SESSION : 2025-26

DPP

DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 1

Topic :-Applications of Derivatives

1 **(b)**

We have,

$f(x) = x(x-1)^2$

$\Rightarrow f''(x) = (x-1)^2 + 2x(x-1)$

$\Rightarrow f''(x) = (x-1)(3x-1)$

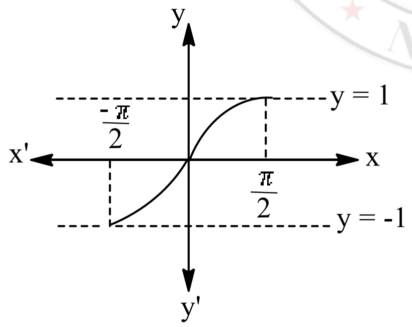
The changes in the signs of $f''(x)$ are shown in diagram

Clearly, $f(x)$ attains a local maximum at $x = \frac{1}{3}$ and a local minimum at $x = 1$

$\therefore$ Maximum value of $f(x) = f\left(\frac{1}{3}\right) = \frac{4}{27}$

2 **(a)**

Since, $2\pi k - \frac{\pi}{2} \leq \sin x \leq 2\pi k + \frac{\pi}{2}$



For $k = 0$

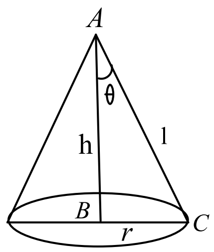
$-\frac{\pi}{2} < \sin x < \frac{\pi}{2}$

Which increase from -1 *to* 1.

Similarly, for other values of k it is increase from -1 *to* 1.

3 **(c)**

Volume of cone, $V = \frac{\pi}{3} r^2 h$



$\Rightarrow \quad V = \frac{\pi}{3} r^2 \sqrt{l^2 - r^2}$

On differentiating w.r.t. r, we get

$\frac{dV}{dr} = \frac{\pi}{3}\left[2r\sqrt{l^2 - r^2} + \frac{r^2}{2\sqrt{l^2 - r^2}}(-2r)\right]$

Put $\frac{dV}{dr} = 0$

$\Rightarrow \quad 2r\sqrt{(l^2 - r^2)} - \frac{r^3}{\sqrt{l^2 - r^2}} = 0$

$\Rightarrow \quad r[2(l^2 - r^2) - r^2] = 0$

$\Rightarrow \quad r = \pm\, l\sqrt{\frac{2}{3}}$

$\therefore$ At $\quad r = l\sqrt{\frac{2}{3}}, \frac{d^2V}{dr^2} < 0$, maxima

$\therefore \quad h = \sqrt{l^2 - \frac{2}{3}l^2} = \frac{l}{\sqrt{3}}$

In $\Delta ABC, \tan\theta = \frac{r}{h} = \frac{l\sqrt{\frac{2}{3}}}{\frac{l}{\sqrt{3}}} = \sqrt{2}$

4 **(c)**

Given that $f(x) = sin\, x - bx + c$

$\therefore f'(x) = \cos x - b$

For decreasing, $f'(x) < 0$, for all $x \in R$.

$\Rightarrow \cos x < b\ for\ all\ x \in R \Rightarrow b > 1$.

5 **(c)**

Given, $f(x) = 2x^3 + 3x^2 - 12x + 1$

$\Rightarrow f'(x) = 6x^2 + 6x - 12$

For $f(x) to$ be decreasing, $f'(x) < 0$

$\Rightarrow 6(x^2 + x - 2) < 0$

$\Rightarrow (x + 2)(x - 1) < 0$

$\Rightarrow x \in (-2,1)$

6 **(a)**

We have,

$f(x) = 2\,x + \cot^{-1} x + \log\left(\sqrt{1 + x^2} - x\right)$

$\therefore f'(x) = 2 - \frac{1}{1 + x^2} + \frac{1}{\sqrt{1 + x^2} - x}\left(\frac{x}{\sqrt{1 + x^2}} - 1\right)$

$\Rightarrow f'(x) = \frac{1 + 2\,x^2}{1 + x^2} - \frac{1}{\sqrt{1 + x^2}}$

$$\Rightarrow f'(x) = \frac{1+2\,x^2}{1+x^2} - \frac{\sqrt{(1+x^2)}}{1+x^2}$$

$$\Rightarrow f'(x) = \frac{x^2 + \sqrt{1+x^2}\{\sqrt{1+x^2}-1\}}{1+x^2} > 0 \text{ for all } x$$

Hence, $f(x)$ is an increasing function on $(-\infty, \infty)$ and in particular on $(0, \infty)$

7 **(c)**

We have,

$$f(x) = 3\cos^2 x + 4\sin^2 x + \cos\frac{x}{2} + \sin\frac{x}{2}$$

$$\Rightarrow f(x) = 4 - \cos^2 x + \cos\frac{x}{2} + \sin\frac{x}{2}$$

$$\Rightarrow f'(x) = \sin 2x - \tfrac{1}{2}\left(\sin\tfrac{x}{2} - \cos\tfrac{x}{2}\right) \quad \text{....(i)}$$

$$\Rightarrow f'(x) = 2\sin x\cos x - \frac{1}{2}\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)$$

$$\Rightarrow f'(x) = 2\sin x\left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right) + \frac{1}{2}\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)$$

$$\Rightarrow f'(x) = \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left\{2\sin x\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right) + \frac{1}{2}\right\}$$

$$\Rightarrow f'(x) = \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left\{2\sqrt{2}\sin x\sin\left(\frac{x}{2} + \frac{\pi}{4}\right) + \frac{1}{2}\right\}$$

For local maximum or minimum, we must have

$f'(x) = 0$

$$\Rightarrow \cos\frac{x}{2} - \sin\frac{x}{2} = 0$$

$$\Rightarrow \cos\frac{x}{2} = \sin\frac{x}{2} \Rightarrow \tan\frac{x}{2} = 1 \Rightarrow \frac{x}{2} = \frac{\pi}{4} \Rightarrow x = \frac{\pi}{2}$$

Now,

$$f''(x) = 2\cos 2x - \tfrac{1}{4}\left(\cos\tfrac{x}{2} + \sin\tfrac{x}{2}\right) \quad \text{[Using (i)]}$$

$$\Rightarrow f''\left(\frac{\pi}{2}\right) = 2\cos\pi - \frac{1}{4}\left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right) = -2 - \frac{1}{2\sqrt{2}} < 0$$

Thus, $f(x)$ attains a local maximum at $x = \frac{\pi}{2}$

Local maximum value $= f\left(\frac{\pi}{2}\right) = 4 + \frac{2}{\sqrt{2}} = 4 + \sqrt{2}$

8 **(c)**

$$\because \quad y = \left(\frac{c^6 - a^2x^4}{b^2}\right)^{\frac{1}{4}}$$

Let $f(x) = xy = \left(\frac{c^6x^4 - a^2x^8}{b^2}\right)^{\frac{1}{4}}$

$$\Rightarrow \quad f'(x) = \frac{1}{4}\left(\frac{c^6x^4 - a^2x^8}{b^2}\right)^{-3/4}$$

$$\left(\frac{4x^3c^6}{b^2} - \frac{8x^7a^2}{b^2}\right)$$

Put $f'(x) = 0$

$$\Rightarrow \quad x = \pm\frac{c^{3/2}}{2^{1/4}\sqrt{a}}$$

$$\therefore \quad f\left(\frac{c^{3/2}}{c^{1/4}\sqrt{a}}\right) = \frac{c^3}{\sqrt{2ab}}$$

9 **(a)**

Let the radius of the circular wave ring by r cm at any time t. Then, $\frac{dr}{dt} = 30$ cm/sec (given)

Let A be the area of the enclosed ring. Then,

$A = \pi r^2$

$\Rightarrow \dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$

$\Rightarrow \dfrac{dA}{dt} = 2\pi \times 50 \times \dfrac{30}{100}\,\text{m}^2\text{sec} = 30\pi^2\ \text{m}^2/\text{sec}$

10 **(b)**

We have,

$x = t\cos t$ and $y = t\sin t$

$\therefore \frac{dx}{dt} = \cos t - t\sin t$ and $\frac{dy}{dx} = \sin t + t\cos t$

At the origin, we have

$x = 0, y = 0 \Rightarrow t\cos t = 0$ and $t\cos t = 0 \Rightarrow t = 0$

The slope of the tangent at $t = 0$ is

$$\frac{dy}{dx} = \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)_{t=0} = \left(\frac{\sin t + t\cos t}{\cos t - t\sin t}\right)_{t=0} = 0$$

So, the equation of the tangent at the origin is

$t - 0 = 0(x - 0) \Rightarrow y = 0$

11 **(c)**

Surface area of sphere $S = 4\pi r^2$ and $\frac{dr}{dt} = 2$

$\therefore\ \dfrac{dS}{dt} = 4\pi \times 2r\dfrac{dr}{dt} = 8\pi r \times 2 = 16\pi r$

$\Rightarrow\ \dfrac{dS}{dt} \propto r$

12 **(c)**

Let $f(x) = \left(\frac{1}{x}\right)^x = x^{-x} = e^{-x\log x}$. Then,

$f'(x) = -\left(\dfrac{1}{x}\right)^x (\log x + 1) = -x^{-x}(\log x + 1)$

Now,

$f'(x) = 0$

$\Rightarrow -x^{-x}(\log x + 1) = 0$

$\Rightarrow \log x + 1 = 0 \Rightarrow \log x = -1 \Rightarrow x = e^{-1}$

Clearly, $f''(x) < 0$ at $x = e^{-1}$

Hence, $f(x) = x^{-x}$ is maximum for $x = e^{-1}$. The maximum value is $e^{1/e}$

13 **(c)**

Given, $f(x) = 2x^3 - 21x^2 + 36x - 30$

$\Rightarrow\quad f'(x) = 6x^2 - 42x + 36$

For maxima or minima, put $f'(x) = 0$

$\Rightarrow\ 6x^2 - 42x + 36 = 0 \quad \Rightarrow \quad x = 6, 1$

And $\quad f''(x) = 12x - 42$

$f''(1) = -30 \quad$ and $\quad f''(6) = 30$

Hence, $f(x)$ has maxima at $x = 1$ and minima at $x = 6$

14 **(a)**

Let $l$be the length of an edge and Vbe the volume of cue at any time t.

$\because \quad V = t^3$

$\because \ \dfrac{dV}{dt} = 3l^2 \dfrac{dl}{dt}$

$= 3 \times 5^2 \times 10 cm^3/s$

$= 750 cm^3/s.$

15 **(c)**

We have, $\frac{dy}{dx} = \frac{-\sin\theta}{1-\cos\theta}$

Clearly, $\frac{dy}{dx} = 0$ for $\theta = (2k+1)\pi$

So, the tangent is parallel to x-axis i.e. $y = 0$

16 **(b)**

We have,

$5x^5 - 10x^3 + x + 2y + 6 = 0 \qquad$ ...(i)

Differentiating with respect to x, we get

$25x^4 - 30x^2 + 1 + 2\dfrac{dy}{dx} = 0$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{1}{2}(25x^4 - 30x^2 + 1)$

$\Rightarrow \left(\dfrac{dy}{dx}\right)_{(0,-3)} = -\dfrac{1}{2}$

The equation of the normal at $(0,-3)$ is

$y + 3 = 2(x - 0) \Rightarrow 2x - y - 3 = 0$

Solving (i) and (ii), we obtain the coordinates of their points of intersection as $P(0,-3), (1,-1)$ and $(-1,-5)$

Hence, the normal at $P(0,-3)$ meets the curve again at $(1,-1)$ and $(-1,-5)$

17 **(b)**

We have,

$\dfrac{dx}{d\theta} = a\,(-\sin\theta + \sin\theta + \theta\cos\theta) = a\,\theta\cos\theta$

and, $\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta) = a\,\theta\sin\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \tan\theta \Rightarrow -\dfrac{1}{\frac{dy}{dx}} = -\cot\theta$

Hence, the slope of the normal varies as θ

The equation of the normal at any point is

$y - a(\sin\theta - \theta\cos\theta) = -\cot\theta\ \{x - a(\cos\theta + \theta\sin\theta)\}$

$\Rightarrow x\cos\ \theta + y\sin\theta = a$

Clearly, it is a line at a constant distance $|a|$ from the origin

18 **(d)**

We have,

$f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \le x \le 2 \\ 37 - x, & 2 < x \le 3 \end{cases}$

Clearly, $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) = f(2)$

So, $f(x)$ is continuous at $x = 2$

Hence, it is continuous on $[-1, 3]$

Thus, option (b) is correct
We find that
$f'(x) = 6x + 12 > 0$ for all $x \in [-1, 2]$
$\Rightarrow f(x)$ is increasing on $[-1, 2]$
Thus, option (a) is correct
Also,
$f'(x) < 0$ for all $x \in (2, 3]$
$\Rightarrow f(x)$ is decreasing on $(2, 3]$
Hence, $f(x)$ is attains the maximum value at $x = 2$
So, option (c) is correct

19 **(b)**
Given, $f(x) = x^3 - 3x^2 + 2x$
$\Rightarrow f'(x) = 3x^2 - 6x + 2$
Now, f(a)=f(0)=0
And $f(b) = f\left(\frac{1}{2}\right)$
$= \frac{1}{2}\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right) = \frac{3}{8}$
By Lagrange's Mean Value Theorem
$$\frac{f(b) - f(a)}{b - a} = f'(c)$$
$$\Rightarrow \frac{\frac{3}{8} - 0}{\frac{1}{2} - 0} = 3c^2 - 6c + 2$$
$\Rightarrow 12c^2 - 24x + 5 = 0$
This is a quadratic equation in c.
$$c = \frac{24 \pm \sqrt{576 - 240}}{24}$$
$$= 1 \pm \frac{\sqrt{21}}{6}$$
But c lies between 0 to$\frac{1}{2}$
$\therefore$ we take, $c = 1 - \frac{\sqrt{21}}{6}$

20 **(a)**
Since,$f(x) = kx - \sin x$is monotonically increasing for all $x \in R$.Therefore,
$f'(x) > 0$for all $x \in R$
$\Rightarrow K - \cos x > 0$
$\Rightarrow K > \cos x$
$\Rightarrow K > 1$ [$\therefore$ maximum value of $\cos x$ is 1]

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	C	C	C	A	C	C	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	C	A	C	B	B	D	B	A

SESSION : 2025-26

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Subject :MATHS
DPP No. : 2

Topic :-Applications of Derivatives

1 **(c)**

From mean value theorem $f'(c) = \frac{f(b)-f(a)}{b-a}$

Given, $a = 0 \Rightarrow f(a) = 0$

and $b = \frac{1}{2} \Rightarrow f(b) = \frac{3}{8}$

Now, $f'(x) = (x-1)(x-2) + x(x-2) + x(x-1)$

$\therefore f'(c) = (c-1)(c-2) + c(c-2) + c(c-1)$

$= c^2 - 3c + 2 + c^2 - 2c + c^2 - c$

$\Rightarrow f'(c) = 3c^2 - 6c + 2$

By definition of mean value theorem

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow 3c^2 - 6c + 2 = \frac{\left(\frac{3}{8}\right) - 0}{\left(\frac{1}{2}\right) - 0} = \frac{3}{4}$$

$$\Rightarrow 3c^2 - 6c + \frac{5}{4} = 0$$

This is a quadratic equation in c

$$\therefore c = \frac{6 \pm \sqrt{36 - 15}}{2 \times 3} = \frac{6 \pm \sqrt{21}}{6} = 1 \pm \frac{\sqrt{21}}{6}$$

Since, $'c'$ lies between $\left[0, \frac{1}{2}\right]$

$$\therefore c = 1 - \frac{\sqrt{21}}{6} \quad \left(\text{neglecting } c = 1 + \frac{\sqrt{21}}{6}\right)$$

2 **(a)**

$$\because f(x) = \frac{1}{4x^2 + 2x + 1}$$

On differentiating w.r.t. x, we get

$f'(x) = \frac{-(8x+2)}{(4x^2+2x+1)^2}$ ...(i)

For maxima or minima put $f'(x) = 0$

$\Rightarrow \quad 8x + 2 = 0 \quad \Rightarrow \quad x = -\frac{1}{4}$

Again differentiating w.r.t. x of Eq. (i), we get

$$f''(x) = -\frac{\begin{bmatrix}(4x^2 + 2x + 1)^2(8) - (8x - 2)2 \times \\ (4x^2 + 2x + 1)(8x + 2)\end{bmatrix}}{(4x^2 + 2x + 1)^4}$$

At $x = -\frac{1}{4}, f''\left(-\frac{1}{4}\right) = -\text{ve}$

$f(x)$ is maximum at $x = -\frac{1}{4}$

$\therefore$ maximum value of $f(x)$

$f\left(-\frac{1}{4}\right)_{\max} = \dfrac{1}{4 \times \frac{1}{16} - 2 \times \frac{1}{4} + 1}$

$= \dfrac{1}{\frac{1}{4} - \frac{2}{4} + 1}$

$= \dfrac{4}{1 - 2 + 4} = \dfrac{4}{3}$

4 **(b)**

Let $f(x) = 2x + 3y$

$\therefore \quad f(x) = 2x + \frac{18}{x} \qquad [\because xy = 6, \text{given}]$

$\Rightarrow \quad f'(x) = 2 - \dfrac{18}{x^2}$

Put $f'(x) = 0$ for maxima or minima

$\Rightarrow \quad 0 = 2 - \dfrac{18}{x^2} \quad \Rightarrow \quad x = \pm 3$

And $\quad f''(x) = \frac{36}{x^3} \qquad \Rightarrow f''(3) = \frac{36}{3^3} > 0$

$\therefore$ At $x = 3, f(x)$ is minimum.

