SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 1

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(b)**

We have,

$-\pi/4 < x < \pi/4$

$\Rightarrow -1 < \tan x < 1 \Rightarrow 0 \leq \tan^2 x < 1 \Rightarrow [\tan^2 x] = 0$

$\therefore f(x) = [\tan^2 x] = 0$ for all $x \in (-\pi/4, \pi/4)$

Thus, $f(x)$ is a constant function on $\in (-\pi/4, \pi/4)$

So, it is continuous on $\in (-\pi/4, \pi/4)$ and $f'(x) = 0$ for all $x \in (-\pi/4, \pi/4)$

2 **(d)**

Since, $f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{-e^x + 2^x}{x} = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{-e^x + 2^x \log 2}{1} = f(0)$ [by L 'Hospital's rule]

$\Rightarrow f(0) = -1 + \log 2$

3 **(b)**

Since $f(x)$ is an even function

$\therefore f(-x) = f(x)$ for all x

$\Rightarrow -f'(-x) = f'(x)$ for all x

$\Rightarrow f'(-x) = -f'(x)$ for all x

$\Rightarrow f'(x)$ is an odd function

4 **(c)**

We have,

$$f(x) = \begin{cases} [\cos \pi x], x < 1 \\ |x-2|, 1 \leq x < 2 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 2-x, & 1 \leq x < 2 \\ -1, & 1/2 < x < 1 \\ 0, & 0 < x \leq 1/2 \\ 1, & x = 0 \\ 0, & -1/2 \leq x < 0 \\ -1, & -3/2 < x < -1/2 \end{cases}$$

It is evident from the definition that $f(x)$ is discontinuous at $x = 1/2$

5 **(b)**

We have,

$$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0} \frac{|-2-h+2|}{\tan^{-1}(-2-h+2)}$$

$$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{h\to 0} \frac{h}{\tan^{-1}(-h)} = \lim_{h\to 0} \frac{-h}{\tan^{-1} h} = -1$$

and,

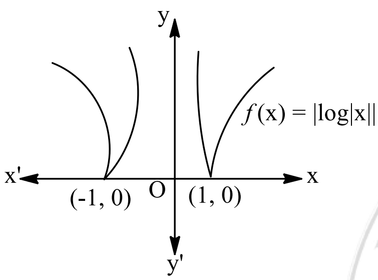
$$\lim_{x\to -2^+} f(x) = \lim_{h\to 0} f(-2+h) = \lim_{h\to 0} \frac{|-2+h+2|}{\tan^{-1}(-2+h+2)}$$

$$\Rightarrow \lim_{x\to -2^+} f(x) = \lim_{h\to 0} \frac{h}{\tan^{-1} h} = 1$$

$$\therefore \lim_{x\to -2^-} f(x) \neq \lim_{\to -2^+} f(x)$$

So, $f(x)$ is neither continuous nor differentiable at $x=-2$

6 **(b)**



From the graph of $f(x) = |\log|x||$ it is clear that $f(x)$ is everywhere continuous but not differentiable at $x = \pm 1$, due to sharp edge

7 **(b)**

We have,

$$\lim_{x\to a} \frac{xf(a) - a\,f(x)}{x-a} = \lim_{x\to a} \frac{x\,f(a) - a\,f(a) - a\big(f(x) - f(a)\big)}{x-a}$$

$$\Rightarrow \lim_{x\to a} \frac{x\,f(a) - a\,f(x)}{x-a} = \lim_{x\to a} \frac{f(a)(x-a)}{x-a} - a \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$\Rightarrow \lim_{x\to a} \frac{x\,f(a) - a\,f(x)}{x-a} = f(a) - a\,f'(a) = 4 - 2a$$

8 **(c)**

Given,$f(x) = x\left(\sqrt{x} + \sqrt{x+1}\right)$. At $x=0$ LHL of $\sqrt{x}$ is not defined, therefore it is not continuous at $x=0$

Hence, it is not differentiable at $x=0$

9 **(a)**

Here, $f'(x) = \begin{cases} 2ax, & b \neq 0, x \leq 1 \\ 2bx + a, & x > 1 \end{cases}$

Since, $f(X)$ is continuous at $x=1$

$$\therefore \lim_{h\to 0} f(x) = \lim_{h\to 1^+} f(x)$$

$$\Rightarrow \quad a+b = b+a+c \ \Rightarrow c = 0$$

Also, $f(x)$ is differentiable at $x=1$

$\therefore$ (LHD at $x=1$)=(RHD at $x=1$)

$$\Rightarrow \quad 2a = 2b(1) + a \ \Rightarrow \ a = 2b$$

10 **(d)**

We have,

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}\left\{\frac{x^2}{4}-\frac{3x}{4}+\frac{13}{4}\right\} = \frac{1}{4}-\frac{3}{2}+\frac{13}{4} = 2$$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} |x-3| = 2$

and, $f(1) = |1-3| = 2$

$\therefore \lim_{x\to 1^-} f(x) = f(1) = \lim_{x\to 1^+} f(x)$

So, $f(x)$ is continuous at $x = 1$

We have,

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3} |x-3| = 0,\ \lim_{x\to 3^+} f(x) = \lim_{x\to 3} |x-3| = 0$

and, $f(3) = 0$

$\therefore \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

So, $f(x)$ is continuous at $x = 3$

Now,

(LHD at $x = 1$)

$$= \left\{\frac{d}{dx}\left(\frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4}\right)\right\}_{x=1} = \left\{\frac{x}{2}-\frac{3}{2}\right\}_{x=1} = \frac{1}{2}-\frac{3}{2} = -1$$

(RHD at $x = 1$) $= \left\{\frac{d}{dx}\left(-(x-3)\right)\right\}_{x=1} = -1$

$\therefore$ (LHD at $x = 1$) $=$ (RHD at $x = 1$)

So, $f(x)$ is differentiable at $x = 1$

11 **(d)**

$$f(x) = \begin{cases} \dfrac{2\sin x - \sin 2x}{2x\cos x} & ,\text{if } x \neq 0, \\ a, & \text{if } x = 0 \end{cases}$$

Now, $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{2\sin x - \sin 2x}{2x\cos x} \quad \left(\frac{0}{0}\ \text{form}\right)$

$$= \lim_{x\to 0} \frac{2\cos x - 2\cos 2x}{2\,(\cos x - x\sin x)}$$

$$= \lim_{x\to 0} \frac{2-2}{2(1-0)} = 0$$

Since, $f(x)$ is continuous at $x = 0$

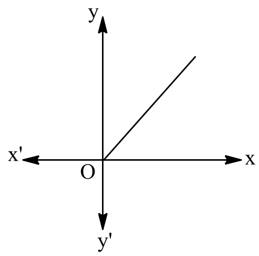
$\therefore \quad f(0) = \lim_{x\to 0} f(x)$

$\Rightarrow a = 0$

12 **(a)**

Given, $f(x) = x + |x|$

$$\therefore \quad f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



It is clear from the graph of $f(x)$ is continuous for every value of x

Alternate

Since, x and $|x|$ is continuous for every value of x, so their sum is also continous for every value of x

13 **(a)**

Since $f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0^-} f(x) = f(0) = \lim_{x\to 0^+} f(x)$

$\Rightarrow \lim_{x\to 0}\{1 + |\sin x|\}^{\frac{a}{|\sin x|}} = b = \lim_{x\to 0} e^{\frac{\tan 2x}{\tan 3x}}$

$\Rightarrow e^a = b = e^{2/3} \Rightarrow a = \frac{2}{3}$ and $a = \log_e b$

14 **(b)**

We have,

$$f(x) = \begin{cases} x^2 + \dfrac{(x^2/1 + x^2)}{1 - (1/1 + x^2)} = x^2 + 1, x \neq 0 \\ \qquad\qquad 0, \qquad x = 0 \end{cases}$$

Clearly, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 1 \neq f(0)$

So, $f(x)$ is discontinuous at $x = 0$

15 **(d)**

LHD$= \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h}$

$= \lim_{h\to 0} \dfrac{1-1}{-h} = 0$

RHD$= \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0} \dfrac{1 + \sin(0+h) - 1}{h} = \lim_{h\to 0} \dfrac{\sin h}{h} = 1$

