = \begin{vmatrix} 2+4\sin 3\theta & \sin^2\theta & 4\sin 3\theta \\ 2+4\sin 3\theta & 1+\sin^2\theta & 4\sin 3\theta \\ 2+4\sin 3\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix}$$

$$\Rightarrow \quad \Delta = (2+4\sin 3\theta)\begin{vmatrix} 1 & \sin^2\theta & 4\sin 3\theta \\ 1 & 1+\sin^2\theta & 4\sin 3\theta \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \quad \Delta = \begin{vmatrix} 1 & \sin^2\theta & 4\sin 3\theta \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \times (2+4\sin 3\theta)$$

$\Rightarrow$ $\Delta = (2 + 4\sin 3\theta)$

For non-trivial solution

$\Delta = 0$

$$\Rightarrow \quad \sin 3\theta = \frac{-1}{2}$$

$$\Rightarrow \quad \theta = \frac{7\pi}{18}$$

81. Answer (1)

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1 \\ 1 & 0 & -\lambda \end{vmatrix} = 0$$

$A^3 - 2A^2 + A = 0$

$\Rightarrow$ $A^2 - A = A^3 - A^2 = A^4 - A^3 = A^5 - A^4 = A^6 - A^5$

$= A^7 - A^6$

So $A^7 - A^2 = A^6 - A$

$\Rightarrow$ $A^8 - A^3 = A^7 - A^2 = A^6 - A$

And So on.

then $A^{2025} - A^{2020} = A^6 - A$

82. Answer (4)

det(2 Adj(2 Adj(Adj·2A))) = 2^{41}

$\Rightarrow$ det(2 Adj(2 Adj(2^2·Adj A))) = 2^{41}

$\Rightarrow$ det·(2 Adj(2^5 Adj(Adj A))) = 2^{41}

$\Rightarrow$ det·(2^{11} Adj(Adj(Adj A))) = 2^{41}

$\Rightarrow$ 2^{33}·det(Adj(Adj(Adj A))) = 2^{41}

$\Rightarrow$ $|A|^8 = 2^8$ $\Rightarrow$ $|A| = 2$

$\Rightarrow$ $|A|^2 = 4$

83. Answer (5)

For infinite solutions

First requirement

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & -1 \\ 3 & 3 & \beta \end{vmatrix} = 0, \Rightarrow \beta = -1$$

Now the equations are :

$2x + y - z = 3$ ...(i)

$x - y - z = \alpha$ ...(ii)

$3x + 3y - z = 3$ ...(iii)

For infinite solutions, one equation should be obtainable as linear combination of other two equation.

Adding (ii) and (iii) and dividing by 2 given LHS of (ii)

$\Rightarrow$ $\dfrac{3+\alpha}{2} = 3 \Rightarrow \alpha = 3.$ Hence $\alpha + \beta - \alpha\beta = 5$

84. Answer (3)

$x + y + z = 4$

$3x + 2y + 5z = 3$

$9x + 4y + (28 + [\lambda])z = [\lambda]$

For unique solution $\Delta \neq 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28+[\lambda] \end{vmatrix} \neq 0$$

$\Rightarrow$ $(56 + 2[\lambda] - 20) - 1(84 + 3[\lambda] - 45) + 1(-6) \neq 0$

$\Rightarrow$ $36 + 2[\lambda] - 39 - 3[\lambda] - 6 \neq 0$

$\Rightarrow$ $[\lambda] \neq -9$

$\Rightarrow$ $\lambda \in (-\infty, -9) \cup [-8, \infty)$

and if $[\lambda] = -9$, $\Delta_x = \Delta_y = \Delta_z = 0$ gives infinite solution.

$\therefore$ for $\lambda \in R$ set of equations have solution.