The minimum value is $f(3) = 12$

5 **(c)**

On differentiating given curve w. r. t. x, we get

$4\,y^3 \dfrac{dy}{dx} = 3ax^2$

$\Rightarrow \left(\dfrac{dy}{dx}\right)_{(a,a)} = \dfrac{3a^3}{4a^3} = \dfrac{3}{4}$

$\therefore$ Equation of normal at point (a, a) is

$y - a = -\dfrac{4}{3}\,(x - a)$

$\Rightarrow \quad 4x + 3y = 7a$

7 **(b)**

Let $f(x) = x^3 - px + q$

Then $\quad f'(x) = 3x^2 - p$

Put $\quad f'(x) = 0 \quad \Rightarrow \quad x = \sqrt{\frac{p}{3}}, \ -\sqrt{\frac{p}{3}}$

Now, $f''(x) = 6x$

$\therefore$ At $\quad x = \sqrt{\frac{p}{3}, \ f''(x) = 6\sqrt{\frac{p}{3}} > 0,}$ minima

And at $x = -\sqrt{\frac{p}{3}},\ \ f''(x) < 0$, maxima

8 **(a)**

For $x = p, y = ap^2bp + c$

And for $x = q,\ \ y = aq^2 + bq + c$

Slope $= \frac{aq^2+bq+c-ap^2-bp-c}{q-p}$

$= a(q+p) + b$

$\frac{dy}{dx} = 2ax + b = a(q+p) + b$

(according to the equation)

$\therefore \quad x = \frac{q+p}{2}$

9 **(c)**

Clearly, $f(x)$ is continuous and differentiable on the intervals $[0, 3]$ and $(0, 3)$ respectively for all $n \in N$

Also, $f(0) = f(3) = 0$

It is given that Rolle's theorem for the function $f(x)$ defined on $[0, 3]$ is applicable with $c = \frac{3}{4}$

$\therefore f'(c) = 0$

$\Rightarrow 2(c-3)^n + 2nc(c-3)^{n-1} = 0$

$[\because f(x) = 2x(x-3)^n \therefore f'(x) = 2(x-3)^n + 2nx(x-3)^{n-1}]$

$\Rightarrow 2(c-3)^{n-1}(c - 3 + nc) = 0$

$\Rightarrow \frac{3}{4} - 3 + \frac{3}{4}n = 0 \Rightarrow n = 3 \quad \left[\because c = \frac{3}{4}\right]$

10 **(a)**

Given,

$y - x = 1$

$\Rightarrow \quad y = x + 1$

$\frac{dy}{dx} = 1$

And $y^2 = x$

$\Rightarrow \quad 2y\frac{dy}{dx} = 1$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2y}$

$\therefore \quad 1 = \frac{1}{2y}$

$\Rightarrow \quad 2y = 1$

$\Rightarrow \quad y = \frac{1}{2}$

$\therefore$ point on the curve is $\left(\frac{1}{4}, \frac{1}{2}\right)$

$\therefore$ required shortest distance

$= \left|\frac{\frac{1}{4} - \frac{1}{2} + 1}{\sqrt{2}}\right| = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

11 **(d)**

Since,$s^3 \propto v \Rightarrow \frac{ds}{dt} = ks^3$.......(i)

$$\Rightarrow \quad \frac{d^2s}{dt^2} = 3ks^2\frac{ds}{dt}$$

$$\Rightarrow \quad \frac{d^2s}{dt} = 3k^2s^5 \qquad [from\ Eq.(i)]$$

Hence, acceleration of particle is proportional to s^5.

12 **(a)**

Given, $s = t^3 + 2t^2 + t$

$\Rightarrow \quad v = \frac{ds}{dt} = 3t^2 + 4t + 1$

Speed of the particle after 1 s

$v_{(t=1)} = \left(\frac{ds}{dt}\right)_{(t=1)}$

$= 3 \times 1^2 + 4 \times 1 + 1 = 3 + 5 = 8$ cm/s

14 **(b)**

Given, $y = a^x \quad \Rightarrow \quad \frac{dy}{dx} = a^x \log a$

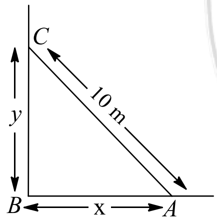
Now, length of subtangent at any point $(x_1,\ y_1)$

$= \frac{y}{dy/dx} = \frac{a^{x_1}}{a^{x_1}\log a} = \frac{1}{\log a}$

16 **(c)**

Let $AB = x$m, $BC = y$m and $AC = 10$m

$\therefore \quad x^2 + y^2 = 100 \quad$ ...(i)



On differentiating w.r.t t,m we get

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\Rightarrow \quad 2x(3) - 2y(4) = 0$

$\Rightarrow \quad x = \frac{4y}{3}$

On putting this value in Eq. (i), we get

$\frac{16}{9}y^2 + y^2 = 100$

$\Rightarrow \quad y^2 = \frac{100\times9}{25} = 36 \quad \Rightarrow \quad y = 6$m

17 **(c)**

Since $f(x)$, which is of degree 3, has relative minimum/maximum at $x = -1$ and $x = \frac{1}{3}$.

Therefore $x = -1, x = \frac{1}{3}$ are roots of $f'(x) = 0$. Thus, $x + 1$ and $3x - 1$ are factors of $f'(x)$

Consequently, we have

$f'(x) = \lambda(x+1)(3\,x-1) = \lambda(3\,x^2 + 2\,x - 1)$

$\Rightarrow f(x) = \lambda(x^3 + x^2 - x) + c$

Now, $f(-2) = 0$ [Given]

$\Rightarrow c = 2\lambda$ ...(i)

We have,

$$\int_{-1}^{1} f(x)dx = \frac{14}{3}$$

$$\Rightarrow \int_{-1}^{1} [\lambda(x^3 + x^2 - x) + c]dx = \frac{14}{3}$$

$\Rightarrow \lambda \int_{-1}^{1} x^2 + \int_{-1}^{1} c\, dx = \frac{14}{3} \Rightarrow \frac{2\lambda}{3} + 2\,c = \frac{14}{3} \Rightarrow \lambda + 3\,c = 7$ ...(ii)

Solving (i) and (ii), we get $\lambda = 1, c = 2$

Hence, $f(x) = x^3 + x^2 - x + 2$

18 **(d)**

$$f(x) = cosx + \frac{1}{2}cos2x - \frac{1}{3}cos3x$$

$\Rightarrow \quad f'(x) = -\sin x - \sin 2x + \sin 3x$

Put $\quad f'(x) = 0$

$$\Rightarrow 2\sin\frac{3x}{2}\cos\frac{x}{2} = 2\sin\frac{3x}{2}\cos\frac{3x}{2}$$

$$\Rightarrow \sin\frac{3x}{2} = 0, \cos\frac{3x}{2} = \cos\frac{x}{2}$$

$\Rightarrow \quad x = \frac{2n\pi}{3}, \frac{3x}{2} = 2n\pi \pm \frac{x}{2}$

$\Rightarrow$ At $x = 0, \frac{2\pi}{3}$,

At $x = 0$

$$f(x) = 1 + \frac{1}{2} - \frac{1}{3} = \frac{7}{6}$$

At $\quad x = \frac{2\pi}{3}$,

$$f(x) = -\frac{1}{2} - \frac{1}{4} - \frac{1}{3} = -\frac{13}{12}$$

$\therefore$ difference$= \frac{7}{6} + \frac{13}{12} = \frac{27}{12} = \frac{9}{4}$

19 **(a)**

Let $f(x) = e^{x-1} + x - 2$. Then,

$f(1) = e^0 + 1 - 2 = 0$

$\Rightarrow x = 1$ is a real roots of the equation $f(x) = 0$

Let $x = \alpha$ be other real root of $f(x) = 0$ such that $\alpha > 1$

Consider the interval $[1, \alpha]$

Clearly, $f(1) = f(\alpha) = 0$

So, by Rolle's theorem $f'(x) = 0$ has a root in $(1, \alpha)$

But, $f'(x) = e^{x-1} + 1 > 1$ for all x

$\therefore f'(x) \neq 0$ for any $x \in (1, \alpha)$

This is a contradiction

Hence, $f(x) = 0$ has no real root other than 1

20 **(d)**

We have,

$f(x) = \sin^6 x + \cos^6 x$

$\Rightarrow f(x) = (\sin^2 x + \cos^2 x)^3 - 3(\sin^2 x + \cos^2 x)(\sin^2 x \cos^2 x)$

$\Rightarrow f(x) = 1 - 3(\sin^2 x \cos^2 x) = 1 - \dfrac{3}{4}\sin^2 2x$

$\therefore f(x) \leq 1$, for all x $\quad \left[\because -\frac{3}{4}\sin^2 2x < 0\right]$

and, $f(x) > 1 - \frac{3}{4} = \frac{1}{4}$ for all x $\quad \left[\because -\frac{3}{4}\sin^2 2x \geq -\frac{3}{4}\right]$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	C	B	C	A	B	A	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	D	B	C	C	C	D	A	D

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

Class : XIth

Solutions

Subject :MATH

Date :

DPP No. : 3

Topic :-Applications of Derivatives

1 **(c)**

From mean value theorem $f'(c) = \frac{f(b)-f(a)}{b-a}$

Given, $a = 0 \Rightarrow f(a) = 0$

and $b = \frac{1}{2} \quad \Rightarrow f(b) = \frac{3}{8}$

Now, $f'(x) = (x-1)(x-2) + x(x-2) + x(x-1)$

$$\therefore \ f'(c) = (c-1)(c-2) + c(c-2) + c(c-1)$$

$$= c^2 - 3c + 2 + c^2 - 2c + c^2 - c$$

$$\Rightarrow \ f'(c) = 3c^2 - 6c + 2$$

By definition of mean value theorem

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad 3c^2 - 6c + 2 = \frac{\left(\frac{3}{8}\right) - 0}{\left(\frac{1}{2}\right) - 0} = \frac{3}{4}$$

$$\Rightarrow \ 3c^2 - 6c + \frac{5}{4} = 0$$

This is a quadratic equation in c

$$\therefore \quad c = \frac{6 \pm \sqrt{36-15}}{2 \times 3} = \frac{6 \pm \sqrt{21}}{6} = 1 \pm \frac{\sqrt{21}}{6}$$

Since, $'c'$ lies between $\left[0, \frac{1}{2}\right]$

$$\therefore \quad c = 1 - \frac{\sqrt{21}}{6} \quad \left(neglecting \ \ c = 1 + \frac{\sqrt{21}}{6}\right)$$

2 **(a)**

$$\because \quad f(x) = \frac{1}{4x^2 + 2x + 1}$$

On differentiating w.r.t. x, we get

$f'(x) = \frac{-(8x+2)}{(4x^2+2x+1)^2}$ ...(i)

For maxima or minima put $f'(x) = 0$

$$\Rightarrow \quad 8x + 2 = 0 \quad \Rightarrow \quad x = -\frac{1}{4}$$

Again differentiating w.r.t. x of Eq. (i), we get

$f''(x) = -\frac{\left[(4x^2+2x+1)^2(8)-(8x-2)2\times(4x^2+2x+1)(8x+2)\right]}{(4x^2+2x+1)^4}$

At $x = -\frac{1}{4}, f''\left(-\frac{1}{4}\right) = -ve$

$f(x)$ is maximum at $x = -\frac{1}{4}$

$\therefore$ maximum value of $f(x)$

$$f\left(-\frac{1}{4}\right)_{max} = \frac{1}{4 \times \frac{1}{16} - 2 \times \frac{1}{4} + 1}$$

$$= \frac{1}{\frac{1}{4} - \frac{2}{4} + 1}$$

$$= \frac{4}{1 - 2 + 4} = \frac{4}{3}$$

4 **(b)**

Let $f(x) = 2x + 3y$

$\therefore \quad f(x) = 2x + \frac{18}{x}$ $\quad$ $[\because xy = 6$, given$]$

$$\Rightarrow \quad f'(x) = 2 - \frac{18}{x^2}$$

Put $f'(x) = 0$ for maxima or minima

$$\Rightarrow \quad 0 = 2 - \frac{18}{x^2} \quad \Rightarrow \quad x = \pm 3$$

And $\quad f''(x) = \frac{36}{x^3} \quad \Rightarrow f''(3) = \frac{36}{3^3} > 0$

$\therefore$ At $x = 3, f(x)$ is minimum.