$\Rightarrow$ LHD$\neq$RHD

16 **(a)**

Given, $f(x) = x - |x - x^2|$

At $x = 1$, $f(1) = 1 - |1 - 1| = 1$

$\lim_{x\to 1^{-1}} f(x) = \lim_{h\to 0}[(1-h) - |(1-h) - (1-h)^2|]$

$= \lim_{h\to 0} [(1-h) - |h - h^2|] = 1$

$\lim_{x\to 1^+} f(x) = \lim_{h\to 0}[(1+h) - |(1+h) - (1+h)^2|]$

$= \lim_{h\to 0} [1 + h - |-h^2 - h|] = 1$

$\because \lim_{x\to 1^{-1}} f(x) = \lim_{x\to 1^+} = f(1)$

17 **(a)**

We have,

$f(x+y+z)=f(x)f(y)f(z)$ for all x,y,z ...(i)

$\Rightarrow f(0)=f(0)f(0)f(0)$ [Putting $x=y=z=0$]

$\Rightarrow f(0)\{1-f(0)^2\}=0$

$\Rightarrow f(0)=1$ [$\because f(0)=0 \Rightarrow f(x)=0$ for all x]

Putting $z=0$ and $y=2$ in (i), we get

$f(x+2)=f(x)f(2)f(0)$

$\Rightarrow f(x+2)=4f(x)$ for all x

$\Rightarrow f'(2)=4f'(0)$ [Putting $x=0$]

$\Rightarrow f'(2)=4\times 3=12$

18 **(b)**

For $x>1$, we have

$$f(x)=|\log|x||=\log x \quad \Rightarrow \quad f'(x)=\frac{1}{x}$$

For $x<-1$, we have

$$f(x)=|\log|x||=\log(-x) \quad \Rightarrow \quad f'(x)=\frac{1}{x}$$

For $0<x<1$, we have

$$f(x)=|\log|x||=-\log x \quad \Rightarrow \quad f'(x)=\frac{-1}{x}$$

For $-1<x<0$, we have

$$f(x)=-\log(-x) \quad \Rightarrow \quad f'(x)=-\frac{1}{x}$$

Hence, $f'(x)=\begin{cases}\frac{1}{x}, & |x|>1\\ -\frac{1}{x}, & |x|<1\end{cases}$

19 **(c)**

Since, $\lim\limits_{x\to 0} f(x)=f(0)$

$$\Rightarrow \lim_{x\to 0}\frac{1-\cos x}{x^2}=k$$

$$\Rightarrow \lim_{x\to 0}\frac{-(-\sin x)}{2x}=k \quad \text{[using L 'Hospital's rule]}$$

$$\Rightarrow \frac{1}{2}\lim_{x\to 0}\frac{\sin x}{x}=k \quad \Rightarrow \quad k=\frac{1}{2}$$

20 **(b)**

Given, $f(X)=|x-1|+|x-2|$

$$=\begin{cases}x-1+x-2, & x\geq 2\\ x-1+2-x, & 1\leq x<2\\ 1-x+2-x, & x<1\end{cases}$$

$$=\begin{cases}2x-3, & x\geq 2\\ 1, & 1\leq x<2\\ 3-2x, & x<1\end{cases}$$

$$f'(x)=\begin{cases}2, & x>2\\ 0, & 1<x<2\\ -1, & x<1\end{cases}$$

Hence, except $x=1$ and $x=2$, $f(x)$ is differentiable everywhere in R

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	B	C	B	B	B	C	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	A	B	D	A	A	B	C	B

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 2

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(b)**

Clearly, $f(x)$ is differentiable for all non-zero values of x. For $x \neq 0$, we have

$$f'(x) = \frac{x\, e^{-x^2}}{\sqrt{1-e^{-x^2}}}$$

Now,

$$(\text{LHD at } x=0) = \lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{x-0}$$

$$\Rightarrow (\text{LHD at } x=0) = \lim_{h\to 0} \frac{\sqrt{1-e^{-h^2}}}{-h} = \lim_{h\to 0} -\frac{\sqrt{1-e^{-h^2}}}{h}$$

$$\Rightarrow (\text{LHD at } x=0) = -\lim_{h\to 0} \sqrt{\frac{e^{h^2}-1}{h^2}} \times \frac{1}{\sqrt{e^{h^2}}} = -1$$

and, $(\text{RHD at } x=0) = \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{\sqrt{1-e^{-h^2}}-0}{h}$

$$\Rightarrow (\text{RHD at } x=0) = \lim_{h\to 0} \sqrt{\frac{e^{h^2}-1}{h^2}} \times \frac{1}{\sqrt{e^{h^2}}} = 1$$

So, $f(x)$ is not differentiable at $x=0$

Hence, the set of points of differentiability of $f(x)$ is $(-\infty, 0) \cup (0, \infty)$

2 **(c)**

Since $f(x)$ is continuous at $x=0$

$$\therefore f(0) = \lim_{x\to 0} x \sin\left(\frac{1}{x}\right) = 0$$

3 **(d)**

For $f(x)$ to be continuous everywhere, we must have,

$$f(0) = \lim_{x\to 0} f(x)$$

$$\Rightarrow f(0) = \lim_{x\to 0} \frac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2} \quad \left[\text{Form } \frac{0}{0}\right]$$

$$\Rightarrow f(0) = \lim_{x\to 0} \frac{\frac{7}{8}(256-7x)^{-\frac{7}{8}}}{(5x+32)^{-4/5}} = \frac{7}{8} \times \frac{2^{-7}}{2^{-4}} = \frac{7}{64}$$

4 **(b)**

We have,

$$f(x) = |x|^3 = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases}$$

$\therefore$ (LHD at $x = 0$) $= \lim\limits_{x \to 0^-} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0} -\dfrac{x^3}{x} = 0$

and,

$\therefore$ (RHD at $x = 0$) $= \lim\limits_{x \to 0^+} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0} \dfrac{x^3}{x} = 0$

Clearly, (LHD at $x = 0$) = (RHD at $x = 0$)

Hence, $f(x)$ is differentiable at $x = 0$ and its derivative at $x = 0$ is 0

5 **(a)**

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(\frac{4^x - 1}{x}\right)^3 \times \frac{\left(\frac{x}{a}\right)}{\sin\left(\frac{x}{a}\right)} \cdot \frac{ax^2}{\log\left(1 + \frac{1}{3}x^2\right)}$$

$$= (\log 4)^3 . 1 . a \lim_{x \to 0} \left(\frac{x^2}{\frac{1}{3}x^2 - \frac{1}{18}x^4 + \ldots}\right)$$

$= 3a\,(\log 4)^3$

$\because \quad \lim\limits_{x \to 0} f(x) = f(0)$

$\Rightarrow \quad 3a\,(\log 4)^3 = 9(\log 4)^3$

$\Rightarrow \quad a = 3$

6 **(d)**

We have,

$f(x) = |[x]x|$ for $-1 < x \leq 2$

$$\Rightarrow f(x) = \begin{cases} -x, & -1 < x < 0 \\ 0, & 0 \leq x < 1 \\ x, & 1 \leq x < 2 \\ 2x, & x = 2 \end{cases}$$

It is evident from the graph of this function that it is continuous but not differentiable at $x = 0$. Also, it is discontinuous at $x = 1$ and non-differentiable at $x = 2$

7 **(c)**

Given, $f(x) = [x^3 - 3]$

Let $g(x) = x^3 - x$ it is in increasing function

$\therefore \quad g(1) = 1 - 3 = -2$

and $g(2) = 8 - 3 = 5$

Here, $f(x)$ is discontinuous at six points

8 **(b)**

Given, $y = \cos^{-1}\cos(x - 1), \; x > 0$

$\Rightarrow \quad y = x - 1, \qquad 0 \leq x - 1 \leq \pi$

$\therefore \quad y = x - 1, \qquad 1 \leq x \leq \pi + 1$

At $x = \frac{5\pi}{4} \in [1, \pi + 1]$

$\Rightarrow \dfrac{dy}{dx} = 1 \quad \Rightarrow \quad \left(\dfrac{dy}{dx}\right)_{x = \frac{5\pi}{4}} = 1$

9 **(d)**

We have,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h} \qquad [\because f(x+y)=f(x)+f(y)]$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(h)}{h} = \lim_{h\to 0}\frac{h^2 g(h)}{h}$$

$$\Rightarrow f'(x) = 0\times g(0) = 0 \qquad \begin{bmatrix}\because g \text{ is continuous} \\ \therefore \lim_{h\to 0} g(h) = g(0)\end{bmatrix}$$