85. Answer (3)

$$|A| = \begin{vmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{vmatrix}$$

$$= \begin{vmatrix} [x]+1 & [x]+2 & [x]+3 \\ [x] & [x]+3 & [x]+3 \\ [x] & [x]+2 & [x]+4 \end{vmatrix}$$

$C_3 \to C_3 - C_2$, $C_2 \to C_2 - C_1$

$$= \begin{vmatrix} [x]+1 & 1 & 1 \\ [x] & 3 & 0 \\ [x] & 2 & 2 \end{vmatrix}$$

(Expanding by C_3)

$= 1(2[x] - 3[x]) + 2(3[x] + 3 - [x]) = -[x] + 2(2[x] + 3)$

$= 3[x] + 6$

$|A| = 192$

$3[x] + 6 = 192$

$[x] = 62$

$x \in [62, 63)$

86. Answer (4)

$$\Delta = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} = 1 - 3a$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 3 \\ 1 & 1 & b \end{vmatrix} = 7 - 3b$$

For no solution, $\Delta = 0$, $\Delta_3 \neq 0$

$1 - 3a = 0$, $7 - 3b \neq 0$

$a = \dfrac{1}{3}$, $b \neq \dfrac{7}{3}$

87. Answer (1)

$$a_r = \cos\frac{2r\pi}{9} + i\sin\frac{2r\pi}{9} = e^{i\frac{2r\pi}{9}} \text{ where r = 1, 2,}$$

3...

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} e^{i\frac{2\pi}{9}} & e^{i\frac{4\pi}{9}} & e^{i\frac{6\pi}{9}} \\ e^{i\frac{8\pi}{9}} & e^{i\frac{10\pi}{9}} & e^{i\frac{12\pi}{9}} \\ e^{i\frac{14\pi}{9}} & e^{i\frac{16\pi}{9}} & e^{i\frac{18\pi}{9}} \end{vmatrix}$$

$$= e^{i\left(\frac{2\pi}{9}+\frac{8\pi}{9}+\frac{14\pi}{9}\right)} \begin{vmatrix} 1 & e^{i\frac{2\pi}{9}} & e^{i\frac{4\pi}{9}} \\ 1 & e^{i\frac{2\pi}{9}} & e^{i\frac{4\pi}{9}} \\ 1 & e^{i\frac{2\pi}{9}} & e^{i\frac{18\pi}{9}} \end{vmatrix} = 0$$

Now, $a_1 \cdot a_9 - a_3 \cdot a_7 = e^{i\frac{2\pi}{9}} \cdot e^{i\frac{18\pi}{9}} - e^{i\frac{6\pi}{9}} \cdot e^{i\frac{14\pi}{9}}$

$$= e^{i\frac{20\pi}{9}} - e^{i\frac{20\pi}{9}}$$

= 0

88. Answer (3)

$$\Delta = \begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$$

$= (1 - \cos^2\alpha) - \cos\gamma(\cos\gamma - \cos\alpha\cos\beta)$

$+ \cos\beta(\cos\alpha\cos\gamma - \cos\beta)$

$= 1 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) + 2\cos\alpha\cos\beta\cos\gamma$

(as $A + B + C = 2\pi$)

$= 1 - (1 - 2\cos\alpha\cos\beta\cos\gamma) + 2\cos\alpha\cos\beta\cos\gamma$

= 0

$\therefore$ System has infinite solution

89. Answer (3)

Given $-x + y + 2z = 0 \Rightarrow x = y + 2z$

$\therefore 3y + 6z - ay + 5z = 1$ ...(i)

$2y + 4z - 2y - az = 7$ ...(ii)

$(3-a)y + 11z = 1$ ...(iii)

and $z = \frac{7}{(4-a)}$ ...(iv)

For no solution (iii) and (iv) represent parallel lines

i.e. $7 \neq \frac{4-a}{11} \Rightarrow a \neq -73$ and $\frac{3-a}{11} = 0$

$\Rightarrow a = 3$

(also $a = 4$ is acceptable)

$\therefore \quad n(S_1) = 2$

For infinite solution lines shall coincide

i.e., $\frac{3-a}{11} = 0$ and $\frac{1}{11} = \frac{7}{4-a} \Rightarrow 4 - a = 77$

$\Rightarrow a = 3$ and $\Rightarrow a = -73$

$\therefore n(S_2) = 0$

90. Answer (4)

Let common ratio of G.P. is R

$\Rightarrow a_2 = a_1R, \quad a_3 = a_1R^2, \ldots a_{10} = a_1R^9$

$C_1 \to C_1 - C_2,\ C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} \ln\left(\frac{a_1^r a_2^k}{a_2^r a_3^k}\right) & \ln\left(\frac{a_2^r a_3^k}{a_3^r a_4^k}\right) & \ln a_3^r a_4^k \\ \ln\left(\frac{a_4^r a_5^k}{a_5^r a_6^k}\right) & \ln\left(\frac{a_5^r a_6^k}{a_6^r a_7^k}\right) & \ln a_6^r a_7^k \\ \ln\frac{a_7^r a_8^k}{a_8^r a_9^k} & \ln\left(\frac{a_8^r a_9^k}{a_9^r a_{10}^k}\right) & \ln a_9^r a_{10}^k \end{vmatrix}$$