The minimum value is $f(3) = 12$

5 **(c)**

On differentiating given curve w. r. t. x, we get

$$4\,y^3\frac{dy}{dx}=3ax^2$$

$$\Rightarrow\left(\frac{dy}{dx}\right)_{(a,a)}=\frac{3a^3}{4a^3}=\frac{3}{4}$$

$\therefore$ Equation of normal at point (a,a) is

$$y-a=-\frac{4}{3}\,(x-a)$$

$$\Rightarrow\quad 4x+3y=7a$$

7 **(b)**

Let $f(x)=x^3-px+q$

Then $f'(x)=3x^2-p$

Put $f'(x)=0 \quad\Rightarrow\quad x=\sqrt{\frac{p}{3}},\ -\sqrt{\frac{p}{3}}$

Now, $f''(x)=6x$

$\therefore$ At $x=\sqrt{\frac{p}{3},\ f''(x)=6\sqrt{\frac{p}{3}}>0,}$ minima

And at $x=-\sqrt{\frac{p}{3},\ f''(x)}<0$, maxima

8 **(a)**

For $x=p, y=ap^2bp+c$

And for $x=q,\ \ y=aq^2+bq+c$

Slope $=\frac{aq^2+bq+c-ap^2-bp-c}{q-p}$

$$=a(q+p)+b$$

$$\frac{dy}{dx}=2ax+b=a(q+p)+b$$

(according to the equation)

$$\therefore\quad x=\frac{q+p}{2}$$

9 **(c)**

Clearly, $f(x)$ is continuous and differentiable on the intervals $[0,3]$ and $(0,3)$ respectively for all $n\in N$

Also, $f(0)=f(3)=0$

It is given that Rolle's theorem for the function $f(x)$ defined on $[0,3]$ is applicable with $c=\frac{3}{4}$

$$\therefore f'(c)=0$$

$$\Rightarrow 2(c-3)^n + 2nc(c-3)^{n-1} = 0$$

$$[\because f(x) = 2x(x-3)^n \therefore f'(x) = 2(x-3)^n + 2nx(x-3)^{n-1}]$$

$$\Rightarrow 2(c-3)^{n-1}(c-3+nc) = 0$$

$$\Rightarrow \frac{3}{4} - 3 + \frac{3}{4}n = 0 \Rightarrow n = 3 \quad \left[\because c = \frac{3}{4}\right]$$

10 **(a)**

Given,

$$y - x = 1$$

$$\Rightarrow \quad y = x + 1$$

$$\frac{dy}{dx} = 1$$

And $y^2 = x$

$$\Rightarrow \quad 2y\frac{dy}{dx} = 1$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2y}$$

$$\therefore \quad 1 = \frac{1}{2y}$$

$$\Rightarrow \quad 2y = 1$$

$$\Rightarrow \quad y = \frac{1}{2}$$

$\therefore$ point on the curve is $\left(\frac{1}{4}, \frac{1}{2}\right)$

$\therefore$ required shortest distance

$$= \left|\frac{\frac{1}{4} - \frac{1}{2} + 1}{\sqrt{2}}\right| = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$$

11 **(d)**

Since,$s^3 \propto v \Rightarrow \frac{ds}{dt} = ks^3$.......(i)

$$\Rightarrow \quad \frac{d^2s}{dt^2} = 3ks^2\frac{ds}{dt}$$

$$\Rightarrow \quad \frac{d^2s}{dt} = 3k^2s^5 \qquad [from\ Eq.(i)]$$

Hence, acceleration of particle is proportional to s^5.

12 **(a)**

Given, $s = t^3 + 2t^2 + t$

$$\Rightarrow \quad v = \frac{ds}{dt} = 3t^2 + 4t + 1$$

Speed of the particle after 1 s

$$v_{(t=1)} = \left(\frac{ds}{dt}\right)_{(t=1)}$$

$= 3 \times 1^2 + 4 \times 1 + 1 = 3 + 5 = 8$ cm/s

14 **(b)**

Given, $y = a^x \quad \Rightarrow \quad \frac{dy}{dx} = a^x \, log \, log \, a$

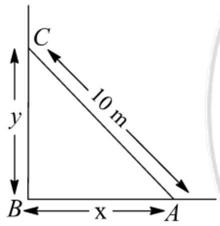
Now, length of subtangent at any point $(x_1, \ y_1)$

$$= \frac{y}{dy/dx} = \frac{a^{x_1}}{a^{x_1} \, log \, log \, a} = \frac{1}{log \, log \, a}$$

16 **(c)**

Let $AB = x$m, $BC = y$m and $AC = 10$m

$\therefore \quad x^2 + y^2 = 100 \quad$ ...(i)



On differentiating w.r.t t,m we get

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\Rightarrow \quad 2x(3) - 2y(4) = 0$$

$$\Rightarrow \quad x = \frac{4y}{3}$$

On putting this value in Eq. (i), we get

$$\frac{16}{9}y^2 + y^2 = 100$$

$\Rightarrow \quad y^2 = \frac{100 \times 9}{25} = 36 \quad \Rightarrow \quad y = 6$m

17 **(c)**

Since $f(x)$, which is of degree 3, has relative minimum/maximum at $x = -1$ and $x = \frac{1}{3}$.

Therefore $x = -1, x = \frac{1}{3}$ are roots of $f'(x) = 0$. Thus, $x + 1$ and $3x - 1$ are factors of $f'(x)$

Consequently, we have

$$f'(x) = \lambda(x + 1)(3\,x - 1) = \lambda(3\,x^2 + 2\,x - 1)$$

$$\Rightarrow f(x) = \lambda(x^3 + x^2 - x) + c$$

Now, $f(-2) = 0$ [Given]
$\Rightarrow c = 2\lambda$ ...(i)
We have,

$$\int_{-1}^{1} f(x)dx = \frac{14}{3}$$

$$\Rightarrow \int_{-1}^{1} [\lambda(x^3 + x^2 - x) + c]dx = \frac{14}{3}$$

$\Rightarrow \lambda \int_{-1}^{1} x^2 + \int_{-1}^{1} c\, dx = \frac{14}{3} \Rightarrow \frac{2\lambda}{3} + 2\, c = \frac{14}{3} \Rightarrow \lambda + 3\, c = 7$ ...(ii)
Solving (i) and (ii), we get $\lambda = 1, c = 2$
Hence, $f(x) = x^3 + x^2 - x + 2$

18 **(d)**

$$f(x) = cosx + \frac{1}{2}cos2x - \frac{1}{3}cos3x$$

$\Rightarrow$ $f'(x)$=$-$sin$x$-sin2x+sin3x
Put $f'(x) = 0$
$\Rightarrow$ $2\, sin\, sin\, \frac{3x}{2}\, cos\, cos\, \frac{x}{2} = 2\, sin\, sin\, \frac{3x}{2}\, cos\, cos\, \frac{3x}{2}$

$$\Rightarrow sin\, sin\, \frac{3x}{2} = 0, cos\, cos\, \frac{3x}{2} = cos\, cos\, \frac{x}{2}$$

$\Rightarrow$ $x = \frac{2n\pi}{3}, \frac{3x}{2} = 2n\pi \pm \frac{x}{2}$
$\Rightarrow$ At $x = 0, \frac{2\pi}{3},$
At $x = 0$

$$f(x) = 1 + \frac{1}{2} - \frac{1}{3} = \frac{7}{6}$$

At $x = \frac{2\pi}{3},$

$$f(x) = -\frac{1}{2} - \frac{1}{4} - \frac{1}{3} = -\frac{13}{12}$$

$\therefore$ difference$= \frac{7}{6} + \frac{13}{12} = \frac{27}{12} = \frac{9}{4}$

19 **(a)**
Let $f(x) = e^{x-1} + x - 2$. Then,

$$f(1) = e^0 + 1 - 2 = 0$$

$\Rightarrow x = 1$ is a real roots of the equation $f(x) = 0$
Let $x = \alpha$ be other real root of $f(x) = 0$ such that $\alpha > 1$
Consider the interval $[1, \alpha]$
Clearly, $f(1) = f(\alpha) = 0$

So, by Rolle's theorem $f'(x) = 0$ has a root in $(1, \alpha)$
But, $f'(x) = e^{x-1} + 1 > 1$ for all x
$\therefore f'(x) \neq 0$ for any $x \in (1, \alpha)$
This is a contradiction
Hence, $f(x) = 0$ has no real root other than 1

20 **(d)**

We have,

$$f(x) = x \;\; + x$$

$$\Rightarrow f(x) = (x \;\; + x\;)^3 - 3(x \;\; + x\;)(x\; x\;)$$

$$\Rightarrow f(x) = 1 - 3(x\; x\;) = 1 - \frac{3}{4} 2\, x$$

$\therefore f(x) \leq 1$, for all x $\left[\because -\frac{3}{4} 2\, x \; < 0\right]$

and, $f(x) > 1 - \frac{3}{4} = \frac{1}{4}$ for all x $\left[\because -\frac{3}{4} 2\, x \; \geq -\frac{3}{4}\right]$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	C	B	C	A	B	A	C	A
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A.	D	A	D	B	B	C	C	D	A	D

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 4

Topic :-Applications of Derivatives

1 **(a)**

Let $y = x^3 - 12x \quad \Rightarrow \frac{dy}{dx} = 3x^2 - 12$

Put $\frac{dy}{dx} = 0, \quad 3x^2 - 12 = 0$

$\Rightarrow \quad x = \pm 2$

At $x = 2, \quad y = 2^3 - 12(2) = -16$

At $x = -2, \quad y = (-2)^3 - 12(-2) = 16$

Hence, option (a) is correct

2 **(c)**

We have,

$xy = a^2$ and $S = b^2x + c^2y$

$\Rightarrow S = b^2x + \frac{c^2a^2}{x}$

$\Rightarrow \frac{dS}{dx} = b^2 - \frac{c^2a^2}{x^2}$ and $\frac{d^3S}{dx^2} = \frac{2c^2a^2}{x^3}$

For local maximum or minimum, we must have

$\frac{dS}{x} = 0 \Rightarrow b^2 - \frac{c^2a^2}{x^2} = 0 \Rightarrow x^2 = \frac{c^2a^2}{b^2} \Rightarrow x = \pm\frac{ca}{b}$

Clearly, $\frac{d^2S}{dx^2} > 0$ for $x = \frac{ca}{b}$

So, $x = \frac{ca}{b}$ is the point of local minimum

Local minimum value of $S = b^2\left(\frac{ca}{b}\right) + c^2\left(\frac{a^2b}{ca}\right) = 2abc$

3 **(b)**

We have,

$g(x) = f(x) + f(1 - x)$

$\therefore g'(x) = f'(x) - f'(1 - x)$ for all $x \in [0,1]$

Now, $f"(x) < 0$ for $0 \le x \le 1$

$\Rightarrow f'(x)$ is a decreasing function on $[0, 1]$

$\Rightarrow f'(x) > f'(1 - x)$ if $x < 1 - x$

and,

$f'(x) < f'(1-x)$ if $x > 1-x$

$\Rightarrow f'(x) - f'(1-x) > 0$ if $x < \frac{1}{2}$

and,

$f'(x) - f'(1-x) < 0$ if $x > \frac{1}{2}$

$\Rightarrow g'(x) > 0$ if $x \in (0, 1/2)$

and,

$\Rightarrow g'(x) < 0$ if $x \in (1/2, 1)$

$\Rightarrow g(x)$ decreases on $\left[\frac{1}{2,1}\right]$ and increases on $\left[\frac{0,1}{2}\right]$

4 **(d)**

Since, $f(x) = x\, e^{1-x}$
$f'(x) = -xe^{1-x} + e^{1-x}$
$= e^{1-x}(1-x)$
$\Rightarrow \quad f'(x) < 0, \ \forall\ (1, \infty)$

5 **(a)**
Consider the function
$\phi(x) = a_0 \dfrac{x^{n+1}}{n+1} + a_1 \dfrac{x^n}{n} + a_2 \dfrac{x^{n-1}}{n-1} + \cdots + a_{n-1} \dfrac{x^2}{2} + a_n x$
Since $\phi(x)$ is a polynomial. Therefore, it is continuous on $[0, 1]$ and differentiable on $(0, 1)$
Also, $\phi(0) = 0$
and, $\phi(1) = \dfrac{a_0}{n+1} + \dfrac{a_1}{n} + \dfrac{a_2}{n-2} + \cdots + a_n = 0$ [Given]
$\therefore \phi(0) = \phi(1)$
Thus, $\phi(x)$ satisfies conditions of Rolle's theorem on $[0, 1]$
Consequently, there exist $c \in (0, 1)$ such that $\phi'(c) = 0$ i.e. $c \in (0, 1)$ is a zero of
$\phi'(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_n = f(x)$

6 **(c)**
Given, $x = at^2 + bt + c$
$\Rightarrow \ (speed) \dfrac{dx}{dt} = 2at + b$
$\Rightarrow \ (\text{acceleration}) \dfrac{d^2x}{dt^2} = 2a$
$\therefore$ The particle will moving with
Uniform acceleration.

9 **(a)**
On differentiating given equation w. r. t. x, we get
$\frac{dx}{dt} = 100 - \frac{25}{2}$.(2t)=100-25t
At maximum height, velocity $\frac{dx}{dt} = 0$

$$\therefore \quad 100 - 25t = 0 \quad \Rightarrow \quad t = 4$$

$$\therefore \quad x = 100 \times 4 - \frac{25 \times 16}{2} = 200m$$

10 **(b)**
We have,
$y = \int_0^x |t|\, dt$ ...(i)
$\Rightarrow \dfrac{dy}{dx} = |x|$

Let $P(x_1, y_1)$ be a point on the curve (i) such that the tangent at P is parallel to the line $y = 2x$

$\therefore$ (Slope of the tangent at P) $= 2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = 2 \Rightarrow |x_1| = 2 \Rightarrow x_1 = \pm 2$

Now, $y = \int_0^x |t|\, dt$

$$y = \begin{cases} \int_0^x t dt = \frac{x^2}{2}, & \text{if } x \geq 0 \\ -\int_0^x t\, dt = -\frac{x^2}{2}, & \text{if } x < 0 \end{cases}$$

$\therefore x_1 = 2 \Rightarrow y_1 = 2$ and $x_1 = -2 \Rightarrow y_1 = -2$

Thus, the two points on the curve are $(2, 2)$ and $(-2, -2)$

The equations of the tangents at these two points are

$y - 2 = 2(x - 2)$ and $y + 2 = 2(x + 2)$ respectively

Or, $2x - y - 2 = 0$ and $2x - y + 2 = 0$ respectively

These tangents cut off intercepts -2 and 2 respectively on y-axis

11 **(a)**

We have,

$f'(x) = \sec^2 x - 1 \geq 0$ for all x [$\because |\sec x| \geq 1$ for all x]

Hence, $f(x)$ always increases

12 **(b)**

Let $P = xy$. Then,

$P = x(8 - x)$ [$\because x + y = 8$ (given)]

$\Rightarrow P = 8x - x^2 \Rightarrow \frac{dP}{dx} = 8 - 2x$ and $\frac{d^2P}{dx^2} = -2$

For maximum and minimum, we must have

$\frac{dP}{dx} = 0 \Rightarrow 8 - 2x = 0 \Rightarrow x = 4$

Clearly, $\frac{d^2P}{dx^2} = -1 < 0$ for all x

Hence, P is maximum when $x = y = 4$. The maximum value of P is given by $P = 4 \times 4 = 16$

13 **(d)**

Given curve is $y = 2x^2 - x + 1$ ...(i)

$\Rightarrow \frac{dy}{dx} = 4x - 1$

Since, tangent to the curve is parallel to the given line$y = 3x + 9$. Then, slopes will be equal

$\therefore 4x - 1 = 3$

$\Rightarrow x = 1$

From Eq. (i), $y = 2(1)^2 - 1 + 1 = 2$

Hence, required point is $(1, 2)$

14 **(d)**

Let $P(x_1, y_1)$ be a point on $y^2 = 2x^3$ such that the tangent at P is perpendicular to the line $4x - 3y + 2 = 0$

$\therefore \left(\frac{dy}{dx}\right)_{(x_1,y_1)} \times \left(\frac{-4}{-3}\right) = -1 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{3}{4}$ ...(i)

Now,

$y^2 = 3x^3$

$\Rightarrow 2y\frac{dy}{dx} = 6x^2 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{3x_1^2}{y_1}$ ...(ii)

From (i) and (ii), we get

$\frac{3x_1^2}{y_1} = \frac{3}{4} \Rightarrow y_1 = 4x_1^2$ ...(iii)

Since (x_1, y_1) lies on $y^2 = 2x^3$

$\therefore y_1^2 = 2x_1^3$ ...(iv)

Solving (iii) and (iv), we get

$(4x_1)^2 = 2x_1^3 \Rightarrow x_1 = 0, x_1 = 1/8$

Putting the values of x_1 in (iv), we get

$y_1 = 0, y_1 = \pm\frac{1}{16}$

Hence, the required points are $(0, 0)$, $(1/8, 1/16)$, $(1/8, -1/16)$

15 **(d)**

We have,

$x = e^t \cos t$ and $y = e^t \sin t$

$\Rightarrow \dfrac{dx}{dt} = e^t(\cos t - \sin t)$ and $\dfrac{dy}{dt} = e^t(\sin t + \cos t)$

$\therefore \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\sin t + \cos t}{\cos t - \sin t} \Rightarrow \left(\dfrac{dy}{dx}\right)_{t=\pi/4} = \infty$