10 **(b)**

Using Heine's definition of continuity, it can be shown that $f(x)$ is everywhere discontinuous

11 **(b)**

For $x \neq -1$, we have

$f(x) = 1 - 2x + 3x^2 - 4x^3 + \cdots\infty$

$$\Rightarrow f(x) = (1+x)^{-2} = \frac{1}{(1+x)^2}$$

Thus, we have

$$f(x) = \begin{cases}\dfrac{1}{(1+x)^2}, & x\neq -1 \\ 1, & x = -1\end{cases}$$

We have, $\lim_{x\to -1^-} f(x) \to \infty$ and $\lim_{x\to -1^-} f(x) \to \infty$

So, $f(x)$ is not continuous at $x = -1$

Consequently, it is not differentiable there at

12 **(b)**

At $x = a$,

LHL= $\lim_{x\to a^-} f(x) = \lim_{x\to a} 2a - x = a$

And RHL= $\lim_{x\to a^+} f(x) = \lim_{x\to a} 3x - 2a = a$

And $f(a) = 3(a) - 2a = a$

$\therefore$ LHL=RHL= $f(a)$

Hence, it is continuous at $x = a$

Again, at $x = a$

LHD= $\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$

$$= \lim_{h\to 0}\frac{2a-(a-h)-a}{-h} = -1$$

and RHD= $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

$$= \lim_{h\to 0}\frac{3(a+h)-2a-a}{h} = 3$$

∴ LHD≠RHD

Hence, it is not differentiable at $x = a$

13 **(b)**

We have,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(x)f(h) - f(x)}{h}$$

$$\Rightarrow f'(x) = f(x) \lim_{h\to 0} \frac{f(h) - 1}{h}$$

$$\Rightarrow f'(x) = f(x) \lim_{h\to 0} \frac{1 + (\sin 2h)g(h) - 1}{h}$$

$$\Rightarrow f'(x) = f(x) \lim_{h\to 0} \frac{\sin 2h}{h} \times \lim_{h\to 0} g(h) = 2f(x)g(0)$$

14 **(c)**

If $-1 \le x \le 1$, then $0 \le x \sin \pi x \le 1/2$

$\therefore f(x) = [x \sin \pi x] = 0$, for $-1 \le x \le 1$

If $1 < x < 1 + h$, where h is a small positive real number, then

$\pi < \pi x < \pi + \pi h \Rightarrow -1 < \sin \pi x < 0 \Rightarrow -1 < x \sin \pi x < 0$

$\therefore f(x) = [x \sin \pi x] = -1$ in the right neighbourhood of $x = 1$

Thus, $f(x)$ is constant and equal to zero in $[-1, 1]$ and so $f(x)$ is differentiable and hence continuous on $(-1,1)$

At $x = 1$, $f(x)$ is discontinuous because

$\Rightarrow \lim_{x\to 1^-} f(x) = 0$ and $\lim_{x\to 1^+} f(x) = -1$

Hence, $f(x)$ is not differentiable at $x = 1$

15 **(d)**

We have,

$$(\text{LHD at } x = 0) = \left\{\frac{d}{dx}(1)\right\}_{x=0} = 0$$

$$(\text{RHD at } x = 0) = \left\{\frac{d}{dx}(1 + \sin x)\right\}_{x=0} = \cos 0 = 1$$

Hence, $f'(x)$ at $x = 0$ does not exist

16 **(c)**

Here, $f'(x) = \begin{cases} 2bx + a, & x \ge -1 \\ 2a, & x < -1 \end{cases}$

Given, $f'(x)$ is continuous everywhere

$\therefore \lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} f(x)$

$\Rightarrow -2b + a = -2a$

$\Rightarrow 3a = 2b$

$\Rightarrow a = 2, \quad b = 3$

or $a = -2, \; b = -3$

17 **(b)**

We have,

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{\log\cos x}{\log(1+x^2)}$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{\log(1-1+\cos x)}{\log(1+x^2)}\cdot\frac{1-\cos x}{1-\cos x}$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{\log\{1-(1-\cos x)\}}{1-\cos x}\cdot\frac{1-\cos x}{\log(1+x^2)}$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=-\lim_{x\to 0}\log\frac{[1-(1-\cos x)]}{-(1-\cos x)}\times\frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}\times\frac{x^2}{\log(1+x^2)}$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=-\frac{1}{2}$$

Hence, $f(x)$ is differentiable and hence continuous at $x=0$

18 **(a)**

Since $f(x)$ is continuous at $x=1$. Therefore,

$\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^+} f(x)\Rightarrow A-B=3\Rightarrow A=3+B$ ...(i)

If $f(x)$ is continuous at $x=2$, then

$\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^+} f(x)\Rightarrow 6=4B-A$ ...(ii)

Solving (i) and (ii) we get $B=3$

As $f(x)$ is not continuous at $x=2$. Therefore, $B\neq 3$

Hence, $A=3+B$ and $B\neq 3$

19 **(a)**

We have,

$$f(x)=\begin{cases} x-4, & x\geq 4\\ -(x-4), & 1\leq x<4\\ (x^3/2)-x^2+3x+(1/2), & x<1\end{cases}$$

Clearly, $f(x)$ is continuous for all x but it is not differentiable at $x=1$ and $x=4$

20 **(a)**

It is given that $f(x)$ is continuous at $x=1$

$\therefore \lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^+} f(x)=f(1)$

$\Rightarrow \lim\limits_{x\to 1^-} a[x+1]+b[x-1]=\lim\limits_{x\to 1^+} a[x+1]+b[x-1]$

$\Rightarrow a-b=2a+0\times b$

$\Rightarrow a+b=0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	D	B	A	D	C	B	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	B	C	D	C	B	A	A	A

SESSION : 2025-26

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 3

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(c)**

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \lambda[x] = 0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 5^{1/x} = 0$

And $f(0) = \lambda[0] = 0$

$\therefore$ f is continuous only whatever λ may be

2 **(b)**

We have,

$y(x) = f(e^x)\, e^{f(x)}$

$\Rightarrow y'(x) = f'(e^x)\cdot e^x \cdot e^{f(x)} + f(e^x)\, e^{f(x)}\, f'(x)$

$\Rightarrow y'(0) = f'(1)e^{f(0)} + f(1)e^{f(0)}f'(0)$

$\Rightarrow y'(0) = 2 \qquad [\because f(0) = f(1) = 0, f'(1) = 2]$

3 **(b)**

Since $f(x)$ is differentiable at $x = 1$. Therefore,

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{a(1-h)^2 - b - 1}{-h} = \lim_{h\to 0} \frac{\frac{1}{|1+h|} - 1}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{(a-b-1) - 2\,ah + ah^2}{-h} = \lim_{h\to 0} \frac{-h}{h(1+h)}$$

$$\Rightarrow \lim_{h\to 0} \frac{-(a-b-1) - 2\,ah - ah^2}{h} = -1$$

$\Rightarrow -(a-b-1) = 0$ and so $\lim_{h\to 0} \frac{2ah - ah^2}{h} = -1$

$\Rightarrow a - b - 1 = 0$ and $2a = -1 \Rightarrow a = -\frac{1}{2}, b = -\frac{3}{2}$

4 **(c)**

We have,

$f(x) = \frac{\sin 4\,\pi[x]}{1+[x]^2} = 0$ for all x $[\because 4\pi[x]$is a multiple of $\pi]$

$\Rightarrow f'(x) = 0$ for all x

5 **(d)**

We have,

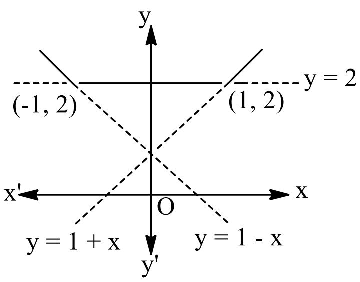
$$\lim_{x\to0} f(x) = \lim_{x\to0} \sin\frac{1}{x}$$

$\Rightarrow \lim_{x\to0} f(x) =$ An oscillating number which oscillates between -1 and 1

Hence, $\lim_{x\to0} f(x)$ does not exist

Consequently, $f(x)$ cannot be continuous at $x = 0$ for any value of k

6 **(c)**



It is clear from the graph that $f(x)$ is continuous everywhere and also differentiable everywhere except $\{-1, 1\}$ due to sharp edge