$$\Delta = \begin{vmatrix} \ln\frac{1}{R^{r+k}} & \ln\frac{1}{R^{r+k}} & \ln a_3^r a_4^k \\ \ln\frac{1}{R^{r+k}} & \ln\frac{1}{R^{r+k}} & \ln a_6^r a_7^k \\ \ln\frac{1}{R^{r+k}} & \ln\frac{1}{R^{r+k}} & \ln a_9^r a_{10}^k \end{vmatrix} = 0 \ \forall\ r, k \in N$$

$\Rightarrow$ No. of elements in S is infinitely many

$\Rightarrow$ Option (4) is correct.

91. Answer (1)

$$B = \left[b_{ij}\right]_{3\times 3} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$\therefore \quad (B) = \begin{bmatrix} 3^0 \cdot a_{11} & 3 \cdot a_{21} & 3^2 \cdot a_{31} \\ 3 \cdot a_{12} & 3^2 \cdot a_{22} & 3^3 \cdot a_{32} \\ 3^2 \cdot a_{13} & 3^3 \cdot a_{23} & 3^4 \cdot a_{33} \end{bmatrix}$$

$$\det(B) = 3.3^2 \cdot 3.3^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$\therefore \quad 3^4 = 3^6 \cdot \det(A)$

$\therefore \quad \det(A) = \dfrac{1}{3^2} = \dfrac{1}{9}$

92. Answer (1)

$$f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

Operate $R_1 \to R_1 + R_3 - 2R_2$ and using $a - 2b + c = 1$

$$f(x) = \begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$\Rightarrow \quad f(x) = 1 \qquad \Rightarrow \quad f(50) = 1$

93. Answer (2)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix}$$

$= 1(10\alpha - 9\alpha) - 1(5\alpha - 3) + 1(3\alpha^2 - 2\alpha)$

$= \alpha - 5\alpha + 3 + 3\alpha^2 - 2\alpha$

$= 3\alpha^2 - 6\alpha + 3$

For inconsistency $\Delta = 0$ i.e. $\alpha = 1$

Now check for $\alpha = 1$

$x + y + z = 1 \quad \ldots$(i)

$x + 2y + 3z = -1 \quad \ldots$(ii)

$x + 3y + 5z = 4 \quad \ldots$(iii)

By (ii) × 2 – (i) × 1

$x + 3y + 5z = -3$

so equations are

inconsistent for $\alpha = 1$

94. Answer (2)

$|A| = a^2 + 1$

$|adj\ A| = (a^2 + 1)^2$

$S = \left\{1, \sqrt{3}, \sqrt{5}, \sqrt{7}, \ldots, \sqrt{49}\right\}$

$\sum_{a \in S} \det(adj\ A) = (1^2 + 1)^2 + (3 + 1)^2 + (5 + 1)^2 + \ldots + (49 + 1)^2$

$= 2^2 (1^2 + 2^2 + 3^2 + \ldots + 25^2)$

$= 4.\dfrac{25 \cdot 26 \cdot 51}{6} = 100 \cdot 221$

$\lambda = 221$

95. Answer (3)

Given system of equations

$x + y + az = 2 \quad \ldots$(i)

$3x + y + z = 4 \quad \ldots$(ii)

$x + 2z = 1 \quad \ldots$(iii)

Solving (i), (ii) and (iii), we get

$x = 1$, $y = 1$, $z = 0$ (and for unique solution $a \neq -3$)

Now, $(\alpha, 1)$, $(1, \alpha)$ and $(1, -1)$ are collinear

$$\therefore \quad \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad \alpha(\alpha + 1) - 1(0) + 1(-1 - \alpha) = 0$