So, tangent at $t = \frac{\pi}{4}$ subtends a right angle with x-axis

16 **(b)**

We have,

$y^2 = 4a\left(x + a\sin\frac{x}{a}\right)$ ...(i)

$\Rightarrow 2y\dfrac{dy}{dx} = 4a\left(1 + \cos\dfrac{x}{a}\right)$

For points at which the tangents are parallel to x-axis, we must have

$\dfrac{dy}{dx} = 0$

$\Rightarrow 4a\ \left(1 + \cos\dfrac{x}{a}\right) = 0 \Rightarrow \cos\dfrac{x}{a} = -1 \Rightarrow \dfrac{x}{a} = (2n+1)\pi$

For these values of x, we have $\sin\frac{x}{a} = 0$

Putting $\sin\frac{x}{a} = 0$ in (i), we get $y^2 = 4ax$

Therefore, all these points lie on the parabola $y^2 = 4ax$

17 **(b)**

Given $y = x^{5/2}$

$\therefore\ \dfrac{dy}{dx} = \dfrac{5}{2}x^{3/2}, \dfrac{d^2y}{dx^2} = \dfrac{15}{4}x^{1/2}$

At $x = 0, \frac{dy}{dx} = 0, \frac{d^2y}{dx^2} = 0$

and $\frac{d^3y}{dx^3}$ is not defined

When $x = 0, y = 0$

$\therefore$ $(0, 0)$ is a point of inflexion

18 **(a)**

Let $f(x) = 2x + 3y$ and $xy = 6$

$\Rightarrow\ \ f(x) = 2x + \dfrac{18}{x}$

On differentiating w.r.t. x, we get

$f'(x) = 2 - \dfrac{18}{x^2}$

Put $f'(x) = 0$ for maxima or minima

$\Rightarrow\ \ 0 = 2 - \dfrac{18}{x^2} \Rightarrow\ x = \pm 3$

and $f''(x) = \frac{36}{x^3}$

$\Rightarrow\ \ f''(3) = \dfrac{36}{3^3} > 0$

$\therefore$ At $x = 3, f(x)$ is minimum

The minimum value of $f(x)$ is

$f(3) = 2(3) + 3(2) = 12$

19 **(b)**

Given, $f(x) = x^2 - 2x + 4$

$f'(x) = 2x - 2$

By applying Mean value theorem

$f'(c) = 2c - 2 = 0$

$\Rightarrow c = 1$

20 **(b)**

By the algebraic meaning of Rolle's theorem between any two roots of a polynomial there is always a root of its derivative

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	D	A	C	A	A	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	D	D	D	B	B	A	B	B

Topic :-Applications of Derivatives

1 **(c)**

$p(t) = 1000 + \frac{1000t}{100 + t^2}$

$\Rightarrow \quad p'(t) = 0 + \frac{(100 + t^2)(1000) - 1000t\,(2t)}{(100 + t^2)^2}$

$= 1000\frac{(100 - t^2)}{(100 + t^2)^2}$

Put $p'(t) = 0$ for maxima or minima

$\Rightarrow \quad 100 - t^2 = 0$

$\Rightarrow \quad t = \pm 10$

Now, $p''(t) = 1000$

$\times \left[\frac{(100 + t^2)^2(-2t) - (100 - t^2)2(100 + t^2)2t}{(100 + t^2)^4}\right]$

$= 1000t\frac{[(100 + t^2)(-2) - (100 - t^2)(4)]}{(100 + t^2)^3}$

$= -1000t\,\frac{[600 - 2t^2]}{(100 + t^2)^3}$

At $t = 10$, $p''(t) < 0$

$\therefore$ The maximum value is

$p(10) = 1000 + \frac{10000}{100 + 100}$

$= 1000 + \frac{10000}{200} = 1050$

2 **(a)**

We have,

$f'(x) = (x - a)^{2n}(x - b)^{2\,m+1}$

$\therefore f'(x) = 0 \Rightarrow x = a, b$

For $x = b - h$, we have

$f'(x) = (b - h - a)^{2n}(-h)^{2m+1} < 0$

and for $x = b + h$, we have

$f'(x) = (b + h - a)^{2n}\,h^{2m+1} > 0$

Thus, as x passes through $b, f'(x)$ changes sign from negative

Hence, $x = b$ is a point of minimum

3 **(d)**

Given equation of curve is

$y = 4 - 2x^2$

$\Rightarrow \dfrac{dy}{dt} = -4x\dfrac{dx}{dt}$

Given $\dfrac{dx}{dt} = -5$, at point $(1,2)$

$\therefore \dfrac{dy}{dt} = -4(1)(-5) = 20\text{unit/s}$

4 **(c)**

Given $y^2 = 2(x-3)$ ...(i)

$\Rightarrow 2y\dfrac{dy}{dx} = 2 \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{y}$

Slope of the normal$= \frac{-1}{(dy/dx)} = -y$

Slope of the given line=2

$\therefore \; y = -2$

From Eq. (i), $x = 5$

$\therefore$ Required point is $(5, -2)$

5 **(a)**

Given, $f(x) = \frac{x}{1+|x|}$

$\therefore \; f'(x) = \dfrac{(1+|x|).1 - x.\frac{|x|}{x}}{(1+|x|)^2}$

$= \dfrac{1}{(1+|x|)^2} > 0 \forall\, x \in R$

$\Rightarrow \; f(x)$ is strictly increasing

6 **(d)**

Given, $f(x) = 2x^2 - 3x^2 + 90x + 174$

$\therefore f'(x)6x^2 - 6x + 90$

Now, $D = b^2 - 4ac = 36 - 4 \times 6 \times 90 < 0$

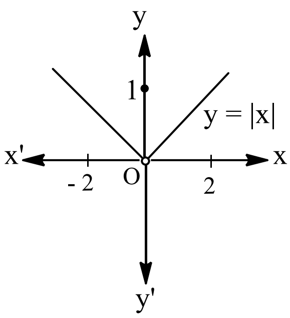
$\therefore f'(x) > 0 \forall x \in (-\infty, \infty)$

7 **(a)**

Given,

$f(x) = \begin{cases} |x|, & \text{for } 0 < |x| \le 2 \\ 1, & \text{for } x = 0 \end{cases}$

It is clear from the graph that $f(x)$ has local maximum.



8 **(a)**

We have,

$f(x) = x^2 + ax + 1$

$\Rightarrow f'(x) = 2x + a$

For $f(x)$ to be increasing on $[1, 2]$, we must have

$f'(x) > 0$ for all $x \in R$

Now,

$f'(x) = 2x + a$

$\Rightarrow f''(x) = 2 > 0$ for all $x \in R$

$\Rightarrow f'(x)$ is increasing for all $x \in R$

$\Rightarrow f'(x)$ is increasing on $[1, 2]$

$\Rightarrow f'(1)$ is the minimum value of $f(x)$ in $[1, 2]$

Thus,

$f'(x) > 0$ for all $x \in [1, 2]$

$\Rightarrow f'(1) > 0$

$\Rightarrow 2 + a > 0 \Rightarrow a > -2 \Rightarrow a \in (-2, \infty)$

9 **(c)**

Since, $\frac{dx}{dt} = \frac{dy}{dt}$ ... (i)

Given equation of curve is

$y = x^2 + 2x$

$$\Rightarrow \frac{dy}{dt} = (2x + 2)\frac{dx}{dt}$$

$\Rightarrow 1 = 2x + 2$ [from Eq.(i)]

$\Rightarrow x = -1/2, \; y = -3/4$

$\therefore$point on the curve is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$.

10 **(b)**

Given, $p(x) = x^4 + ax^3 + bx^2 + cx + d$

$\Rightarrow p'(x) = 4x^3 + 3ax^2 + 2bx + c$

$\therefore$ $x = 0$ *is* a solution for p'(x)=0,

$\Rightarrow c = 0$

$\therefore$ $p(x) = x^4 + ax^3 + bx^2 + d$ ...(i)

Also , we have p(−1)<p(1)

$\Rightarrow 1 - a + b + d < 1 + a + b + d$

$\Rightarrow a > 0$

$\therefore$ $p'(x) = 0$, *only* when x=0

and $p(x)$*is differentiable in* $(-1,1)$, *we should have the maximum and minimum at the*

points $x = -1,0$ and 1 only

Also ,we have $p(-1) < p(1)$

$\therefore$ Maximum of $p(x) = \max\{p(0), p(1)\}$

And minimum of P(x)=Min {P(-1), P(0)}

In the interval [0,1]

$p'(x) = 4x^3 + 3ax^2 + 2bx$

$= x(4x^2 + 3ax + 2b)$

$\therefore p'(x) has\ only\ one\ root x = 0, then\ 4x^2 + 3ax + 2b = 0\ has$

No real roots.

$\therefore \quad (3a)^2 - 32b < 0$

$\Rightarrow \dfrac{3a^2}{32} < b$

$\therefore \quad b > 0$

Thus , we have a>0 and b>0

$\therefore \quad p'(x) = 4x^3 + 4ax^2 + 2bx > 0, \forall x \in (0,1)$

Hence ,$p(x) = p(1)$

Similarly,$p(x)$ is decreasing in [-1,0].

Therefore, Minimum $p(x)$does not occur at $x = -1$.

11 **(c)**

$p(t) = 1000 + \frac{1000t}{100 + t^2}$

$\Rightarrow \quad p'(t) = 0 + \frac{(100 + t^2)(1000) - 1000t\,(2t)}{(100 + t^2)^2}$

$= 1000\frac{(100 - t^2)}{(100 + t^2)^2}$

Put $p'(t) = 0$ for maxima or minima

$\Rightarrow \quad 100 - t^2 = 0$

$\Rightarrow \quad t = \pm 10$

Now, $p''(t) = 1000$

$\times \left[\frac{(100 + t^2)^2(-2t) - (100 - t^2)2(100 + t^2)2t}{(100 + t^2)^4}\right]$

$= 1000t\frac{[(100 + t^2)(-2) - (100 - t^2)(4)]}{(100 + t^2)^3}$

$= -1000t\ \frac{[600 - 2t^2]}{(100 + t^2)^3}$

At $t = 10$, $p''(t) < 0$

∴ The maximum value is

$p(10) = 1000 + \frac{10000}{100 + 100}$

$= 1000 + \frac{10000}{200} = 1050$

12 **(a)**

We have,

$f'(x) = (x - a)^{2n}(x - b)^{2\,m+1}$

$\therefore f'(x) = 0 \Rightarrow x = a, b$

For $x = b - h$, we have

$f'(x) = (b - h - a)^{2n}(-h)^{2m+1} < 0$

and for $x = b + h$, we have

$f'(x) = (b + h - a)^{2n}\ h^{2m+1} > 0$

Thus, as x passes through b, $f'(x)$ changes sign from negative

Hence, $x = b$ is a point of minimum

13 **(d)**

Given equation of curve is

$y = 4 - 2x^2$

$\Rightarrow \ \frac{dy}{dt} = -4x\frac{dx}{dt}$

Given $\ \frac{dx}{dt} = -5$, at point (1,2)

$\therefore \dfrac{dy}{dt} = -4(1)(-5) = 20\text{unit/s}$

14 **(c)**

Given $\quad y^2 = 2(x-3) \qquad$ ...(i)

$\Rightarrow \quad 2y\dfrac{dy}{dx} = 2 \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1}{y}$

Slope of the normal= $\frac{-1}{(dy/dx)} = -y$

Slope of the given line=2

$\therefore \quad y = -2$

From Eq. (i), $x = 5$

$\therefore$ Required point is $(5, -2)$

15 **(a)**

Given, $f(x) = \frac{x}{1+|x|}$

$$\therefore \quad f'(x) = \frac{(1+|x|).1 - x.\dfrac{|x|}{x}}{(1+|x|)^2}$$

$$= \frac{1}{(1+|x|)^2} > 0 \forall\, x \in R$$

$\Rightarrow \quad f(x)$ is strictly increasing

16 **(d)**

Given, $f(x) = 2x^2 - 3x^2 + 90x + 174$

$\therefore f'(x)6x^2 - 6x + 90$

Now, $D = b^2 - 4ac = 36 - 4 \times 6 \times 90 < 0$

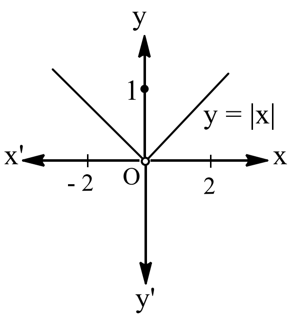
$\therefore f'(x) > 0 \forall x \in (-\infty, \infty)$

17 **(a)**

Given,

$$f(x) = \begin{cases} |x|, & \text{for } 0 < |x| \le 2 \\ 1, & \text{for } x = 0 \end{cases}$$

It is clear from the graph that $f(x)$ has local maximum.



18 **(a)**

We have,

$f(x) = x^2 + ax + 1$

$\Rightarrow f'(x) = 2x + a$

For $f(x)$ to be increasing on $[1, 2]$, we must have

$f'(x) > 0$ for all $x \in R$
Now,
$f'(x) = 2x + a$
$\Rightarrow f''(x) = 2 > 0$ for all $x \in R$
$\Rightarrow f'(x)$ is increasing for all $x \in R$
$\Rightarrow f'(x)$ is increasing on $[1, 2]$
$\Rightarrow f'(1)$ is the minimum value of $f(x)$ in $[1, 2]$
Thus,
$f'(x) > 0$ for all $x \in [1, 2]$
$\Rightarrow f'(1) > 0$
$\Rightarrow 2 + a > 0 \Rightarrow a > -2 \Rightarrow a \in (-2, \infty)$

19 **(c)**

Since, $\frac{dx}{dt} = \frac{dy}{dt}$ ... (i)

Given equation of curve is

$y = x^2 + 2x$

$\Rightarrow \frac{dy}{dt} = (2x + 2)\frac{dx}{dt}$

$\Rightarrow 1 = 2x + 2$ [from Eq.(i)]

$\Rightarrow x = -1/2, \; y = -3/4$

$\therefore$point on the curve is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$.

20 **(b)**

Given, $p(x) = x^4 + ax^3 + bx^2 + cx + d$

$\Rightarrow \quad p'(x) = 4x^3 + 3ax^2 + 2bx + c$

$\therefore \quad x = 0$ is a solution for p'(x)=0,

$\Rightarrow \quad c = 0$

$\therefore \quad p(x) = x^4 + ax^3 + bx^2 + d$ ...(i)

Also , we have p(−1)<p(1)

$\Rightarrow \; 1 - a + b + d < 1 + a + b + d$

$\Rightarrow \; a > 0$

$\therefore \; p'(x) = 0, only$ when x=0

$and\ p(x) is\ differentiable\ in\ (-1,1), we\ should\ have\ the\ maximum\ and\ minimum$
$at\ the$
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Also ,we have $p(-1) < p(1)$

$\therefore$ Maximum of $p(x) = \max\{p(0), p(1)\}$

And minimum of P(x)=Min {P(-1), P(0)}

In the interval [0,1]

$p'(x) = 4x^3 + 3ax^2 + 2bx$

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No real roots.

$\therefore \quad (3a)^2 - 32b < 0$

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$\therefore \quad b > 0$

Thus , we have a>0 and b>0

$\therefore \quad p'(x) = 4x^3 + 4ax^2 + 2bx > 0, \forall x \in (0,1)$

Hence ,$p(x) = p(1)$

Similarly,$p(x)$ is decreasing in [-1,0].