7 **(d)**

We have,

$\log\left(\frac{x}{y}\right) = \log x - \log y$ and $\log(e) = 1$

$\therefore f(x) = \log x$

Clearly, $f(x)$ is unbounded because $f(x) \to -\infty$ as $x \to 0$ and $f(x) \to +\infty$ as $x \to \infty$

We have,

$$f\left(\frac{1}{x}\right) = \log\left(\frac{1}{x}\right) = -\log x$$

As $x \to 0, f\left(\frac{1}{x}\right) \to \infty$

Also,

$$\lim_{x\to0} xf(x) = \lim_{x\to0} x\log x = \lim_{x\to0} \frac{\log x}{1/x}$$

$$\Rightarrow \lim_{x\to0} xf(x) = \lim_{x\to0} \frac{1/x}{-1/x^2} = -\lim_{x\to0} x = 0$$

9 **(c)**

Since $g(x)$ is the inverse of $f(x)$. Therefore,

$fog(x) = x$, for all x

$\Rightarrow \frac{d}{dx}\{fog\,(x)\} = 1$, for all x

$\Rightarrow f'\big(g(x)\big)\, g'(x) = 1$, for all x

$\Rightarrow \frac{1}{1+\{g(x)\}^3} \times g'(x) = 1$ for all x $\quad \left[\because f'(x) = \frac{1}{1+x^3}\right]$

$\Rightarrow g'(x) = 1 + \{g(x)\}^3$, for all x

10 **(d)**

We have,

$f(x) = |x^2 - 4x + 3|$

$$\Rightarrow f(x) = \begin{cases} x^2 - 4x + 3, & \text{if } x^2 - 4x + 3 \geq 0 \\ -(x^2 - 4x + 3), & \text{if } x^2 - 4x + 3 < 0 \end{cases}$$

$\Rightarrow f(x) = \begin{cases} x^2 - 4x + 3, & \text{if } x \le 1 \text{ or } x \ge 3 \\ -x^2 + 4x - 3, & \text{if } 1 < x < 3 \end{cases}$

Clearly, $f(x)$ is everywhere continuous

Now,

$$(\text{LHD at } x = 1) = \left(\frac{d}{dx}(x^2 - 4x + 3)\right)_{\text{at } x=1}$$

$\Rightarrow (\text{LHD at } x = 1) = (2x - 4)_{\text{at } x=1} = -2$

and,

$$(\text{RHD at } x = 1) = \left(\frac{d}{dx}(-x^2 + 4x - 3)\right)_{\text{at } x=1}$$

$\Rightarrow (\text{RHD at } x = 1) = (-2x + 4)_{\text{at } x=1} = 2$

Clearly, $(\text{LHD at } x = 1) \neq (\text{RHD at } x = 1)$

So, $f(x)$ is not differentiable at $x = 1$

Similarly, it can be checked that $f(x)$ is not differentiable at $x = 3$ also

ALITER We have,

$f(x) = |x^2 - 4x + 3| = |x - 1|\,|x - 3|$

Since, $|x - 1|$ and $|x - 3|$ are not differentiable at 1 and 3 respectively

Therefore, $f(x)$ is not differentiable at $x = 1$ and $x = 3$

11 **(c)**

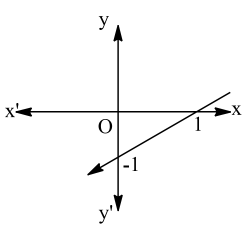
The point of discontinuity of $f(x)$ are those points where $\tan x$ is infinite.

ie, $\tan x = \tan \infty$

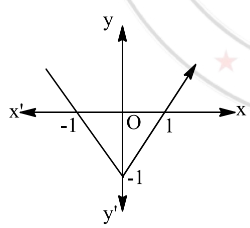
$\Rightarrow \quad x = (2n + 1)\frac{\pi}{2}, \qquad n \in I$

12 **(a)**

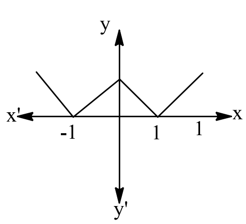
Using graphical transformation



(i) y = x - 1



(ii) y = |x| - 1



(iii) y = ||x| - 1|

As, we know the function is not differentiable at6 sharp edges and in figure (iii) $y = \big||x| - 1\big|$ we have 3 sharp edges at $x = -1, 0, 1$

$\therefore f(x)$ is not differentiable at $\{0, \pm 1\}$

13 **(c)**

$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} 2(0-h) = 0$

And $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} 2(0+h) + 1 = 1$

$\because \quad \lim_{x\to 0^-} f(x) \neq \lim_{x\to 1^+} f(x)$

$\therefore \quad f(x)$ is discontinuous at $x = 0$

14 **(b)**

Draw a rough sketch of $y = f(x)$ and observe its properties

15 **(c)**

$$\lim_{x\to\pi} \frac{(1+\cos x) - \sin x}{(1+\cos x) + \sin x}$$

$$= \lim_{x\to\pi} \frac{2\cos^2 x/2 - 2(\sin x/2)\cos\ x/2}{2\cos^2 x/2 + 2(\sin x/2)\cos\ x/2}$$

$$= \lim_{x\to\pi} \tan\left(\frac{\pi}{4} - \frac{\pi}{2}\right) = -1$$

Since, $f(x)$ is continuous at $x = \pi$

$\therefore \quad f(\pi) = \lim_{x\to\pi} f(x) = -1$

16 **(d)**

$$f'(1^-) = \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h\to 0} \frac{(1-h-1).\sin\left(\frac{1}{1-h-1}\right) - 0}{-h}$$

$$= -\lim_{h\to 0} \sin\frac{1}{h}$$

And $f'(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$$= \lim_{h\to 0} \frac{(1+h-1)\sin\left(\frac{1}{1+h-1}\right) - 0}{h}$$

$$= \lim_{h\to 0} \sin\frac{1}{h}$$

$\therefore \quad f'(1^-) \neq f'(1^+)$

f is not differentiable at $x = 1$

Again, now

$$f'(0^+) = \lim_{h\to 0} \frac{(0+h-1)\sin\left(\frac{1}{0+h-1}\right) - \sin 1}{h}$$

$$= \lim_{h\to 0} \frac{\left[-\left\{(h-1)\cos\left(\frac{1}{h-1}\right) \times \left(\frac{1}{(h-1)^2}\right)\right\} + \sin\left(\frac{1}{h-1}\right)\right]}{1}$$

[using L 'Hospital's rule]

$= \cos 1 - \sin 1$

And $f'(0^-) = \lim_{h\to 0} \frac{(0-h-1)\sin\left(\frac{1}{0-h-1}\right) - \sin 1}{-h}$

$$= \lim_{h\to 0} \frac{(-h-1)\cos\left(\frac{1}{-h-1}\right)\left(\frac{1}{(-h-1)^2}\right) - \sin\left(\frac{1}{-h-1}\right)}{-1}$$

[using L 'Hospital's rule]

$= \cos 1 - \sin 1$

$\Rightarrow \quad f'(0^-) = f'(0^+)$

$\therefore \quad f$ is differentiable at $x = 0$

17 **(c)**

As $f(x)$ is continuous at $x = \frac{\pi}{2}$

$\therefore \lim_{x\to \frac{\pi}{2}^-} f(x) = \lim_{x\to \frac{\pi}{2}^+} f(x)$

$$\Rightarrow m\frac{\pi}{2} + 1 = \sin\frac{\pi}{2} + n \Rightarrow m\frac{\pi}{2} + 1 = 1 + n \Rightarrow n = \frac{m\pi}{2}$$

18 **(d)**

Since, $\frac{f(6)-f(1)}{6-1} \geq 2 \quad \left[\because \quad f'(x) = \frac{y_2-y_1}{x_2-x_1}\right]$