$\Rightarrow \quad \alpha^2 - 1 = 0$

$\therefore \quad \alpha = \pm 1$

$\therefore \quad$ Sum of absolute values of $\alpha = 1 + 1 = 2$

96. Answer (2)

Let $A=\begin{bmatrix} a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$

$$A\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\0\end{bmatrix}\Rightarrow\begin{bmatrix} a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\0\end{bmatrix}\Rightarrow\begin{matrix}a+b=1\\d+e=1\\g+h=0\end{matrix}$$

$$A\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}-1\\0\\1\end{bmatrix}\Rightarrow\begin{bmatrix} a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}-1\\0\\1\end{bmatrix}\Rightarrow\begin{matrix}a+c=-1\\d+f=0\\g+i=1\end{matrix}$$

$$A\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}1\\1\\2\end{bmatrix}\Rightarrow\begin{bmatrix} a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}1\\1\\2\end{bmatrix}\Rightarrow\begin{matrix}c=1\\f=1\\i=2\end{matrix}$$

Solving will get

$a=-2$, $b=3$, $c=1$, $d=-1$, $e=2$, $f=1$, $g=-1$, $h=1$, $i=2$

$$A=\begin{bmatrix}-2 & 3 & 1\\ -1 & 2 & 1\\ -1 & 1 & 2\end{bmatrix}\Rightarrow A=2I=\begin{bmatrix}-4 & 3 & 1\\ -1 & 0 & 1\\ -1 & 1 & 0\end{bmatrix}$$

$$(A-2I)x=\begin{bmatrix}4\\1\\1\end{bmatrix}$$

$\Rightarrow\ -4x_1+3x_2+x_3=4$ ...(i)

$-x_1+x_3=1$ ...(ii)

$-x_1+x_2=1$ ...(iii)

So 3(iii) + (ii) = (i)

$\therefore$ Infinite solution

97. Answer (4)

The system may be inconsistent if

$$\begin{vmatrix}-k & 3 & -14\\ -15 & 4 & -k\\ -4 & 1 & 3\end{vmatrix}=0\Rightarrow k=\pm 11$$

Hence if system is consistent then

$k\in R-\{11,-11\}$

98. Answer (3)

$$|\text{adj}(24A)|=|\text{adj}(3\,\text{adj}(2A))|$$

$$\Rightarrow\ |24A|^2=|3\ \text{adj}(2A)|^2$$

$$\Rightarrow\ \left(24^3\right)^2\cdot|A|^2=\left(3^3\right)^2|\text{adj}(2A)|^2$$

$$\Rightarrow\ 24^6\cdot|A|^2=3^6|2A|^4$$

$$\Rightarrow\ 24^6|A|^2=3^6\cdot(2^3)^4|A|^4$$

$$\Rightarrow\ |A|^2=\frac{24^6}{3^6\cdot 2^{12}}=\frac{2^{18}\cdot 3^6}{3^6\cdot 2^{12}}=2^6$$

99. Answer (3)

$$\begin{vmatrix}3 & -2 & 1\\ 5 & -8 & 9\\ 2 & 1 & a\end{vmatrix}=0\Rightarrow -14a-42=0\Rightarrow a=-3$$

Now 3(equation (1)) – (equation (2)) – 2(equation (3)) is

$3(3x-2y+z-b)-(5x-8y+9z-3)-2(2x+y+az+1)=0$

$\Rightarrow\ -3b+3-2=0\ \Rightarrow\ b=\frac{1}{3}$

So for no solution $a=-3$ and $b\neq\frac{1}{3}$

100. Answer (3)

Given system of equations

$\alpha x+y+z=5$

$x+2y+3z=4$, has infinite solution

$x+3y+5z=\beta$

$$\therefore\ \Delta=\begin{vmatrix}\alpha & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & 5\end{vmatrix}=0\Rightarrow\alpha(1)-1(2)+1(1)=0$$

$$\Rightarrow\ \boxed{\alpha=1}$$

and $\Delta_1=\begin{vmatrix}5 & 1 & 1\\ 4 & 2 & 3\\ \beta & 3 & 5\end{vmatrix}=0$

$\Rightarrow \quad \beta = 3$

And $\Delta_2 = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 4 & 3 \\ 1 & \beta & 5 \end{vmatrix} = 0 \Rightarrow (20 - 3\beta) - 5(2) + 1(\beta - 4) = 0$

$\Rightarrow -2\beta + 6 = 0$

$\Rightarrow \beta = 3$

Similarly,

$\Rightarrow \quad \Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 4 \\ 1 & 3 & \beta \end{vmatrix} = 0 \Rightarrow \beta = 3$