Therefore, Minimum $p(x)$does not occur at $x = -1$.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	D	C	A	D	A	A	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	A	B	B	A	A	B	B	A

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 6

Topic :-Applications of Derivatives

1 **(c)**

We have, $x = at^2 \Rightarrow \frac{dx}{dt} = 2at$

and $y = 2at \Rightarrow \frac{sy}{st} = 2a$

$\therefore$ Slope of tangent $\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

$\Rightarrow \frac{1}{t} = \infty$

$\Rightarrow t = 0 \Rightarrow$ Point of contact is (0, 0)

2 **(d)**

Curve is $y^2 = px^3 + q$

$\therefore 2y\frac{dy}{dx} = 3px^2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)} = \frac{3p.4}{2.3}$

$\Rightarrow 4 = 2p$

$\Rightarrow p = 2$

Also, curve is passing through (2, 3)

$\therefore 9 = 8p + q$

$\Rightarrow q = -7$

$\therefore (p, q)$ is $(2, -7)$

3 **(b)**

Given, $x = t - 6t^2 + t^3$

On differentiating, w. r. t. x, we get

$\frac{dx}{dt} = 1 - 12t + 3t^2$

Again, differentiating, we get

$$\frac{d^2x}{dt^2} = -12 + 6t$$

When $\frac{d^2x}{dt^2} = 0 \Rightarrow t = 2$units time

4 **(a)**

Given equation of curve is

$y^2 = px^3 + q$

$\therefore 2y\frac{dy}{dx} = 3px^2$

$\therefore \quad \left(\frac{dy}{dx}\right)_{(2,3)} = \frac{3p\,(2)^2}{2.3} = 2p$

The equation of tangent at (2, 3) is

$(y-3) = 2p(x-2)$

$\Rightarrow \quad y = 2px - (4p-3)$

This is similar to $y = 4x - 5$

$\therefore \quad 2p = 4$ and $4p - 3 = 5$

$\Rightarrow\ p = 2$ and $p = 2$

Since, the point (2, 3) lies on the curve

$\therefore \quad 9 = 8p + q \quad \Rightarrow \quad q = -7 \quad [\because p = 2]$

5 **(c)**

Given, $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}, for\ u \in (-\infty, \infty)$

Now, $g(-u) = 2\tan^{-1}(e^{-u}) - \frac{\pi}{2}$

$= 2(\cot^{-1}(e^u)) - \frac{\pi}{2}$

$= 2\left(\frac{\pi}{2} - \tan^{-1}(e^u)\right) - \frac{\pi}{2}$

$= -g(u)$

$\therefore \quad g(u)$ Is an odd function.

Also, $g'(u) = 2\frac{1}{1+(e^u)^2}.e^u - 0 > 0$

Which is strictly increasing in $(-\infty, \infty)$

6 **(b)**

Given curve is $y = 2x^2 - x + 1$

Let the coordinate of P are (h, k)

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = 4x - 1$

At the point (h, k), the slope

$= \left(\frac{dy}{dx}\right)_{(h,k)} = 4h - 1$

Since, the tangent is parallel to the given line $y = 3x + 4$

$\Rightarrow \quad 4h - 1 = 3 \quad \Rightarrow \quad h = 1, k = 2$

$\therefore$ Coordinates of point P are (1, 2)

7 **(a)**

Let $f(x) = ax^2 + \frac{b}{x^2} - c$, where $a, b, c > 0$ and $x > 0$

Then, $f'(x) = 2ax - \frac{2b}{x^3}$ and $f''(x) = 2a + \frac{6b}{x^4}$

For local maximum or minimum, we must have

$f'(x) = 0 \Rightarrow x^4 = \frac{b}{a} \Rightarrow x = \pm\left(\frac{b}{a}\right)^{1/4}$

Clearly, $f''\left\{\pm\left(\frac{b}{a}\right)^{1/4}\right\} = 2a + 6a > 0$

Therefore, $x = \pm\left(\frac{b}{a}\right)^{1/4}$ are points of local minimum

Local minimum value of $f(x)$ is given by

$$f\left\{\left(\frac{b}{a}\right)^{1/4}\right\} = a\left(\frac{b}{a}\right)^{1/2} + b\left(\frac{a}{b}\right)^{1/2} - c = 2\sqrt{ab} - c$$

But, it is given that

$f(x) \geq 0$ for all x $\left[\because ax^2 + \frac{b}{x^2} \geq c\right]$

$\therefore 2\sqrt{ab} - c \geq 0 \Rightarrow 4ab \geq c^2$

8 **(a)**

Since $f(x) = x^4 - 62\,x^2 + ax + 9$ attains its maximum at $x = 1$

$\therefore f'(x) = 0$ at $x = 1$

$\Rightarrow f'(x) = 4\,x^3 - 124\,x + a = 0$ at $x = 1$

$\Rightarrow 4 - 124 + a = 0 \Rightarrow a = 120$

9 **(b)**

On differentiating given curve w.r.t. x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}.\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

The normal is parallel to x-axis, if

$$-\left(\frac{dx}{dy}\right)_{(x_1,y_1)} = 0 \quad \Rightarrow \quad \sqrt{\frac{x_1}{y_1}} = 0 \quad \Rightarrow \quad x_1 = 0 \qquad \text{...(ii)}$$

Since, the point (x_1, y_1) lies on a curve

$\therefore \quad \sqrt{x_1} + \sqrt{y_1} = \sqrt{a}$

$\Rightarrow \quad 0 + \sqrt{y_1} = \sqrt{a}$ [from Eq. (ii)]

$\Rightarrow \quad y_1 = a$

$\therefore$ Required point is $(0, a)$

10 **(a)**

$y = (2x - 1)e^{2(1-x)}$

$$\Rightarrow \quad \frac{dy}{dx} = 2e^{2(1-x)} - 2(2x - 1)e^{2(1-x)}$$

$= 2e^{2(1-x)}(2 - 2x)$

$= 4e^{2(1-x)}(1 - x)$

Put, $\frac{dy}{dx} = 0 \Rightarrow x = 1$

Now, $\frac{d^2y}{dx^2} = -8e^{2(1-x)}(1 - x) - 4e^{2(1-x)}$

$$\Rightarrow \quad \left(\frac{d^2y}{dx^2}\right)_{x=1} = -4 < 0$$

So, y is maximum at $x = 1$, when $x = 1, y = 1$.

Thus, the point of maximum is $(1,1)$.

The equation of the tangent at $(1,1)$ is

$y - 1 = 0(x - 1) \Rightarrow y = 1$

11 **(c)**

If $f'''(a) > 0$, then it is not an extreme point

12 **(b)**

Given function is $f(x) = x + \sin x$

On differentiating w.r.t. x, we get

$f'(x) = 1 + \cos x$

For maxima or minima put $f'(x) = 0$

$\Rightarrow \quad 1 + \cos x = 0 \quad \Rightarrow \ \cos x = -1 \quad \Rightarrow \quad x = \pi$

Again differentiating w.r.t. x, we get

$f''(x) = -\sin x\,, at \ \ x = \pi f''(\pi) = 0$

Again differentiating w.r.t. x, we get $f'''(x) = -\cos x$,

$f'''(\pi) = 1$

At $x = \pi, f(x)$ is minimum

13 **(b)**

Let $V = \frac{4}{3}\pi r^3 \ \Rightarrow \ \frac{dV}{dt} 4\pi r^2 \frac{dr}{dt}$

$$\Rightarrow \frac{dr}{dt} = \frac{30}{4 \times \pi \times 15 \times 15} = \frac{1}{30\pi} ft/\min \left[\therefore \frac{dV}{dt} = 30, r = 15\right]$$

14 **(a)**

Given, $\frac{x^2}{3} - \frac{y^2}{2} = 1$ ...(i)

$$\Rightarrow \ \frac{2x}{3} - y\frac{dy}{dx} = 0$$

$$\Rightarrow \ \frac{dy}{dx} = \frac{2x}{3y}$$

Now, slope of the line $y = x$ is 1

Since, tangent is parallel to given line, then

$\frac{2x}{3y} = 1 \quad \Rightarrow \quad x = \frac{3y}{2}$ ...(ii)

From Eqs. (i) and (ii), we get $y = \pm 2$

$\therefore$ Points of contact are (3, 2) and $(-3, -2)$

$\therefore$ equations of tangents are

$y - 2 = (x - 3) \quad \Rightarrow \quad x - y - 1 = 0$

and $\ y + 2 = x + 3 \quad \Rightarrow \quad x - y + 1 = 0$

15 **(c)**

The two curves are

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i) and, $\frac{x^2}{l^2} - \frac{y^2}{m^2} = 1$ ...(ii)

Differentiating with respect to x, we get

$$\left(\frac{dy}{dx}\right)_{C_1} = -\frac{b^2}{a^2}\frac{x}{y}, \left(\frac{dy}{dx}\right)_{C_2} = \frac{m^2}{l^2}\frac{x}{y}$$

The two curves intersect orthogonally, iff.

$$\left(\frac{dy}{dx}\right)_{C_1}\left(\frac{dy}{dx}\right)_{C_2} = -1$$

$$\Rightarrow -\frac{b^2}{a^2}\frac{x}{y} \times \frac{m^2}{l^2}\frac{x}{y} = -1$$

$\Rightarrow m^2 b^2 x^2 = a^2 l^2 y^2$ ...(iii)

Subtracting (ii) from (i), we get

$x^2\left(\frac{1}{a^2} - \frac{1}{l^2}\right) + y^2\left(\frac{1}{b^2} + \frac{1}{m^2}\right) = 0$ ...(iv)

From (iii) and (iv), we get

$$\frac{1}{m^2 b^2}\left(\frac{1}{a^2} - \frac{1}{l^2}\right) = -\frac{1}{a^2 l^2}\left(\frac{1}{b^2} + \frac{1}{m^2}\right)$$

$\Rightarrow l^2 - a^2 = -b^2 - m^2 \Rightarrow a^2 - b^2 = l^2 + m^2$

16 **(b)**

We have,

$\frac{dy}{dt} > \frac{dx}{dt}, \frac{dy}{dt} > 0$ and $\frac{dx}{dt} > 0$

Now, $12y = x^3$

$\Rightarrow 12\frac{dy}{dt} = 3x^2\frac{dx}{dt} \Rightarrow 3\left(x^2\frac{dx}{dt} - 4\frac{dy}{dt}\right) = 0 \Rightarrow x^2\frac{dx}{dt} = 4\frac{dy}{dt}$

We have,

$\frac{dy}{dt} > \frac{dx}{dt}$

$\Rightarrow \frac{x^2}{4}\frac{dx}{dt} > \frac{dx}{dt}$

$\Rightarrow (x^2 - 4)\frac{dx}{dt} > 0 \Rightarrow x^2 - 4 > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

17 **(b)**

Let the centre of circle on y-axis be (0.k).

Let $d = \sqrt{(7-0)^2 + (3-k)^2}$

$\Rightarrow \quad d^2 = 7^2 + (3-k)^2 = D \quad$ (say)

On differentiating w. r. t. k , we get

$\frac{dD}{dk} = 0 + 2(3-k)(-1)$

Put $\frac{dD}{dk} = 0 \Rightarrow k = 3$

Now, $\frac{d^2D}{dk^2} = 2 > 0\ minima,$

$\therefore Minimum$ value at k=3 is d=7

Hence, minimum circle of radius 7.

18 **(b)**

$f(x) = e^x \sin x$

$f'(x) = e^x cosx + sin\,xe^x$

$f''(x) = -e^x sin\,x + \cos xe^x + e^x \cos x + e^x \sin x$

$2e^x cosx$

For maximum slope

$f''(x) = 0$

$\Rightarrow \ 2e^x \cos x = 0$

$\Rightarrow \quad \cos x = 0$

$\Rightarrow \ x = \frac{\pi}{2}, \frac{3\pi}{2}, \forall x \in [0, 2\pi]$

$f'''(x) = 2 \quad [-e^x \sin x + e^x \cos x]$

$at\ \ x = \frac{\pi}{2}\ , f'''(x) < 0$

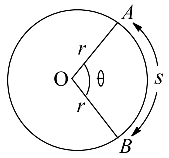
$and\ \ at\ \ \ x = \frac{3\pi}{2} f'''(x) > 0$

$\therefore$ slop is maximum at $x = \frac{\pi}{2}$

19 **(d)**

$\because$ Perimeter of a sector= p

Let AOB be the sector with radius r



If angle of the sector be θ radius, then area of sector,

$A = \frac{1}{2} r^2 \theta$ ...(i)

And length of arc, $s = r\theta$

$\Rightarrow \quad \theta = \frac{s}{r}$

$\therefore$ perimeter of the sector

$p = r + s + r = 2r + s$ ...(ii)

On subtracting $\theta = \frac{s}{r}$ in Eq. (i), we get

$A = \left(\frac{1}{2} r^2\right)\left(\frac{s}{r}\right) = \frac{1}{2} rs$

$\Rightarrow \quad s = \frac{2A}{r}$

Now, on substituting the value of s in Eq. (ii), we get

$p = 2r + \left(\frac{2A}{r}\right) \Rightarrow 2A = pr - 2r^2$

On differentiating w.r.t. r, we get

$2\frac{dA}{dr} = p - 4r$

For the maximum area, put

$2\frac{dA}{dr} = 0 \Rightarrow p - 4r = 0 \Rightarrow r = \frac{p}{4}$

20 **(b)**

Given, $y^2 = x^3$

On differentiating w.r.t x, we get

$$2y\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3x^2}{2y} \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,1)} = \frac{3}{2}$$

$\therefore$ Equation of normal at point (1, 1) is

$$y - 1 = -\frac{2}{3}(x - 1)$$

$$\Rightarrow \quad 2x + 3y = 5$$

Hence, the equation of line parallel to above line will be in option (b), ie, $2x + 3y = 7$

ANSWER-KEY										
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Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	B	A	C	B	B	B	D	B

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 7

Topic :-Applications of Derivatives

1 **(a)**

Given that, $s=\sqrt{t}$

$\therefore \ \dfrac{ds}{dt}=\dfrac{1}{2\sqrt{t}}$

$\Rightarrow \ v=\dfrac{1}{2\sqrt{t}} \ \Rightarrow \ \dfrac{dv}{dt}=-\dfrac{1}{2.2t^{3/2}}$

$\Rightarrow \ a=-\dfrac{2}{(2\sqrt{t})^3}$

$\Rightarrow \ a=-2v^3$

$\Rightarrow \ a\propto v^3$

2 **(d)**

Let $PQ=a$ and $PR=b$, then $\Delta=\frac{1}{2}ab\sin\theta$

$\because \ -1\le\sin\theta\le 1$

$\therefore$ Area is maximum when $\sin\theta=1\Rightarrow \ \theta=\frac{\pi}{2}$

3 **(a)**

$f'(x)=-\text{a}\sin x+b\sec^2 x+1$

Now, $f'(0)=0$ and $f'\left(\frac{\pi}{6}\right)=0$

$\Rightarrow \ b+1=0$ and $-\frac{a}{2}+\frac{4b}{3}+1=0$

$\Rightarrow \ b=-1, a=-\dfrac{2}{3}$

4 **(c)**

Given curve is

$y=e^{2x}+x^2$

At $x=0, y=1$

$\therefore$ Any point on the curve is

$\dfrac{dy}{dx}=2e^{2x}+2x$

Slope of normal at (0,1)$=-\frac{1}{2+0}=-\frac{1}{2}$

$\therefore$ Equation of normal is

$y-1=-\dfrac{1}{2}(x-0)$

$\Rightarrow \ 2y-2=-x$

$\Rightarrow \ x+2y-2=0$

Required distance$=\left|\frac{0+0-2}{\sqrt{1+4}}\right|$

$= \frac{2}{\sqrt{5}}$

5 **(d)**

We have, $f(x) = x\, e^{1-x}$

$\Rightarrow f'(x) = e^{1-x}(1-x) < 0$ for all $x \in (1, \infty)$

So, $f(x)$ strictly decreases in $(1, \infty)$

6 **(a)**

Let $x_1, x_2 \in R$ such that $x_1 < x_2$. Then,

$x_1 < x_2$

$f(x_1) > f(x_2)$ $\quad$ [$\because f$ is a decreasing function]

$\Rightarrow g(f(x_1)) < g(f(x_2))$ [$\because g$ is a decreasing function]