$\Rightarrow \quad f(6) - f(1) \geq 10$

$\Rightarrow f(6) + 2 \geq 10$

$\Rightarrow \quad f(6) \geq 8$

19 **(b)**

We have,

$\lim_{x\to a^-} f(x)\ g(x) = \lim_{x\to a^-} f(x) . \lim_{x\to a^-} g(x) = m \times l = ml$

and,

$\lim_{x\to a^+} f(x)\ g(x) = \lim_{x\to a^+} f(x) \lim_{x\to a^+} g(x) = lm$

$\therefore \lim_{x\to a^-} f(x)\ g(x) = \lim_{x\to a^+} f(x)\ g(x) = lm$

Hence, $\lim_{x\to a} f(x)\ g(x)$ exists and is equal to lm

20 **(c)**

We have,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(x)f(h) - f(x)}{h}$$

$$\Rightarrow f'(x) = f(x)\lim_{h\to 0}\frac{f(h) - 1}{h} \qquad [\because f(x+y) = f(x)f(y)]$$

$$\Rightarrow f'(x) = f(x)\left\{\lim_{h\to 0}\frac{1 + h\,g(h) - 1}{h}\right\} \qquad [\because f(x) = 1 + x\,g(x)]$$

$$\Rightarrow f'(x) = f(x)\lim_{h\to 0} g(h) = f(x)\cdot 1 = f(x)$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	B	C	D	C	D	B	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	C	B	C	D	C	D	B	C

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 4

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(a)**

We have, $f(x)=\begin{cases} x^2, & x\geq 0 \\ -x^2, & x<0 \end{cases}$

Clearly, $f(x)$ is differentiable for all $x>0$ and for all$x<0$. So, we check the differentiable at $x=0$

Now, (RHD at $x=0$)

$\left(\dfrac{d}{dx}(x)^2\right)_{x=0}=(2x)_{x=0}=0$

And (LHD at $=0$)

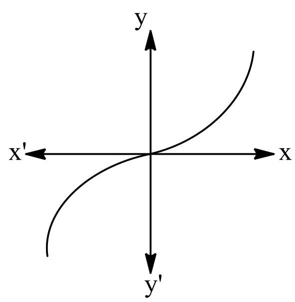
$\left(\dfrac{d}{dx}(-x)^2\right)_{x=0}=(-2x)_{x=0}=0$

$\therefore$ (LHD at $x=0$)=(RHD at $x=0$)

So, $f(x)$ is differentiable for all x ie, the set of all points where $f(x)$ is differentiable is $(-\infty,\infty)$

Alternate

It is clear from the graph $f(x)$ is differentiable everywhere.



2 **(a)**

Since, $f'(0)=\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x-0}=10$

$\Rightarrow \lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=10$

$\Rightarrow f(0)\left(\lim\limits_{h\to 0}\frac{f(h)-1}{h}\right)=10$ ...(i)

[$\because$ $f(0+h)=f(0)f(h)$, given]

Now, $f(0)=f(0)f(0)$

$\Rightarrow f(0)=1$

$\therefore$ From Eq. (i)

$\lim\limits_{h\to 0}\frac{f(h)-1}{h}=10$ ...(ii)

Now, $f'(6)=\lim_{h\to 0}\frac{f(6+h)-f(6)}{h}$

$=\lim_{x\to 0}\left(\frac{f(h)-1}{h}\right)f(6)$ [from Eq. (ii)]

$=10\times 3=30$

3 **(a)**

We have,

$$f'(a^+)=\lim_{x\to a^+}\frac{f(x)-f(0)}{x-a}$$

$$\Rightarrow f'(a^+)=\lim_{x\to a^+}\frac{|x-a|\phi(x)}{x-a}$$

$$\Rightarrow f'(a^+)=\lim_{x\to a}\frac{(x-a)}{(x-a)}\phi(x)\quad [\because x>a\ \therefore |x-a|=x-a]$$

$$\Rightarrow f'(a^+)=\lim_{x\to a}\phi(x)$$

$\Rightarrow f'(a^+)=\phi(a)$ [$\because \phi(x)$ is continuous at $x=a$]

and,

$$f'(a^-)=\lim_{x\to a^-}\frac{f(x)-f(0)}{x-a}$$

$$\Rightarrow f'(a^-)=\lim_{x\to a^-}\frac{|x-a|\phi(x)}{x-a}$$

$$\Rightarrow f'(a^-)=\lim_{x\to a}\frac{(x-a)\phi(x)}{(x-a)}\quad [\because x<a\ \therefore |x-a|=-(x-a)]$$

$$\Rightarrow f'(a^-)=-\lim_{x\to a}\phi(x)$$

$\Rightarrow f'(a^-)=-\phi(a)$ [$\because \phi(x)$ is continuous at $x=a$]

4 **(b)**

LHL= $\lim_{h\to 0}(0-h)_e{}^{-\left(\frac{1}{|-h|}+\frac{1}{(-h)}\right)}=\lim_{h\to 0}(-h)=0$

RHL= $\lim_{h\to 0}(0+h)_e{}^{-\left(\frac{1}{|h|}+\frac{1}{(h)}\right)}=\lim_{h\to 0}\frac{h}{e^{2/h}}=0$

LHL=RHL= $f(0)$

Therefore, $f(x)$ is continuous for all x

Differentiability at $x=0$

$$Lf'(0)=\lim_{h\to 0}\frac{(-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}}{(-h)-0}=1$$

$$Rf'(0)=\lim_{h\to 0}\frac{he^{-\left(\frac{1}{h}+\frac{1}{h}\right)-0}}{h-0}$$

$$=\lim_{h\to 0}\frac{1}{e^{2/h}}=0$$

$\Rightarrow\ Rf'(0)Lf'(0)$

Therefore, $f(x)$ is not differentiable at $x=0$

5 **(d)**

We have,

$$f(x)=\begin{cases}3, & x<0\\ 2x+1, & x\geq 0\end{cases}$$

Clearly, f is continuous but not differentiable at $x=0$

Now,

$f(|x|) = 2|x| + 1$ for all x

Clearly, $f(|x|)$ is everywhere continuous but not differentiable at $x = 0$

7 **(c)**

We have,

$f(x) = |x - 0.5| + |x - 1| + \tan x, 0 < x < 2$

$$\Rightarrow f(x) = \begin{cases} -2x + 1.5 + \tan x, & 0 < x < 0.5 \\ 0.5 + \tan x, & 0.5 \leq x < 1 \\ 2x - 1.5 + \tan x, & 1 \leq x < 2 \end{cases}$$

It is evident from the above definition that

$Lf'(0.5) \neq Rf'(0.5)$ and $Lf'(1) \neq Rf'(1)$

Also, the function is not continuous at $= \pi/2$. So, it cannot be differentiable thereat

8 **(d)**

Given, $f(x) = \begin{cases} \log_{(1-3x)}(1+3x), \text{for } x \neq 0 \\ k, \quad \text{for } x = 0 \end{cases}$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\log(1+3x)}{\log(1-3x)}$$

$$= -\lim_{x\to 0} \frac{\log(1+3x)}{3x} \cdot \frac{(-3x)}{\log(1-3x)}$$

$= -1$

And $f(0) = k$

$\because f(x)$ is continuous at $x = 0$

$\therefore \quad k = -1$

9 **(d)**

Since $f(x)$ is differentiable at $x = c$. Therefore, it is continuous at $x = c$

Hence, $\lim_{x\to c} f(x) = f(c)$

10 **(a)**

Given, $f(x) = ae^{|x|} + b\,|x|^2$

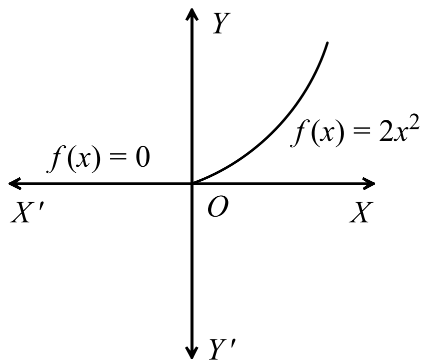
We know $e^{|x|}$ is not differentiable at $x = 0$ and $|x|^2$ is differentiable at $x = 0$

$\therefore \quad f(x)$ is differentiable at $x = 0$, if $a = 0$ and $b \in R$

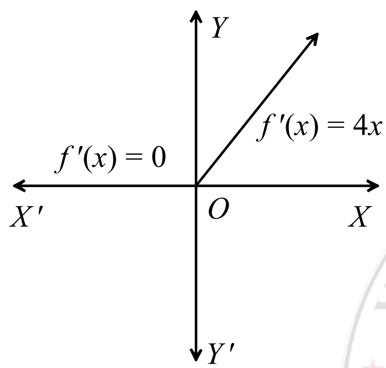
11 **(a)**

We have,

$$f(x) = \begin{cases} (x - x)(-x) = 0, x < 0 \\ (x + x)x = 2x^2, x \geq 0 \end{cases}$$



(i)



(ii)