$\therefore \quad (\alpha, \beta) = (1, 3)$

101. Answer (4)

$x + 2y + z = 2$

$\alpha x + 3y - z = \alpha$

$-\alpha x + y + 2z = -\alpha$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 1(6+1) - 2(2\alpha - \alpha) + 1(\alpha + 3\alpha)$$

$= 7 + 2\alpha$

$\Delta = 0 \;\Rightarrow \alpha = -\dfrac{7}{2}$

$$\Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 14 + 2\alpha \neq 0 \text{ for } \alpha = -\frac{7}{2}$$

$\therefore$ For no solution $\alpha = -\dfrac{7}{2}$

102. Answer (14)

$$|adj(adj(A))| = |A|^{2^2} = |A|^4$$

$$\therefore \quad |A|^4 = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix}$$

$$= (14)^3 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= (14)^3 (3 - 2(-5) - 1(-1))$$

$$|A|^4 = (14)^4 \Rightarrow |A| = 14$$

103. Answer (3)

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}, \; a \in R$$

$f(x) = a(a^2 + ax) + 1(a^2x + ax^2)$

$= a(x + a)^2$

$f'(x) = 2a(x + a)$

Now, $2f'(10) - f'(5) + 100 = 0$

$\Rightarrow \quad 2 \cdot 2a(10 + a) - 2a(5 + a) + 100 = 0$

$\Rightarrow \quad 2a(a + 15) + 100 = 0$

$\Rightarrow \quad a^2 + 15a + 50 = 0$

$\Rightarrow \quad a = -10, -5$

$\therefore \quad$ Sum of squares of values of $a = 125$.

104. Answer (3)

A and B are two matrices of order 3×3.

and $AB = I$, $\quad |A| = \dfrac{1}{8}$

Now, $|A|\,|B| = 1$

$|B| = 8$

$\therefore \quad |adj(B(adj(2A))| = |B(adj(2A))|^2$

$= |B|^2|adj(2A)|^2$

$= 2^6|2A|^{2\times2}$

$= 2^6 \cdot 2^{12} \cdot \dfrac{1}{2^{12}} = 64$

105. Answer (2)

$$\Delta = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & |\lambda| \end{vmatrix} = 0 \Rightarrow |\lambda| = 7$$

But at $\lambda = 7$, $D_x = D_y = D_z = 0$

$P_1 : 2x + 3y - z = -2$

$P_2 : x + y + z = 4$

$P_3 : x - y + |\lambda|z = 4\lambda - 4$

So clearly $5P_2 - 2P_1 = P_3$, so at $\lambda = 7$, system of equation is having infinite solutions.

So $\lambda = -7$ is correct answer.

106. Answer (1)

$|A| = 2$

$||A|\,\text{adj}(5\,\text{adj}\,A^3)|$

$= |25|A|\,\text{adj}(\text{adj}\,A^3)|$

$= 25^3\,|A|^3 \cdot |\text{adj}\,A^3|^2$

$= 25^3 \cdot 2^3 \cdot |A^3|^4$

$= 25^3 \cdot 2^3 \cdot 2^{12} = 10^6 \cdot 512$

107. Answer (58)

If $2x - 3y = \gamma + 5$ and $\alpha x + 5y = \beta + 1$ have infinitely many solutions then

$$\frac{2}{\alpha} = \frac{-3}{5} = \frac{\gamma+5}{\beta+1}$$

$$\Rightarrow \quad \alpha = -\frac{10}{3} \text{ and } 3\beta + 5\gamma = -28$$

So $|9\alpha + 3\beta + 5\gamma| = |-30 - 28| = 5$

108. Answer (2)

$2x + y - z = 7$

$x - 3y + 2z = 1$

$x + 4y + \delta z = k$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = -7\delta - 21 = 0$$

$$\boxed{\delta = -3}$$

$$\Delta_1 = \begin{vmatrix} 7 & 1 & -1 \\ 1 & -3 & 2 \\ k & 4 & -3 \end{vmatrix}$$

$\Rightarrow \quad 6 - k = 0 \Rightarrow k = 6$

$\delta + k = -3 + 6 = 3$

109. Answer (3)

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \Rightarrow adj(A) \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$B = I - {}^5C_1(\text{adj}A) + {}^5C_2(\text{adj}A)^2 + \ldots.. + {}^5C_5(\text{adj}A)^5$

$$= (I - \text{adj}A)^5 = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}\right)^5 = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}^5$$