$\Rightarrow gof(x_1) < gof(x_2)$

7 **(d)**

$\because \quad y^2 = 4ax$

$\therefore \ \frac{dy}{dx} = \frac{2a}{y}$

Length of subnormal$= y\frac{dy}{dx} = y\frac{2a}{y} = 2a$

8 **(c)**

Given, $\quad x = at^2$ and $y = 2at$

$\therefore \quad \frac{dx}{dt} = 2at$ and $\frac{dy}{dt} = 2a$

$\therefore$ Slope of tangent, $\left(\frac{dy}{dx}\right) = \frac{2a}{2at} = \frac{1}{t}$

$\Rightarrow \ \frac{1}{t} = \infty, \quad \Rightarrow \quad t = 0$ $\quad$ [given]

$\therefore$ Point of contact is $(0, 0)$

9 **(b)**

We have,

$ay^2 = x^3$

$\Rightarrow 2\, ay\, \frac{dy}{dx} = 3\, x^2$

$\Rightarrow \frac{dy}{dx} = \frac{3\, x^2}{2ay}$

Let (x_1, y_1) be a point on $ay^2 = x^3$. Then,

$ay_1^2 = x_1^3 \quad$ ...(i)

The equation of the normal at (x_1, y_1) is

$y - y_1 = -\frac{2ay_1}{3x_1^2}(x - x_1)$

This meets the coordinate axes at

$A\left(x_1 + \frac{3x_1^2}{2a}, 0\right)$ and $B\left(0, y_1 + \frac{2ay_1}{3x_1}\right)$

Since the normal cuts off equal intercepts with the coordinate axes

$\therefore x_1 + \frac{3\, x_1^2}{2\, a} = y_1 + \frac{2\, ay_1}{3\, x_1}$

$\Rightarrow x_1 \frac{(2a + 3x_1)}{2a} = y_1 \frac{(3x_1 + 2a)}{3x_1}$

$\Rightarrow 3x_1^2 = 2ay_1$

$\Rightarrow 9x_1^4 = 4a^2y_1^2$ ...(ii)

From (i) and (ii), we get

$$9x_1^4 = 4a^2\left(\frac{x_1^3}{a}\right) \Rightarrow x_1 = \frac{4a}{9}$$

10 **(d)**

Given, $y = 2x^2 - 6x - 4 \quad \Rightarrow \quad \frac{dy}{dx} = 4x - 6$

Since, $\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = 4x - 6 = 0 \quad \Rightarrow \quad x = \frac{3}{2}$

$$\Rightarrow \quad y = 2.\frac{9}{4} - 6.\frac{3}{2} - 4 = -\frac{17}{2}$$

$\therefore$ Required point is $\left(\frac{3}{2}, -\frac{17}{2}\right)$

11 **(d)**

$\because f(x) = x^3 - 6x^2 + ax + b$

On differentiating w.r.t. x, we get

$f'(x) = 3x^2 - 12x + a$

By the definition of Rolle's theorem

$$f'(c) = 0 \quad \Rightarrow \quad f'\left(2 + \frac{1}{\sqrt{3}}\right) = 0$$

$$\Rightarrow \quad 3\left(2 + \frac{1}{\sqrt{3}}\right)^2 = -12\left(2 + \frac{1}{\sqrt{3}}\right) + a = 0$$

$$\Rightarrow \quad 3\left(4 + \frac{1}{3} + \frac{4}{\sqrt{3}}\right) - 12\left(2 + \frac{1}{\sqrt{3}}\right) + a = 0$$

$\Rightarrow \quad 12 + 1 + 4\sqrt{3} - 24 - 4\sqrt{3} + a = 0$

$\Rightarrow \quad a = 11$

12 **(b)**

For the point $(3, \log 2)$, we take $y = g(x) = \log(x - 1)$

$$\frac{dy}{dx} = \frac{1}{(x-1)} \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{(3,\log 2)} = \frac{1}{2}$$

$\therefore$ Equation of normal is

$y - \log 2 = -2(x - 3)$

$\Rightarrow \quad y + 2x = 6 + \log 2$

14 **(c)**

Let T be the processing time. The number of batches is $\frac{N}{x}$ and the processing time of one batch is $\alpha + \beta\, x^2$ seconds

$$\therefore T = \frac{N}{x}(\alpha + \beta\, x^2) = N\left(\frac{\alpha}{x} + \beta\, x\right) \Rightarrow \frac{dT}{dx} = N\left(-\frac{\alpha}{x^2} + \beta\right)$$

For fast processing T must be least for which $\frac{dT}{dx} = 0$

$$\therefore N\left(-\frac{\alpha}{x^2} + \beta\right) = 0 \Rightarrow x = \sqrt{\frac{\alpha}{\beta}}$$

Clearly, $\frac{d^2T}{dx^2} = N\frac{2\alpha}{x^3} > 0$ for $x = \sqrt{\frac{\alpha}{\beta}}$

Hence, T is least when $x = \sqrt{\frac{\alpha}{\beta}}$

15 **(d)**

Let (x, y) be the point on the curve $2x^2 + y^2 - 2x = 0$. Then its distance from $(a, 0)$ is given by

$S = \sqrt{(x-a)^2 + y^2}$

$\Rightarrow S^2 = x^2 - 2\,ax + a^2 + 2\,x - 2\,x^2$ [Using $2\,x^2 + y^2 - 2\,x = 0$]

$\Rightarrow S^2 = -x^2 + 2\,x(1-a) + a^2$ ...(i)

$\Rightarrow 2\,S\,\dfrac{dS}{dx} = -2\,x + 2(1-a)$

For S to be maximum, we must have,

$\dfrac{dS}{dx} = 0 \Rightarrow -2x + 2(1-a) = 0 \Rightarrow x = 1 - a$

It can be easily checked that $\frac{d^2S}{dx^2} < 0$ for $x = 1 - a$

Hence, S is maximum for $x = 1 - a$

Putting $x = 1 - a$ in (i), we get $S = \sqrt{1 - 2\,a + 2\,a^2}$

16 **(b)**

Given curve is

$y = x^2 - 3x + 2$

$\dfrac{dy}{dx} = 2x - 3$

$(2x - 3) \times 1 = -1$

$\Rightarrow \quad 2x - 3 = -1$

$\Rightarrow \quad 2x = 2$

$\Rightarrow \quad x = 1$

$\therefore \quad y = 0$

$\Rightarrow$ Required point is$(1,0)$.

17 **(c)**

The equations of the tangent and normal to $y^2 = 4ax$ at $P(at^2, 2at)$ are

$ty = x + at^2$ ...(i)

and, $y + tx = 2\,at + at^3$ ...(ii)

Liens (i) and (ii) meet the x-axis at $T(-at^2, 0)$ and $G(2\,a + at^2, 0)$ respectively

Since PT is perpendicular to PG. Therefore, TG is the diameter of the circle through P, T, G

Hence, the equation of the circle is

$(x + at^2)(x - 2a - at^2) + (y - 0)(y - 0) = 0$

$\Rightarrow x^2 + y^2 - 2ax - at^2(2a + at^2) = 0$

$\Rightarrow 2x + 2y\,\dfrac{dy}{dx} - 2a = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{a - x}{y}$

$\Rightarrow m_1 = \left(\frac{dy}{dx}\right)_P = \frac{a - at^2}{2at} = \frac{1 - t^2}{2t}$ ...(i)

$\Rightarrow y^2 = 4ax$

$\Rightarrow \left(\dfrac{dy}{dx}\right) = \dfrac{2a}{y}$

$\Rightarrow m_2 = \left(\frac{dy}{dx}\right)_P = \frac{2a}{2at} = \frac{1}{t}$ ...(ii)

Let θ be the angle between the tangents at P to the parabola and the circle. Then,

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{\frac{1}{t} - \frac{1 - t^2}{2t}}{1 + \frac{1 - t^2}{2t^2}} = t \Rightarrow \theta = \tan^{-1} t$$

18 **(a)**

Given, $x = a(\cos\theta + \theta\sin\theta)$

And $y = a(\sin\theta - \theta\cos\theta)$

$\Rightarrow \dfrac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$

$\Rightarrow \dfrac{dx}{d\theta} = a\theta\cos\theta$

And $\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta)$

$\Rightarrow \dfrac{dy}{d\theta} = a\theta\sin\theta$

$\therefore \dfrac{dy}{dx} = \tan\theta$

So, equation of normal is

$\Rightarrow \quad y - a\sin\theta + a\theta\cos\theta = -\dfrac{\cos\theta}{\sin\theta}(x - a\cos\theta - a\theta\sin\theta)$

$y\sin\theta - a\sin^2\theta + a\theta\cos\theta\sin\theta$

$= -x\cos\theta + a\cos^2\theta + a\theta\sin\theta\cos\theta$

$\Rightarrow \quad x\cos\theta + y\sin\theta = a$

It is always at a constant distance 'a' from origin.

19 **(a)**

$\because \ f(x) = xe^{x(1-x)}$

On differentiating w.r.t. x, we get

$f'(x) = e^{x(1-x) + x.e^{x(1-x)}}.(1 - 2x)$

$= e^{x(1-x)}\{1 + x(1 - 2x)\}$

$= e^{x(1-x)}.(-2x^2 + x + 1)$

It is clear that $e^{x(1-x)} > 0$ for all x

Now, by sign rule for $-2x^2 + x + 1$

$f'(x) > 0$, if $x \in \left[-\frac{1}{2}, 1\right]$

So, $f(x)$ is increasing on $\left[-\frac{1}{2}, 1\right]$

20 **(a)**

$\because \ f(x) = 2x^3 - 15x^2 + 36x + 4$

On differentiating w.r.t. x, we get

$f'(x) = 6x^2 - 30x + 36$ ...(i)

For maxima or minima $f'(x) = 0$

$\Rightarrow \quad 6x^2 - 30x + 36 = 0$

$\Rightarrow \quad x^2 - 5x + 6 = 0$

$\Rightarrow \quad (x - 2)(x - 3) = 0 \quad \Rightarrow \quad x = 2, 3$

Again differentiating Eq. (i), we get

$f''(x) = 12x - 30$

$\Rightarrow \quad f''(2) = 24 - 30 = -6 < 0$

Therefore, $f(x)$ is maximum at, $x = 2$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	A	C	D	A	D	C	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	B	B	C	D	B	C	A	A	A

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 8

Topic :-Applications of Derivatives

1 **(b)**

We have,

$y^2 = 4ax$

$\Rightarrow 2y\dfrac{dy}{dt} = 4a\dfrac{dx}{dt}$

$\Rightarrow y\dfrac{dy}{dt} = 2a\dfrac{dx}{dt}$

$\Rightarrow y\dfrac{d^2y}{dt^2} + \left(\dfrac{dy}{dt}\right)^2 = 2a\dfrac{d^2x}{dt^2}$

$\Rightarrow y \times 0 + (\text{Constant})^2 = 2a\dfrac{d^2x}{dt^2}\ \left[\because \dfrac{dy}{dt} = \text{Constant}\right]$

$\Rightarrow \dfrac{d^2x}{dt^2} = \text{Constant}$

$\Rightarrow$ Projection $(x, 0)$ of any point (x, y) on X-axis moves with constant acceleration

2 **(b)**

We have,

$\phi(x) = \int_1^x e^{-t^2/2}(1-t^2)dt \Rightarrow \phi'(x) = e^{-x^2/2}(1-x^2)$

Now,

$\phi'(x) = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$

Hence, $x = \pm 1$ are points of extremum of $\phi(x)$

3 **(b)**

Given, $x = 80t - 16t^2$

$$\Rightarrow \quad \frac{dx}{dt} = 80 - 32t$$

At maximum height $\frac{dx}{dt} = 0$

$$\therefore \quad t = 25s$$

4 **(a)**

Let $f(x) = ax^2 + bx + 4$

On differentiating w. r. t., we get

$f'(x) = 2ax + b$

For minimum, put $f'(x) = 0 \Rightarrow \quad x = -\frac{b}{2a}$

Since, it is given that at $x = 1$ minimum value is -1

$\therefore \quad 1 = -\frac{b}{2a} \quad \Rightarrow \quad 2a + b = 0 \quad$ ...(i)

And $\quad f(1) = a + b + 4 = -1$

$\Rightarrow \quad a + b + 5 = 0 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get $a = 5, b = -10$

5 **(a)**

Given, $g(x)1 + x) - \frac{2x}{2+x}$

$$\therefore f'(x) = \frac{1}{1+x} - \frac{(2+x)2 - 2x}{(2+x)^2} = \frac{x^2}{(1+x)(x+2)^2}$$

Clearly $f'(x) > 0$ for all $x > 0$.

6 **(c)**

Let m be the slope of the curve $y = f(x)$. Then,

$$m = \frac{dy}{dx}$$

$\Rightarrow m = e^x(\sin x + \cos x)$

$$\Rightarrow \frac{dm}{dx} = e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$$

$$\Rightarrow \frac{dm}{dx} = 2e^x \cos x$$

$$\Rightarrow \frac{d^2m}{dx^2} = 2e^x(\cos x - \sin x)$$

For maximum/minimum value of m, we must have

$\frac{dm}{dx} = 0 \Rightarrow \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ etc.

Clearly, $\frac{d^2m}{dx^2} < 0$ for $x = \frac{\pi}{2}$

Hence, m is maximum when $x = \frac{\pi}{2}$

7 **(b)**

We have,

$y = \sqrt{9 - x^2} \quad$ ...(i)

Clearly, y is positive and defined for $x \in [-3, 3]$

For the points whose ordinates and abscissae are same i.e. $y = x$, we have

$$x = \sqrt{9 - x^2} \Rightarrow 2x^2 = 9 \Rightarrow x = \pm\frac{3}{\sqrt{2}}$$

$$\therefore y = \pm\frac{3}{\sqrt{2}} \quad [\because y = x]$$

But, $y > 0$. Therefore, $xy = y = \frac{3}{\sqrt{2}}$

So, the point is $P\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

Now,

$$y = \sqrt{9 - x^2} \Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{9 - x^2}} \Rightarrow \left(\frac{dy}{dx}\right)_p = -1$$

8 **(a)**

We know that $\cos x$ is decreasing on $(0, \pi/2)$ and

$\sin x < x$ for $0 < x < \frac{\pi}{2}$

$\therefore \cos(\sin x) > \cos x$ for $0 < x < \frac{\pi}{2}$

Also,

$0 < \cos x < 1 < \frac{\pi}{2}$ for $0 < x < \frac{\pi}{2}$

and, $\sin x < x$ for $0 < x < \frac{\pi}{2}$

$\therefore \sin(\cos x) < \cos x$ for $0 < x < \frac{\pi}{2}$

Hence, $\cos(\sin x) > \cos x$ and $\sin(\cos x) < \cos x$ for

$0 < x < \frac{\pi}{2}$

9 **(d)**

Given, $f(b) - f(a) = (b-a)f'(c)$

$\Rightarrow f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{3-2}{9-4}$

$\Rightarrow f'(c) = \frac{1}{5}$

$\Rightarrow \frac{1}{2}c^{-1/2} = \frac{1}{5}$

$\Rightarrow c = \left(\frac{5}{2}\right)^2$=6.25

10 **(a)**

We have,

$f(x) = (2a-3)(x + 2\sin 3) + (a-1)(\sin^4 x + \cos^4 x) + \log 2$

$\Rightarrow f'(x) = 2a - 3 + 4(a-1)\sin x \cos x\,(\sin^2 x - \cos^2 x)$

$\Rightarrow f'(x) = 2a - 3 - (a-1)\sin 4x$

If $f(x)$ does not have critical points, then

$f'(x) = 0$ must not have any solution in R

$\Rightarrow (2a-3) - (a-1)\sin 4x = 0$ must have no solution in R

$\Rightarrow \sin 4x = \frac{2a-3}{a-1}$ must have no solution in R

$\Rightarrow \left|\frac{2a-3}{a-1}\right| > 1$

$\Rightarrow \frac{2a-3}{a-1} < -1$ or, $\frac{2a-3}{a-1} > 1$

$\Rightarrow \frac{3a-4}{a-1} < 0$ or, $\frac{a-2}{a-1} > 0$

$\Rightarrow a \in (1, 4/3)$ or, $a \in (-\infty, 1) \cup (2, \infty)$

$\Rightarrow a \in (-\infty, 1) \cup (1, 4/3) \cup (2, \infty)$

For $a = 1$, we have

$f'(x) = -1 \neq 0$

$\Rightarrow f(x)$ has no critical point for $a = 1$

Hence, $a \in (-\infty, 4/3) \cup (2, \infty)$

12 **(d)**

Given, curve is $y = x^2 \;\; \Rightarrow \frac{dy}{dx} = 2x$

$\Rightarrow \;\; \left(\frac{dy}{dx}\right)_{(1,1)} = 2 = m_1$ (say)

And $x = y^2 \;\; \Rightarrow \;\; 1 = 2y\frac{dy}{dx}$

$\Rightarrow \;\; \frac{dy}{dx} = \frac{1}{2y} \;\; \Rightarrow \;\; \left(\frac{dy}{dx}\right)_{(1,1)} = \frac{1}{2} = m_2$ (say)

$\therefore$ Angle of intersection at the point (1, 1) is given by

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}} = \frac{3}{4}$$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{3}{4}\right)$

13 **(c)**

Let volume of sphere $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi r^2.(2) \quad [\because \frac{dr}{dt} = 2]$$

$$\therefore \frac{dV}{dt} = 8\pi(5)^2 = 200\pi \, cm^3/\min \; [\because r = 5cm]$$

14 **(c)**

Let area of circle,$A = \pi r^2$

$$\Rightarrow \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \Rightarrow \frac{dA}{dt} = 2\pi.20.2$$

$$\Rightarrow \quad \frac{dA}{dt} = 80\pi \, cm^2/s$$

15 **(d)**

Given, $\quad y = x^3 - 3x^2 - 9x + 5$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 6x - 9$

We know that, this equation gives the slope of the tangent to the curve. The tangent is parallel to x-axis.