As is evident from the graph of $f(x)$ that it is continuous and differentiable for all x
Also, we have
$$f''(x) = \begin{cases} 0, x < 0 \\ 4x, x \geq 0 \end{cases}$$
Clearly, $f''(x)$ is continuous for all x but it is not differentiable at $x = 0$

12 **(b)**

Given, $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & x \neq 1 \\ -\frac{1}{3}, & x = 1 \end{cases}$

$$f(x) = \begin{cases} \frac{1}{2x-5}, & x \neq 1 \\ -\frac{1}{3}, & x = 1 \end{cases}$$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2(1+h)-5} - \left(-\frac{1}{3}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2h-3} + \frac{1}{3}}{h} = \lim_{h \to 0} \frac{3 + 2h - 3}{3h(2h-3)} = -\frac{2}{9}$$

$$Lf'(1) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2(1-h)-5} - \left(-\frac{1}{3}\right)}{-h}$$

$$= \lim_{h\to 0} -\frac{2}{3(2h+3)} = -\frac{2}{9}$$

$$\therefore \ f'(1) = -\frac{2}{9}$$

13 **(b)**

$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{f(1+h)}{h} - \lim_{h\to 0} \frac{f(1)}{h}$$

Given, $\lim_{h\to 0} \frac{f(1+h)}{h} = 5$

So, $\lim_{h\to 0} \frac{f(1)}{h}$ must be finite as $f'(1)$ exist and $\lim_{h\to 0} \frac{f(1)}{h}$ can be finite only, if $f(1) = 0$ and $\lim_{h\to 0} \frac{f(1)}{h} = 0$

So, $f'(1) = \lim_{h\to 0} \frac{f(1+h)}{h} = 5$

14 **(c)**

Since, $f(x)$ is continuous for every value of R except {-1, -2}. Now, we have to check that points

At $x = -2$

LHL$= \lim_{h\to 0} \frac{(-2-h)+2}{(-2-h)^2+3(-2-h)+2}$

$$= \lim_{h\to 0} \frac{-h}{h^2+h} = -1$$

RHL$= \lim_{h\to 0} \frac{(-2+h)+2}{(-2+h)^2+3(-2+h)+2}$

$$= \lim_{h\to 0} \frac{h}{h^2-h} = -1$$

$\Rightarrow$ LHL=RHL=$f(-2)$

$\therefore$ It is continuous at $x = -2$

Now, check for $x = -1$

LHL$= \lim_{h\to 0} \frac{(-1-h)+2}{(-1-h)^2+3(-1-h)+2}$

$$= \lim_{h\to 0} \frac{1-h}{h^2-h} = \infty$$

RHL$= \lim_{h\to 0} \frac{(-1+h)+2}{(-1+h)^2+3(-1+h)+2}$

$$= \lim_{h\to 0} \frac{1+h}{h^2+h} = \infty$$

$\Rightarrow$ LHL=RHL$\neq f(-1)$

$\therefore$ It is not continuous at $x = -1$

The required function is continuous in $R - \{-1\}$

15 **(d)**

$$f(0) = \lim_{x\to 0} \frac{(e^x-1)^2}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x}{4}\right)}$$

$$\Rightarrow \qquad \lim_{x\to 0} \left(\frac{e^x-1}{x}\right)^2 . \frac{\frac{x}{a}.a}{\sin\frac{x}{a}} . \frac{\frac{x}{4}.4}{\log\left(1+\frac{x}{4}\right)} = 12$$

$\Rightarrow \quad 1^2.a.4 = 12$

$\Rightarrow \quad a = 3$

16 **(b)**

We have,

$f(x) = \frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \cdots \infty$

$\Rightarrow f(x) = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{x}{\left((r-1)x+1\right)(rx+1)}$, for $x \neq 0$

$\Rightarrow f(x) = \lim_{n\to\infty} \sum_{r=1}^{n} \left\{\frac{1}{(r-1)x+1} - \frac{1}{rx+1}\right\}$, for $x \neq 0$

$\Rightarrow f(x) = \lim_{n\to\infty} \left\{1 - \frac{2}{nx+1}\right\} = 1$, for $x \neq 0$

For $x = 0$, we have $f(x) = 0$

Thus, we have $f(x) = \begin{cases} 1, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Clearly, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) \neq f(0)$

So, $f(x)$ is not continuous at $x = 0$

17 **(b)**

If possible, let $f(x) + g(x)$ be continuous. Then, $\{f(x) + g(x)\} - f(x)$ must be continuous

$\Rightarrow g(x)$ must be continuous

This is a contradiction to the given fact that $g(x)$ is discontinuous

Hence, $f(x) + g(x)$ must be discontinuous

18 **(c)**

We have,

$f(x+y) = f(x)f(y)$ for all $x, y \in R$

$\therefore f(0) = f(0)f(0)$

$\Rightarrow f(0)\{f(0) - 1\} = 0$

$\Rightarrow f(0) = 1 \quad [\because f(0) \neq 1]$

Now,

$f'(0) = 0$

$\Rightarrow \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = 2$

$\Rightarrow \lim_{h\to 0} \frac{f(h)-1}{h} = 2 \quad [\because f(0) = 1] \quad$ ....(i)

$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(x)f(h) - f(x)}{h} \quad [\because f(x+y) = f(x)f(y)]$

$\Rightarrow f'(x) = f(x) \left\{\lim_{h\to 0} \frac{f(h)-1}{h}\right\} = 2f(x) \quad$ [Using (i)]

19 **(b)**

We have,

$$f(x) = \begin{cases} \dfrac{x^2}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \dfrac{x^2}{2} = x, & x > 0 \\ 0, & x = 0 \\ \dfrac{x^2}{-x} = -x, & x < 0 \end{cases}$$

$\Rightarrow \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0} -x = 0$, $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0} x = 0$ and $f(0) = 0$

So, $f(x)$ is continuous at $x = 0$. Also, $f(x)$ is continuous for all other values of x

Hence, $f(x)$ is everywhere continuous

Clearly, $Lf'(0) = -1$ and $Rf'(0) = 1$

Therefore, $f(x)$ is not differentiable at $x = 0$

20 **(b)**

Since $f(x)$ is continuous at $x = 0$

$\therefore \lim\limits_{x\to 0} f(x) = f(0) \Rightarrow f(0) = 2$ ...(i)

Now, using L' Hospital's rule, we have

$$\lim_{x\to 0} \frac{\int_0^x f(u)\ du}{x} = \lim_{x\to 0} \frac{f(x)}{1} = f(0) \quad [\because f(x) \text{ is continuous at } x = 0]$$

$$\Rightarrow \lim_{x\to 0} \frac{\int_0^x f(u)\, du}{x} = 2 \qquad \text{[Using (i)]}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	A	B	D	A	C	D	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	B	C	D	B	B	C	B	B

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 5

Topic :- CONTINUITY AND DIFFERENTIABILITY

2 **(c)**

$f'(2^+) = \lim_{x\to 2^+}\left(\frac{f(x)-f(2)}{x-2}\right)$

$= \lim_{x\to 2^+}\frac{3x+4-(6+4)}{x-2} = \lim_{x\to 2}\frac{3x-6}{x-2} = 3$

3 **(a)**

Here, $f(x) = \begin{cases} \sin x, x > 0 \\ 0,\ x = 0 \\ -\sin x,\ x < 0 \end{cases}$

RHD$= \lim_{h\to 0}\frac{\sin|0+h|-\sin(0)}{h}$

$= \lim_{h\to 0}\frac{\sin h}{h} = 1$

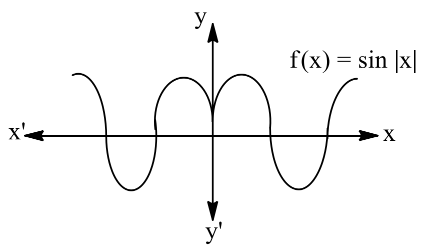
LHD$= \lim_{h\to 0}\frac{\sin|(0-h)|-\sin(0)}{-h}$

$= \frac{-\sin h}{h} = -1$

$\therefore$ LHD$\neq$RHD at $x = 0$

$\therefore$ $f(x)$ is not derivable at $x = 0$

Alternate



It is clear from the graph that $f(x)$ is not differentiable at $x = 0$

4 **(b)**

We have,

$f(x) = \sum_{n=0}^{\infty}\frac{x^n}{n!}(\log_e a)^n$

$\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{(x \log_e a)^n}{n!} = \sum_{n=0}^{\infty} \frac{(\log_e a^x)^n}{n!}$