Let $P = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} \Rightarrow B = P^5$

$$P^2 = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ 0 & -1 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} -1 & -3 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -5 \\ 0 & 1 \end{bmatrix} = B$$

Sum of elements = –1 –5 – 1 + 0 = –7

110. Answer (2)

$$\Delta = \begin{vmatrix} 3\sin 3\theta & -1 & 1 \\ 3\cos 2\theta & 4 & 3 \\ 6 & 7 & 7 \end{vmatrix}$$

$= 3\sin3\theta(7) + 1(21\cos2\theta - 18) + 1(21\cos2\theta - 24)$

$\Delta = 21\sin3\theta + 42\cos2\theta - 42$

For no solution

$\sin3\theta + 2\cos2\theta = 2$

$\Rightarrow \quad \sin3\theta = 2\times2\sin^2\theta$

$\Rightarrow \quad 3\sin\theta - 4\sin^3\theta = 4\sin^2\theta$

$\Rightarrow \quad \sin\theta(3 - 4\sin\theta - 4\sin^2\theta) = 0$

$\sin\theta = 0$ OR $\sin\theta = \frac{1}{2}$

$$\theta = \pi, 2\pi, 3\pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

111. Answer (24)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = I + B$$

$$B^2 = \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & ab \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$B^3 = 0$

$\therefore \quad A^n = (1 + B)^n = {}^nC_0 I + {}^nC_1 B + {}^nC_2 B^2 + {}^nC_3 B^3 + \ldots.$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & na & na \\ 0 & 0 & nb \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \dfrac{n(n-1)ab}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & na & na + \dfrac{n(n-1)}{2}ab \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 48 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing we get $na = 48$, $nb = 96$ and

$$na + \frac{n(n-1)}{2}ab = 2160$$

$\Rightarrow \quad a = 4,\ n = 12$ and $b = 8$

$n + a + b = 24$

112. Answer (4)

$$\Delta = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix}$$

$= 8(3) - 1(-\lambda) + 4(-3 - \lambda)$

$= 24 + \lambda - 12 - 4\lambda$

$= 12 - 3\lambda$

So for $\lambda = 4$, it is having infinitely many solutions.

$$\Delta_x = \begin{vmatrix} -2 & 1 & 4 \\ 0 & 1 & 1 \\ \mu & -3 & 0 \end{vmatrix}$$

$= -2(3) - 1(-\mu) + 4(-\mu)$

$= -6 - 3\mu = 0$

For $\mu = -2$

Distance of $\left(4, -2, \dfrac{-1}{2}\right)$ from $8x + y + 4z + 2 = 0$

$$= \frac{32 - 2 - 2 + 2}{\sqrt{64 + 1 + 16}} = \frac{10}{3} \text{ units}$$

113. Answer (2)

$|(A + I)(\text{adj } A + I)| = 4$

$\Rightarrow \quad |A \text{ adj } A + A + \text{adj } A + I| = 4$

$\Rightarrow \quad |(A)I + A + \text{adj } A + I| = 4$

$|A| = -1 \Rightarrow |A + \text{adj } A| = 4$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad adj\ A = \begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$$

$$\Rightarrow \quad \begin{vmatrix} (a + d) & 0 \\ 0 & (a + d) \end{vmatrix} = 4$$

$\Rightarrow \quad a + d = \pm 2$

114. Answer (42)

$$\det(A) = \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta + \gamma & \gamma + \alpha & \alpha + \beta \end{vmatrix}$$

R_3 ! $R_3 + R_1$

$$\Rightarrow \quad (\alpha + \beta + \gamma)\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$\therefore \quad \det(A) = (\alpha + \beta + \gamma)(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$

Also, det (adj (adj (adj (adj (A)))))

$$= (\det(A))^{2^4} = (\det(A))^{16}$$

$$\therefore \quad \frac{(\alpha + \beta + \gamma)^{16}(\alpha - \beta)^{16}(\beta - \gamma)^{16}(\gamma - \alpha)^{16}}{(\alpha - \beta)^{16}(\beta - \gamma)(\gamma - \alpha)^{16}} = (4.3)^{16}$$