$\therefore \frac{dy}{dx} = 0 \quad \Rightarrow \quad 3x^2 - 6x - 9 = 0$

$$\Rightarrow \quad x = -1, 3$$

16 **(b)**

$\because \; f(x) = x^3 - 6x^2 + 9x + 3$

On differentiating w.r.t. x, we get

$f'(x) = 3x^2 - 12x + 9$

$\Rightarrow \quad f'(x) = 3(x^2 - 4x + 3)$

For decreasing, $f'(x) < 0$

$\Rightarrow \quad (x - 3)(x - 1) < 0,$

$\therefore \quad x \in (1, 3)$

17 **(a)**

We have,

$y = ax^3 + bx^2 + cx \qquad$ ...(i)

$\Rightarrow \frac{dy}{dx} = 3ax^2 + 2bx + c \quad$ ...(ii)

It is given that

$\left(\frac{dy}{dx}\right)_{(0,0)} = \tan 45° \Rightarrow c = 1$

Also,

$\left(\frac{dy}{dx}\right)_{(1,0)} = 0 \Rightarrow 3a + 2b + c = 0 \Rightarrow 3a + 2b + 1 = 0 \; [\because c = 1]$

Clearly, $a = 1$ and $b = -2$ satisfy this equation

Hence, $a = 1, b = -2$ and $c = 1$

18 **(c)**

Let $f(x) = 1 + x\log_e(x + \sqrt{x^2+1}) - \sqrt{1+x^2}$

Clearly, $f(x)$ is defined for all $x \in R$

Now,

$f'(x) = \log_e\left(x + \sqrt{x^2+1}\right) + \frac{x}{\sqrt{x^2+1}} - \frac{x}{\sqrt{x^2+1}}$

$\Rightarrow f'(x) = \log_e\left(x + \sqrt{x^2+1}\right)$

$\Rightarrow f'(x) > 0$ for all $x \in R$ [$\because x + \sqrt{x^2+1} \geq 1$ for all $x \in R$]

$\Rightarrow f(x)$ is increasing on R

$\Rightarrow f(x) \geq f(0)$ for all $x \geq 0$

$\Rightarrow 1 + x\log_e\left(x + \sqrt{x^2+1}\right) - \sqrt{1+x^2} \geq 0$ for all $x \geq 0$

$\Rightarrow 1 + x\log_e\left(x + \sqrt{x^2+1}\right) \geq \sqrt{1+x^2}$ for all $x \geq 0$

19 **(c)**

Given, $f(x) = x^{-x}$

$\Rightarrow \quad \log f(x) = -x\log x$

On differentiating w.r.t. x, we get

$\frac{1}{f(x)}.f'(x) = -\log x - 1$

$\Rightarrow \quad f'(x) = -f(x)(1 + \log x)$

Put $f'(x) =)0$

$\Rightarrow \quad \log x = -1 \quad \Rightarrow \quad x = e^{-1}$

$\therefore \quad f''(x) = -f'(x)(1 + \log x) - f(x)\frac{1}{x}$

$= f(x)(1 + \log x)^2 - \frac{f(x)}{x}$

At $x = \frac{1}{e}$,

$f''(x) = -ef\left(\frac{1}{e}\right) < 0$, maxima

Hence, at $x = \frac{1}{e}$, $f(x)$ is maximum.

20 **(b)**

$f(x) = x(x-1)^2$

$f'(x) = 2x(x-1) + (x-1)^2$

$= (x-1)(2x + x - 1) = (x-1)(3x-1)$

$\therefore \quad f'(c) = \frac{f(2) - f(0)}{2 - 0}$

$\Rightarrow \quad (c-1)(3c-1) = \frac{2-0}{2} = 1$

$\Rightarrow \quad 3c^2 - 4c = 0$

$\Rightarrow \quad c(3c-4) = 0$

$\Rightarrow \quad c = 0 \quad or \quad c = \frac{4}{3}$

$\therefore$ The value of c in $(0, 2)$ is $\frac{4}{3}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	A	A	C	B	A	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	C	C	D	B	A	C	C	B

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 9

Topic :-Applications of Derivatives

1 **(c)**

Let r be the base radius and h be the height of the cone. Then, $2r = h$. Let V be the volume of the cone. Then,

$V = \frac{1}{3}\pi r^2 h = \frac{4\pi r^3}{3}$

$\Rightarrow \frac{dV}{dr} = 4\pi r^2$

$\therefore \Delta V = \frac{dV}{dr}\Delta r$

$\Rightarrow \Delta V = 4\pi r^2 \, \Delta r$

$\Rightarrow \frac{\Delta V}{V} \times 100 = \frac{4\pi r^2 \, \Delta r}{\frac{4}{3}\pi r^3} \times 100 = 3\left(\frac{\Delta r}{r} \times 100\right) = 3\lambda$

2 **(b)**

Given, $f'(x) < 0, \forall\, x \in R$

$\Rightarrow \sqrt{3}\cos x + \sin x - 2a < 0, \forall\, x \in R$

$\Rightarrow \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x < a, \forall\, x \in R$

$\Rightarrow \sin\left(x + \frac{\pi}{3}\right) < a, \forall\, x \in R$

$\Rightarrow a \geq 1 \left[\because \sin\left(x + \frac{\pi}{3}\right) \leq 1\right]$

3 **(d)**

We have,

$f(x) = \cos\left(\frac{\pi}{x}\right) \Rightarrow f'(x) = \frac{\pi}{x^2}\sin\left(\frac{\pi}{x}\right)$

For $f(x)$ to be increasing, we must have

$f'(x) > 0$

$\Rightarrow \frac{\pi}{x^2}\sin\left(\frac{\pi}{x}\right) > 0$

$\Rightarrow \sin\left(\frac{\pi}{x}\right) > 0$

$\Rightarrow 2\,n\,\pi < \frac{\pi}{x} < (2\,n + 1)\,\pi$

$\Rightarrow \frac{1}{2\,n} > x > \frac{1}{2\,n+1} \Rightarrow x \in \left(\frac{1}{2\,n+1}, \frac{1}{2\,n}\right)$

4 **(d)**

Given curves are $\frac{x^2}{a^2}+\frac{y^2}{12}=1$

$\Rightarrow \quad \frac{2x}{a^2}+\frac{2y}{12}\cdot\frac{dy}{dx}=0$

$\Rightarrow \quad \frac{dy}{dx}=-\frac{12x}{a^2y}=m_1 \qquad$ (say)

And $y^3=8x \Rightarrow 3y^2\frac{dy}{dx}=8$

$\Rightarrow \quad \frac{dy}{dx}=\frac{8}{3y^2}=m_2 \qquad$ (say)

For $\theta=\frac{\pi}{2}, 1+m_1m_2=0$

$\Rightarrow \quad 1+\left(\frac{-12x}{a^2y}\right)\left(\frac{8}{3y^2}\right)=0$

$\Rightarrow \quad 3a^2(8x)-96x=0$

$\Rightarrow \quad a^2=4$

5 **(b)**

Consider the function $\phi(x)=f(x)-2\,g(x)$ defined on $[0,1]$

As $f(x)$ and $g(x)$ are differentiable for $0\le x\le 1$. Therefore, $\phi(x)$ is differentiable on $(0,1)$ and continuous on $[0,1]$

We have,

$\phi(0)=f(0)-2\,g(0)=2-0=2$

$\phi(1)=f(1)-2\,g(1)=6-2\,g(1)$

Now, $\phi'(x)=f'(x)-2\,g'(x)$

$\Rightarrow \phi'(c)=f'(c)-2\,g'(c)=0$ [Given]

Thus, $\phi(x)$ satisfies Rolle's theorem on $[0,1]$

$\therefore \phi(0)=\phi(1)$

$\Rightarrow 2=6-2\,g(1)\Rightarrow g(1)=2$

6 **(b)**

$\phi'(x)=f'(x)+a$

$\because \quad \phi'(0)=0$

$\Rightarrow \quad f'(0)+a=0$

$\Rightarrow \quad a=0 \quad (\because f'(0)=0)$

Also, $\phi'(0)>0 \quad (\because f''(0)>0)$

$\Rightarrow$ $\phi'(x)$ has relative minimum at $x=0$ for all b if $a=0$

7 **(b)**

Given curve is $y=2x^2-x+1$

On differentiating w.r.t. x, we get

$\frac{dy}{dx}=4x-1$

Since, this is parallel to the given curve $y=3x+9$

$\therefore$ These slopes are equal

$\Rightarrow \quad 4x-1=3 \quad \Rightarrow \quad x=1$

At $x=1,\ y=2(1)^2-1+1 \quad \Rightarrow y=2$

Thus, the point is (1, 2).

8 **(a)**

Given, $y=-x^3+3x^2+2x-27$

$\Rightarrow \quad \frac{dy}{dx}=-3x^2+6x+2$

Let slope $z = \frac{dy}{dx} = -3x^2 + 6x + 2$

Then, $\frac{dz}{dx} = -6x + 6$

For maximum or minimum put $\frac{dz}{dx} = 0$

$\Rightarrow \quad -6x + 6 = 0$

$\Rightarrow \quad x = 1$

Now, $\frac{d^2z}{dx^2} = -6 < 0$, maxima

The maximum value of z at $x = 1$, is given by

$z = -3 + 6 + 2 = 5$

9 **(b)**

For the curve $y^n = a^{n-1}x$

$$ny^{n-1}y.\frac{dy}{dx} = a^{n-1}$$

$\therefore$Length of subnormal$=y = \frac{dy}{dx}$

$$= y \times \frac{a^{n-1}}{ny^{n-1}} = \frac{a^{n-1}}{ny^{n-2}}$$

For constant subnormal, n should be 2

10 **(d)**

We have,

$f(x) = 2\,x^2 - \log|x|$

$$\Rightarrow f(x) = \begin{cases} 2\,x^2 - \log x\,, & x > 0 \\ 2\,x^2 - \log(-x), & x < 0 \end{cases}$$

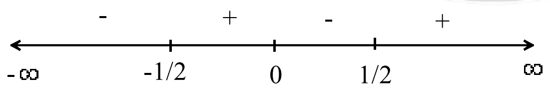
$$\Rightarrow f''(x) = 4\,x - \frac{1}{x} \text{ for all } x \neq 0$$

For $f(x)$ to be increasing, we must have

$f''(x) > 0$

$$\Rightarrow 4\,x - \frac{1}{x} > 0$$

$$\Rightarrow \frac{4\,x^2 - 1}{x} > 0$$



$$\Rightarrow \frac{(2x-1)(2x+1)}{x} > 0$$

$\Rightarrow x(2x-1)(2x+1) > 0$

$\Rightarrow x \in (-1/2, 0) \cup (1/2, \infty)$

11 **(b)**

Let $P(x_1, y_1)$ be the point on the curve $ay^2 = x^3$ where the normal cuts off equal intercepts from the coordinate axes. Therefore,

Slope of the normal at $P = \pm 1$

$$\Rightarrow -\frac{1}{\left(\frac{dy}{dx}\right)_P} = \pm 1$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_P = \pm 1$$

$\Rightarrow \dfrac{3x_1^2}{2ay_1} = \pm 1 \qquad \left[\because ay^2 = x^3 \Rightarrow 2ay\dfrac{dy}{dx} = 3x^2\right]$

$\Rightarrow 9x_1^4 = 4a^2y_1^2$

$\Rightarrow 9x_1^4 = 4ax_1^3 \quad [\because (x_1, y_1)$ lies on $ay^2 = x^3 = 0 \ \therefore ay_1^2 = x_1^3]$

$\Rightarrow x_1 = 0, x_1 = \dfrac{4a}{9}$

At $(x_1 = 0, y_1 = 0)$, the normal is y-axis

So, the required point is (x_1, y_1), where $x_1 = \frac{4a}{9}$

12 **(d)**

$\because \ f(x) = \sin x - \cos x$

On differentiating w.r.t. x, we get

$f'(x) = \cos x + \sin x$

$= \sqrt{2}\left(\dfrac{1}{\sqrt{2}}\cos x + \dfrac{1}{\sqrt{2}}\sin x\right)$

$= \sqrt{2}\left(\cos\dfrac{\pi}{4}\cos x + \sin\dfrac{\pi}{4}\sin x\right)$

$= \sqrt{2}\left[\cos\left(x - \dfrac{\pi}{4}\right)\right]$

For decreasing, $f'(x) < 0$

$\frac{\pi}{2} < \left(x - \frac{\pi}{4}\right) < \frac{3\pi}{2}$(within $0 \le x \le 2\pi$)

$\Rightarrow \dfrac{\pi}{2} + \dfrac{\pi}{4} < \left(x - \dfrac{\pi}{4} + \dfrac{\pi}{4}\right) < \dfrac{3\pi}{2} + \dfrac{\pi}{4}$

$\Rightarrow \dfrac{3\pi}{4} < x < \dfrac{7\pi}{4}$

13 **(d)**

Since, $f(x) = \frac{x}{2} + \frac{2}{x}$

$\therefore \quad f'(x) = \dfrac{1}{2} - \dfrac{2}{x^2}$

For maxima or minima, put $f'(x) = 0$

$\therefore \quad \dfrac{1}{2} - \dfrac{2}{x^2} = 0 \quad \Rightarrow \quad x = \pm 2$

Now, $f''(x) = \frac{4}{x^3}$

$\Rightarrow \quad f''(2) = \frac{4}{8} = \frac{1}{2} > 0$, minima

And $\quad f''(-2) = -\frac{4}{8} = -\frac{1}{2} < 0$, maxima

Hence, $f(x)$ has local minimum at $x = 2$

14 **(b)**

We have,

$f(x) = \tan^{-1} x - \dfrac{1}{2}\log_e x$

$\Rightarrow f'(x) = \dfrac{1}{1+x^2} - \dfrac{1}{2x}$

$\Rightarrow f'(x) = \dfrac{2x - 1 - x^2}{2x(1+x^2)}$

$\therefore f'(x) = 0 \Rightarrow 2x - 1 - x^2 = 0 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1$