$\Rightarrow f(x) = e^{\log_e a^x} = a^x$, which is everywhere continuous and differentiable

5 **(c)**

$f(x) = [x] \cos \left[\frac{2x-1}{2}\right] \pi$

Since, $[x]$ is always discontinuous at all integer value, hence $f(x)$ is discontinuous for all integer value

6 **(c)**

The function f is clearly continuous for $|x| > 1$

We observe that

$\lim_{x \to -1^+} f(x) = 1, \lim_{x \to -1^-} f(x) = \frac{1}{4}$

Also, $\lim_{x \to \frac{1}{n}^+} f(x) = \frac{1}{n^2}$ and, $\lim_{x \to \frac{1}{n}^-} f(x) = \frac{1}{(n+1)^2}$

Thus, f is discontinuous for $x = \pm \frac{1}{n}, n = 1, 2, 3, \ldots$

7 **(c)**

Since, $|f(x) - f(y)| \leq (x-y)^2$

$\Rightarrow \lim_{x \to y} \frac{|f(x) - f(y)|}{|x - y|} \leq \lim_{x \to y} |x - y|$

$\Rightarrow |f'(y)| \leq 0$

$\Rightarrow f'(y) = 0$

$\Rightarrow f(y) =$constant

$\Rightarrow f(y) = 0 \quad \Rightarrow \quad f(1) = 0 \quad [\because \; f(0) = 0, \text{ given}]$

8 **(b)**

Since $\phi(x) = 2x^3 - 5$ is an increasing function on $(1, 2)$ such that $\phi(1) = -3$ and $\phi(2) = 11$

Clearly, between -3 and 11 there are thirteen points where $f(x) = [2x^3 - 5]$ is discontinuous

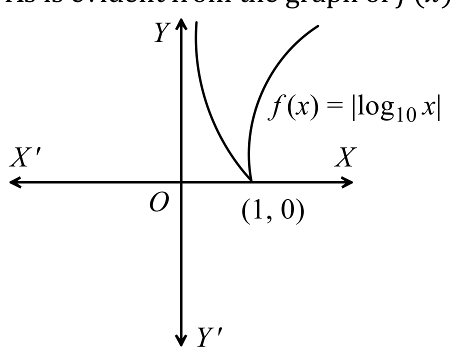
9 **(c)**

Clearly, $[x^2 + 1]$ is discontinuous at $x = \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}$

Note that it is right continuous at $x = 1$ but not left continuous at $x = 3$

10 **(a)**

As is evident from the graph of $f(x)$ that it is continuous but not differentiable at $x = 1$



Now,

$$f''(1^+) = \lim_{x\to1^+} \frac{f(x)-f(1)}{x-1}$$
$$\Rightarrow f''(1^+) = \lim_{h\to0} \frac{f(1+h)-f(1)}{h}$$
$$\Rightarrow f''(1^+) = \lim_{h\to0} \frac{\log_{10}(1+h)-0}{h}$$
$$\Rightarrow f''(1^+) = \lim_{h\to0} \frac{\log(1+h)}{h.\log_e 10} = \frac{1}{\log_e 10} = \log_{10} e$$
$$f''(1^-) = \lim_{x\to1^+} \frac{f(x)-f(1)}{x-1}$$
$$\Rightarrow f''(1^-) = \lim_{h\to0} \frac{f(1-h)-f(1)}{h}$$
$$\Rightarrow f''(1^-) = \lim_{h\to0} \frac{\log_{10}(1-h)}{h} = \lim_{h\to0} \frac{\log_e(1-h)}{h\log_e 10} = -\log_{10} e$$

11 **(b)**

It can be easily seen from the graph of $f(x) = |\cos x|$ that it is everywhere continuous but not differentiable at odd multiples of $\pi/2$

12 **(d)**

We have,

$$\lim_{x\to4^-} f(x) = \lim_{h\to0} f(4-h) = \lim_{h\to0} \frac{4-h-4}{|4-h-4|} + a$$
$$\Rightarrow \lim_{x\to4^-} f(x) = \lim_{h\to0} -\frac{h}{h} + a = a-1$$
$$\Rightarrow \lim_{x\to4^-} f(x) = \lim_{h\to0} f(4+h) = \lim_{h\to0} \frac{4+h-4}{|4+h-4|} + b = b+1$$

and, $f(4) = a+b$

Since $f(x)$ is continuous at $x=4$. Therefore,

$$\lim_{x\to4^-} f(x) = f(4) = \lim_{x\to4^+} f(x)$$
$$\Rightarrow a-1 = a+b = b+1 \Rightarrow b = -1 \text{ and } a = 1$$

13 **(b)**

We have,

$$f(x) = \begin{cases} \dfrac{2^x-1}{\sqrt{1+x}-1}, -1 \le x < \infty, & x \ne 0 \\ k, & x = 0 \end{cases}$$

Since, $f(x)$ is continuous everywhere

$\therefore \quad \lim_{x\to0^-} f(x) = f(0) \quad$ ...(i)

Now, $\lim_{x\to0^-} f(x) = \lim_{h\to0} \frac{2^{(0-h)}-1}{\sqrt{1+(0-h)}-1}$

$$= \lim_{h\to0} \frac{2^{-h}-1}{\sqrt{1-h}-1}$$
$$= \lim_{h\to0} \frac{-2^{-h}\log_e 2}{\frac{-1}{2\sqrt{1-h}}} \text{ [by L' Hospital's rule]}$$
$$= 2\lim_{h\to0} 2^{-h}\log_e 2\sqrt{1-h}$$

$= 2\log_e 2$

From Eq. (i),

$f(0) = 2\log_e 2 = \log_e 4$

15 **(b)**

We have,

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(-h) = \lim_{h\to 0} \frac{e^{-1/h} - 1}{e^{-1/h} + 1} = -1$$

and,

$$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(h) = \lim_{x\to 0} \frac{e^{1/h} - 1}{e^{1/h} + 1} = \lim_{h\to 0} \frac{e^{-1/h}}{e^{-1/h}} = 1$$

$\therefore \lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

Hence, $f(x)$ is not continuous at $x = 0$

16 **(c)**

LHL$= \lim_{x\to 2^-} f(x) = \lim_{h\to 0} 1 + (2 - h) = 3$

RHL$= \lim_{x\to 2^+} f(x) = \lim_{h\to 0} 5 - (2 + h) = 3,\quad f(2) = 3$

Hence, f is continuous at $x = 2$

Now, $Rf''(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$

$$= \lim_{h\to 0} \frac{5 - (2 + h) - 3}{h} = -1$$

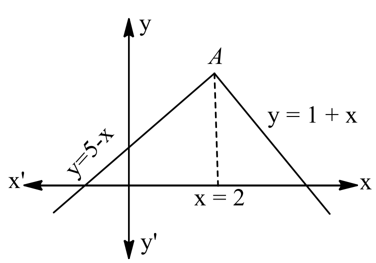
$$Lf''(2) = \lim_{h\to 0} \frac{f(2 - h) - f(2)}{-h}$$

$$= \lim_{h\to 0} \frac{1 + (2 - h) - 3}{-h} = 1$$

$\therefore\ Rf''(2) \neq Lf''(2)$

$\therefore\ f$ is not differentiable at $x = 2$

Alternate



It is clear from the graph that $f(x)$ is continuous everywhere also it is differentiable everywhere except at $x = 2$

17 **(d)**

We have,

$f(x + y) = f(x)f(y)$ for all $x, y \in R$

Putting $x = 1, y = 0$, we get

$f(0) = f(1)f(0) \Rightarrow f(0)\big(1 - f(1)\big) = 0$

$\Rightarrow f(1) = 1 \qquad [\because f(0) \neq 0]$

Now,

$f'(1) = 2$

$\Rightarrow \lim_{h\to 0} \dfrac{f(1+h) - f(1)}{h} = 2$

$\Rightarrow \lim_{h\to 0} \dfrac{f(1)f(h) - f(1)}{h} = 2$

$\Rightarrow f(1) \lim_{h\to 0} \dfrac{f(h) - 1}{h} = 2$

$\Rightarrow \lim_{h\to 0} \frac{f(h)-1}{h} = 2$ [Using $f(1) = 1$] ...(i)