$\Rightarrow \quad \alpha + \beta + \gamma = 12$

$\Rightarrow \quad (\alpha, \beta, \gamma)$ distinct natural triplets

$= {}^{11}C_2 - 1 - {}^3C_2(4) = 55 - 1 - 12$

$= 42$

115. Answer (3)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \quad A^3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Now $B_0 = A^{49} + 2A^{98} = \left(A^3\right)^{16} \cdot A + 2\left(A^3\right)^{32} \cdot A^2$

$B_0 = A + 2A^2 =$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

$|B_0| = 9$

Since, $B_n = \text{Adj } |B_{n-1}| \quad \Rightarrow \quad |B_n| = |B_{n-1}|^2$

Hence $|B_4| = |B_3|^2 = |B_2|^4 = |B_1|^8 = |B_0|^{16}$

$= |3^2|^{16} = 3^{32}$

116. Answer (3)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 1(15 - 2\alpha) - 1(6 - \alpha) + 1(-1)$$

$= 15 - 2\alpha - 6 + \alpha - 1$

$= 8 - \alpha$

For infinite solutions, $\Delta = 0 \Rightarrow \alpha = 8$

$$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ \beta & 5 & 8 \\ 14 & 2 & 3 \end{vmatrix} = 6(-1) - 1(3\beta - 112) + 1(2\beta - 70)$$

$= -6 - 3\beta + 112 + 2\beta - 70$

$= 36 - \beta$

$\Delta_x = 0 \Rightarrow$ for $\beta = 36$

$\alpha + \beta = 44$

117. Answer (2)

AB is zero matrix

$\Rightarrow \quad |A| = |B| = 0$

So neither A nor B is invertible

If $|A| = 0$

$\Rightarrow \quad |\text{adj } A| = 0$ so adj A is not invertible

$AX = 0$ is homogeneous system and $|A| = 0$

So, it is having infinitely many solutions

118. Answer (50)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{then } A^2 = \begin{bmatrix} a^2 + bc & b(a+d) \\ c(a+d) & bc + d^2 \end{bmatrix}$$

For A^{-1} must exist $ad - bc \neq 0$ …(i)

and $A = A^{-1} \Rightarrow A^2 = I$

$\therefore \quad a^2 + bc = d^2 + bc = 1$ …(ii)

and $b(a + d) = c(a + d) = 0$ …(iii)

Case I : When $a = d = 0$, then possible values of (b, c) are (1, 1), (–1, 1) and (1, –1) and (–1, 1). Total four matrices are possible.

Case II : When $a = -d$ then (a, d) be (1, –1) or (–1, 1).

Then total possible values of (b, c) are $(12 + 11) \times 2 = 46$.

$\therefore$ Total possible matrices = 46 + 4 = 50.

119. Answer (04)

$$\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$$

$$= p! \cdot (p+1)! \cdot (p+2)! \begin{vmatrix} 1 & p+1 & (p+1)(p+2) \\ 1 & (p+2) & (p+2)(p+3) \\ 1 & (p+3) & (p+3)(p+4) \end{vmatrix}$$

$$= p! \cdot (p+1)! \cdot (p+2)! \begin{vmatrix} 1 & p+1 & p^2 + 3p + 2 \\ 0 & 1 & 2p + 4 \\ 0 & 1 & 2p + 6 \end{vmatrix}$$

$$= 2(p!) \cdot ((p+1)!) \cdot ((p+2)!) \cdot$$

$= 2(p+1)\cdot(p!)^2\cdot((p+2)!)\cdot$

$= 2(p+1)^2\cdot(p!)^3\cdot((p+2)!)\cdot$

$\therefore$ Maximum value of α is 3 and β is 1.

$\therefore$ $\alpha + \beta = 4$

120. Answer (3)

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 3 & -1 \\ 3 & -1 & \lambda^2 - |\lambda| \end{vmatrix} = 2\left(3\lambda^2 - 3|\lambda| - 1\right) + 3\left(\lambda^2 - |\lambda| + 3\right) + 5(-1-9)$$

$= 9\lambda^2 - 9|\lambda| - 43$

$= 9|\lambda|^2 - 9|\lambda| - 43$

$\Delta = 0$ for 2 values of $|\lambda|$ out of which one is –ve and other is +ve

So, 2 values of λ satisfy the system of equations to obtain no solution

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