Now,

$f(1)=\tan^{-1}1=\frac{\pi}{4}, f\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}-\frac{1}{2}\log_e\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}+\frac{1}{4}\log_e 3$

and, $f(\sqrt{2})=\frac{\pi}{3}-\frac{1}{4}\log_e 3$

Hence, the least value of $f(x)$ is $\frac{\pi}{3}-\frac{1}{4}\log_e 3$

15 **(b)**

The given equation of curve is $xy=1$

$\Rightarrow \quad x\frac{dy}{dx}=-y \quad \Rightarrow \quad \frac{dy}{dx}=-\frac{y}{x}$

Let $\left(t,\frac{1}{t}\right)$ be any point on the curve at which normal to the curve can be drawn

$\therefore \quad \left(\frac{dy}{dx}\right)_{\left(t,\frac{1}{t}\right)}=-\frac{1}{t^2}$

So, slope of normal= t^2

Therefore, the given line $ax+bx+c=0$ will be normal to the curve, if

$t^2=\frac{-b}{a}$

Since, $\quad t^2>0$

$\therefore$ Either $\quad b>0, \quad a<0$

or $\quad a>0, \quad b<0$

16 **(b)**

Given, $\frac{dy}{dt}\propto y, where$ y is the position of village

$$\Rightarrow \frac{1}{y}dy=k\,dt$$

$\Rightarrow \quad \log y=\log c+kt \qquad$ [on integrating]

$$\Rightarrow \quad \log\frac{y}{c}=kt \Rightarrow y=ce^{kt}$$

17 **(a)**

Given , $x^2-2xy+y^2+2x+y-6=0$

On differentiating w.r.t.x, we get

$2x-2\left(y+x\frac{dy}{dx}\right)+2y\frac{dy}{dx}+2+\frac{dy}{dx}=0$

At (2,2)

$4-2\left(2+2\frac{dy}{dx}\right)+4\frac{dy}{dx}+2+\frac{dy}{dx}=0$

$\Rightarrow \quad \frac{dy}{dx}=-2$

$\therefore$ Equation of tangent at (2,2) is

$(y-2)=-2(x-2)$

$\Rightarrow \quad 2x+y=6$

18 **(d)**

Given curves are

$x=a(\theta+sin\theta), y=a(1-cos\theta)$

$\therefore \quad \frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}$

$=\frac{a(\sin\theta)}{a(1+cos\theta)}$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$= tan\frac{\theta}{2}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{\left(\theta=\frac{\pi}{2}\right)} = \tan\frac{\pi}{4} = 1$$

At $\theta = \frac{\pi}{2}, y = a\left(1 - cos\frac{\pi}{2}\right) = a$

$\therefore$ length of normal $= y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$= a\sqrt{1 + (1)^2} = \sqrt{2}a$

19 **(a)**

Given,

$f(x) = \sin x\,(1 + cosx)$

$$\Rightarrow \quad f(x) = \sin x + \frac{1}{2}\sin 2x$$

On differentiating w.r.t. x, we get

$f'(x) = \cos x + \cos 2x$

Put $f'(x) = 0$

$\cos x + \cos 2x = 0$

$$\Rightarrow \quad 2\cos\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right) = 0$$

$$\Rightarrow \quad \cos\frac{x}{2} = 0, \quad \cos\frac{3x}{2} = 0$$

$$\Rightarrow \quad x = \pi, x = \frac{\pi}{3}$$

Now, $f''(x) = -\sin x - 2\sin 2x$

At $x = \frac{\pi}{3}, f''(x) = -\sin\frac{\pi}{3} - 2\sin\frac{2\pi}{3}$

$= -\frac{\sqrt{3}}{2} - \sqrt{3} < 0$, maxima

$\therefore$ maximum value

$$f\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)\left(1 + \cos\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{3}}{2}\left(1 + \frac{1}{2}\right)$$

$$= \frac{3\sqrt{3}}{4} = \frac{3^{3/2}}{4}$$

20 **(c)**

Let $y = 2x^2 + x - 1$

$y = 4x + 1$

For maxima or minima, put $y' = 0$

$$\Rightarrow \quad x = -\frac{1}{4}$$

Now, $y'' = 4 = +\text{ve}$

y is minimum at $x = -\frac{1}{4}$

Thus, minimum value $= 2\left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right) - 1 = -\frac{9}{8}$

Alternate

Here $a > 0$

$\therefore$ Minimum value$= \frac{4ac-b^2}{4a}$

$$= \frac{4 \times 2(-1) - 1}{4 \times 2} = -\frac{9}{8}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	D	D	B	B	B	A	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	D	B	B	B	A	D	A	C

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 10

Topic :-Applications of Derivatives

1 **(a)**

$\because \quad x = t^2$ and $y = 2t$

$\therefore$ At $t = 1,\ x = 1$ and $y = 2$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2}{2t} = \frac{1}{t}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,2)} = 2$

$\therefore$ Equation of normal is

$y - 2 = -1(x - 1)$

$\Rightarrow \quad x + y - 3 = 0$

3 **(d)**

Let $y = a\sec\theta - b\tan\theta$

$\Rightarrow \quad \frac{dy}{d\theta} = a\, sec\theta tan\theta - bsec^2\theta$

Put $\frac{dy}{d\theta} = 0 \Rightarrow sec\theta(a\, tan\theta - bsec\theta) = 0$

$\Rightarrow sin\theta = \frac{b}{a} \qquad (\because sec\theta \neq 0)$

Now, $\frac{d^2y}{d\theta^2} > 0, at\ sin\theta = \frac{b}{a}$

$\therefore$minimum value is

$y = a\frac{a}{\sqrt{a^2 - b^2}} - b\frac{b}{\sqrt{a^2 - b^2}}$

$=$

$\sqrt{a^2 - b^2}$

4 **(d)**

We have,

$y = x^n$

$\Rightarrow \log y = n \log x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{n}{x} \Rightarrow \frac{dy}{dx} = \frac{ny}{x}$

$\therefore \Delta y = \frac{dy}{dx}\Delta x$

$\Rightarrow \Delta y = \frac{ny}{x}\Delta x \Rightarrow \frac{\Delta y}{y} = \left(\frac{\Delta x}{x}\right) \times n \Rightarrow \frac{\Delta y}{y} \div \frac{\Delta x}{x} = n$

5 **(c)**

Let $f(x)=x^7+14x^5+16x^3+30x-560$

$f'(x)=7x^6+70x^4+48x^2+30>0, \forall x \in R$

$\therefore f(x)$ is increasing

$\therefore \quad f(x)=0$ has only one solution

6 **(c)**

Given, $f(x)=x^3+ax^2+bx+c, a^2 \leq 3b$

On differentiating w.r.t. x, we get

$f'(x)=3x^2+2ax+b$

Put $\quad f'(x)=0$

$\Rightarrow \quad 3x^2+2ax+b=0$

$$\Rightarrow \quad x=\frac{-2a \pm \sqrt{4a^2-12b}}{2 \times 3}$$

$$=\frac{-2a \pm 2\sqrt{a^2-3b}}{6}$$

Since, $\quad a^2 \leq 3b$,

$\therefore \quad x$ Has an imaginary value.

Hence, no extreme value of x exist.

8 **(c)**

We have,

$$xy^n=a^{n+1} \Rightarrow y^n+n\,xy^{n-1}\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{y}{nx}$$

Let (x_1, y_1) be a point on $x\,y^n=a^{n+1}$. The equation of the tangent at (x_1, y_1) is

$$y=y_1=-\frac{y_1}{nx_1}(x-x_1)$$

This meets with the coordinate axes at $A\big((n+1)x_1, 0\big)$ and $B\left(0, \frac{(n+1)y_1}{n}\right)$

$\therefore$ Area of $\Delta OAB=\frac{1}{2}(n+1)x_1 \cdot \left(\frac{n+1}{n}\right)y_1$

$\Rightarrow$ Area of $\Delta OAB=\frac{1}{2}\frac{(n+1)^2}{n}x_1y_1 \quad$ ...(i)

Since (x_1, y_1) lies on $xy^n=a^{n+1}$

$$\therefore x_1y_1^n=a^{n+1}(x_1, y_1) \Rightarrow x_1=\frac{a^{n+1}}{y_1^n}$$

Putting the value of x_1 in (i), we get

Area of $\Delta OAB=\frac{1}{2}\frac{(n+1)^2}{n}a^{n+1}y_1^{-n+1}$

This will be a constant, if $n=1$

9 **(a)**

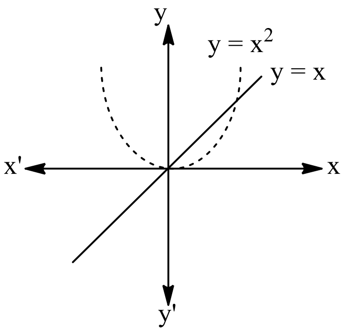
The area of circular plate is $A=\pi r^2$

$$\Rightarrow \frac{dA}{dt}=2\pi r \cdot \frac{dr}{dt}$$

$\Rightarrow \frac{dA}{dt}=2\pi(12)(0.01)=0.24\pi$ sq cm/s

10 **(a)**

$\therefore g(x)=\min(x, x^2)$



It is clear from the graph that g(x) is an increasing function

11 **(d)**

The point of intersection of given curve is (0, 1). On differentiating given curves, we get

$\frac{dy}{dx} = a^x \log a, \frac{dy}{dx} = b^x \log b$

$\Rightarrow \quad m_1 = a^x \log a, m_2 = b^x \log b$

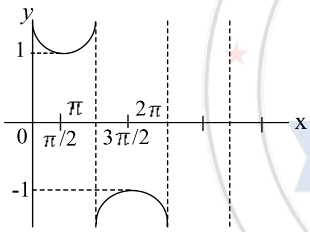
At (0,1) $m_1 = \log a, m_2 = \log b$

$\therefore \quad \tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$$\Rightarrow \quad \theta = \tan^{-1}\left|\frac{\log a - \log b}{1 + \log a \log b}\right|$$

12 **(a)**

The graph of cosec x is opposite in interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$



13 **(b)**

Given, $f(x) = x^{25}(1-x)^{75}$

$\Rightarrow \quad f'(x) = 25x^{24}(1-x)^{75} - 75x^{25}(1-x)^{74}$

$= 25x^{24}(1-x)^{74}(1-4x)$

Put, $f'(x) = 0 \quad \Rightarrow \quad x = 0, 1, \frac{1}{4}$

If $x < \frac{1}{4}$, then

$f'(x) = 25x^{24}(1-x)^{74}(1-4x) > 0$

And if $x > \frac{1}{4}$, then

$f'(x) = 25x^{24}(1-x)^{24}(1-4x) < 0$

Thus, $f'(x)$ changes its sign from positive to negative as x passes through 1/4 from left to right.

Hence, $f(x)$ attains its maximum at x=1/4

14 **(d)**

Given, $f(x) = e^x \sin x, \ x \in [0, \pi]$

At $x = 0, \ f(0) = 0$

And at $x = \pi, \ f(\pi) = 0$

Also, it is continuous and differentiable in the given interval.

Hence, it satisfies the Rolle's theorem.

Hence, option (d) is the required answer

15 **(b)**

Given curve is

$xy = c^2 \quad \Rightarrow y = \dfrac{c^2}{x^2}$

Let $f(x) = ax + by = ax + \frac{bc^2}{x^2}$

On differentiating w.r.t. x, we get

$f'(x) = a - \dfrac{bc^2}{x^2}$

For a maximum or minima, put $f'(x) = 0$

$\Rightarrow \quad ax^2 - bc^2 = 0$

$\Rightarrow \quad x^2 = \dfrac{bc^2}{a} \Rightarrow \quad x = \pm c\sqrt{\dfrac{b}{a}}$

Again on differentiating, we get $f''(x) = \frac{2bc^2}{x^3}$

At6 $x = c\sqrt{\frac{b}{a}}, \; f''(x) > 0$

$\therefore \quad f(x)$ is minimum at $x = c\sqrt{\frac{b}{a}}$

The minimum value at $x = c\sqrt{\frac{b}{a}}$ is

$\therefore \; f\left(c\sqrt{\dfrac{b}{a}}\right) = a.c\sqrt{\dfrac{b}{a}} + \dfrac{bc^2}{c}.\sqrt{\dfrac{a}{b}}$

$= \dfrac{abc + abc}{\sqrt{ab}} = \dfrac{2abc}{\sqrt{ab}} = 2c\sqrt{ab}$

16 **(a)**

Given, $\quad x - 2y = 4$

Let $\quad A = xy \quad \Rightarrow \quad A = 2y^2 + 4y$

$\Rightarrow \quad \dfrac{dA}{dy} = 4 + 4y$

For extremum value, $\frac{dA}{dy} = 0$

$\Rightarrow \quad y = -1$

Now, $\frac{d^2A}{dy^2} = 4 > 0$, minima

At $\; y = -1$,

$x = 4 + 2(-1) = 2$

$\therefore \quad A = xy = 2(-1) = -2$

$\therefore$ Minimum value of xy is -2

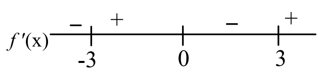
17 **(a)**

Given, $\; f(x) = (9 - x^2)^2$

$\Rightarrow \quad f''(x) = 2(9 - x^2)(-2x)$

Now, put $\; f''(x) = 0$

$\Rightarrow \quad 2(9 - x^2)(-2x) = 0 \quad \Rightarrow \quad x = 0, \pm 3$



$\therefore \quad f''(x)$ is increasing in $(-3,0) \cup (3,\infty)$

18 **(b)**

Let $f(x) = x + \frac{1}{x} \quad \Rightarrow \quad f'(x) = 1 - \frac{1}{x^2}$

For maxima and minima, put $f'(x) = 0$

$\Rightarrow \quad 1 - \dfrac{1}{x^2} = 0 \quad \Rightarrow \quad x = \pm 1$

Now, $\quad f''(x) = \frac{2}{x^3}$

At $x = 1, \quad f''(x) = \frac{2}{x^3}$

At $x = 1, \; f''(x) = +$ ve, minima

And at $x = -1, \; f''(x) = -$ve, maxima

Thus, $f(x)$ attains minimum value at $x = 1$

19 **(a)**

We have,

$12y = x^3$

$\Rightarrow 12\dfrac{dy}{dt} = 3x^2\dfrac{dx}{dt}$

$\Rightarrow 12 = 3x^2 \qquad \left[\because \dfrac{dy}{dt} = \dfrac{dx}{dt}\right]$

$\Rightarrow x = \pm 2$

$\therefore 12y = x^3 \Rightarrow y = \pm\dfrac{2}{3}$

Hence, the points are $(2, 2/3)$ and $(-2, -2/3)$

20 **(b)**

Let $R(x_1, y_1)$ be the point on the parabola $y^2 = 2x$ such that tangent at R is parallel to the chord PQ

$\therefore \left(\dfrac{dy}{dx}\right)_R = \text{Slope of } PQ$

$\Rightarrow \dfrac{1}{y_1} = \dfrac{-1-2}{\frac{1}{2} - 2} \quad \left[\because y^2 = 2x \Rightarrow 2y\dfrac{dy}{dx} = 2 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{y}\right]$

$\Rightarrow \dfrac{1}{y_1} = 2 \Rightarrow y_1 = \dfrac{1}{2}$

Since (x_1, y_1) lies on $y^2 = 2x$

$\therefore y_1^2 = 2x_1 \Rightarrow x_1 = \dfrac{1}{8} \quad [\because y = 1/2]$

Hence, the required point is $(1/8, 1/2)$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	D	D	C	C	A	C	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	B	D	B	A	A	B	A	B