$\therefore f'(4) = \lim_{h\to 0} \dfrac{f(4+h) - f(4)}{h}$

$\Rightarrow f'(4) = \lim_{h\to 0} \dfrac{f(4)f(h) - f(4)}{h}$

$\Rightarrow f'(4) = \left\{\lim_{h\to 0} \dfrac{f(h) - 1}{h}\right\} f(4)$

$\Rightarrow f'(4) = 2\, f(4)$ [From (i)]

$\Rightarrow f'(4) = 2 \times 4 = 8$

18 **(d)**

We have,

$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^+} g(x) = 1$ and $g(1) = 0$

So, $g(x)$ is not continuous at $x = 1$ but $\lim_{x\to 1} g(x)$ exists

We have,

$\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0} [1-h] = 0$

and,

$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0} [1+h] = 1$

So, $\lim_{x\to 1} f(x)$ does not exist and so $f(x)$ is not continuous at $x = 1$

We have, $gof(x) = g\big(f(x)\big) = g([x]) = 0$, for all $x \in R$

So, gof is continuous for all x

We have,

$fog(x) = f\big(g(x)\big)$

$\Rightarrow fog(x) = \begin{cases} f(0), & x \in Z \\ f(x^2), & x \in R - Z \end{cases}$

$\Rightarrow fog(x) = \begin{cases} 0, & x \in Z \\ [x^2], & x \in R - Z \end{cases}$

Which is clearly not continuous

19 **(d)**

At $x = 1$,

RHD$= \lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h}$

$= \lim_{h\to 0} \dfrac{2 - (1+h) - (2-1)}{h} = -1$

LHD$= \lim_{h\to 0^-} \frac{f(1-h)-f(1)}{-h}$

$$= \lim_{h\to 0} \frac{(1-h)-(2-1)}{-h} = 1$$

$\therefore$ LHD≠RHD

20 **(d)**

Given, $f(x) = |x| + \frac{|x|}{x}$

Let $f_1(x) = |x|,\ f_2(x) = \frac{|x|}{x}$

1. LHL= $\lim_{x\to 0^-} f_1(x) = \lim_{x\to 0^-} |x| = 0$

And RHL $\lim_{x\to 0^+} f_1(x) = \lim_{x\to 0^+} |x| = 0$

Here, LHL=RHL=$f(0)$, $f_1(x)$ is continuous

2. LHL= $\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{h\to 0} \frac{|0-h|}{0-h} = -1$

RHL= $\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{h\to 0} \frac{|0+h|}{h} = 1$

$\therefore$ LHL≠RHL, $f_2(x)$ is discontinuous

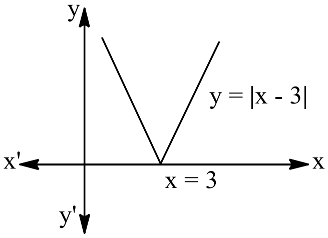
Hence, $f(x)$ is discontinuous at $x = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	A	B	C	C	C	B	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	B	C	B	C	D	D	D	D

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(a)**

From the graph it is clear that $f(x)$ is continuous everywhere but not differentiable at $x = 3$



2 **(b)**

Given, $f(x) = \begin{cases} \frac{2x-3}{2x-3}, & \text{if } x > \frac{3}{2} \\ \frac{-(2x-3)}{2x-3}, & \text{if } x < \frac{3}{2} \end{cases}$

$= \begin{cases} 1, & \text{if } x > \dfrac{3}{2} \\ -1, & \text{if } x < \dfrac{3}{2} \end{cases}$

Now, RHL$= \lim_{x \to \frac{3}{2}^+} f(x) = \lim_{x \to \frac{3}{2}^+} 1 = 1$

And LHL$= \lim_{x \to \frac{3}{2}^-} f(x) = \lim_{x \to \frac{3}{2}^-} (-1) = -1$

$\because$ RHL$\neq$LHL

$\therefore$ $f(x)$ is discontinuous at $x = \frac{3}{2}$

3 **(c)**

Since the functions $(\log t)^2$ and $\frac{\sin t}{t}$ are not defined on $(-1, 2)$. Therefore, the functions in options (a) and (b) are not defined on $(-1, 2)$

The function $g(t) = \frac{1-t+t^2}{1+t+t^2}$ is continuous on $(-1, 2)$ and

$f(x) = \int_0^x \frac{1-t+t^2}{1+t+t^2} dt$ is the integral function of $g(t)$

Therefore, $f(x)$ is differentiable on $(-1, 2)$ such that $f'(x) = g(x)$

4 **(c)**

Since, $f(x) = \frac{1-\tan x}{4x-\pi}$

Now, $\lim_{x \to \pi/4} f(x) = \lim_{x \to \pi/4} \left(\frac{1-\tan x}{4x-\pi}\right)$

$= \lim_{x\to\pi/4}\left(\frac{-\sec^2 x}{4}\right) = -\frac{1}{2}$

Since, $f(x)$ is continuous at

$x = \frac{\pi}{4}$

$\therefore \lim_{x\to\pi/4} f(x) = f\left(\frac{\pi}{4}\right) = -\frac{1}{2}$

5 **(a)**

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1-\cos x}{x} = \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}.x = 0$$

Also, $f(0) = k$

For, $\lim_{x\to 0} f(x) = f(0) \Rightarrow k = 0$

6 **(a)**

We have,

$f(x) = |x| + |x-1|$

$$\Rightarrow f(x) = \begin{cases} -2x+1, & x<0 \\ x-x+1, & 0\le x<1 \\ x+x-1, & x\ge 1\end{cases}$$

$$\Rightarrow f(x) = \begin{cases} -2x+1, & x<0 \\ 1, & 0\le x<1 \\ 2x-1, & x\ge 1\end{cases}$$

Clearly, $\lim_{x\to 0^-} f(x) = 1 = \lim_{x\to 0^+} f(x)$ and $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

So, $f(x)$ is continuous at $x = 0, 1$

7 **(d)**

$$f(0) = \lim_{x\to 0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$$

$$= \lim_{x\to 0}\frac{2-\frac{\sin^{-1}x}{x}}{2+\frac{\tan^{-1}x}{x}}$$

$$= \frac{2-1}{2+1} = \frac{1}{3}$$

9 **(b)**

$$f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{1+h-1}{2(1+h)^2-7(1+h)+5} - \left(\frac{1}{3}\right)}{h}$$

$$= \lim_{h\to 0}\frac{\left(\frac{1}{2h-3}+\frac{1}{3}\right)}{h} = \lim_{h\to 0}\left(\frac{2h}{3h(2h-3)}\right) = -\frac{2}{9}$$

10 **(a)**

LHL$= \lim_{h\to 0} f\left(-\frac{\pi}{2}-h\right) = \lim_{h\to 0} 2\cos\left(-\frac{\pi}{2}-h\right) = 0$

RHL$= \lim_{h\to 0} f\left(-\frac{\pi}{2}+h\right) = \lim_{h\to 0} 2\, a \sin\left(-\frac{\pi}{2}+h\right) + b$

$$= -a + b$$

Since, function is continuous.

$\therefore$ RHL=LHL $\Rightarrow$ $a = b$

From the given options only (a) ie, $\left(\frac{1}{2}, \frac{1}{2}\right)$ satisfies this condition

11 **(a)**

We have,

$f'(0) = 3$

$\Rightarrow \lim_{x\to 0} \dfrac{f(x) - f(0)}{x - 0} = 3$

$\Rightarrow \lim_{h\to 0} \dfrac{f(0+h) - f(0)}{h} = 3$ [Using: (RHD at $x = 0$) $= 3$]

$\Rightarrow \lim_{h\to 0} \dfrac{f(0)f(h) - f(0)}{h} = 3$ $\begin{bmatrix} \because f(x+y) = f(x)f(y) \\ \therefore f(0+h) = f(0)f(h) \end{bmatrix}$

$\Rightarrow f(0)\left(\lim_{h\to 0} \frac{f(h)-1}{h}\right) = 3$ ...(i)

Now, $f(x+y) = f(x)f(y)$ for all $x, y \in R$

$\Rightarrow f(0) = f(0)f(0)$

$\Rightarrow f(0)\{1 - f(0)\} = 0 \Rightarrow f(0) = 1$

Putting $f(0) = 1$ in (i), we get

$\lim_{h\to 0} \frac{f(h)-1}{h} = 3$ ...(ii)

Now,

$f'(5) = \lim_{h\to 0} \dfrac{f(5+h) - f(5)}{h}$

$\Rightarrow f'(5) = \lim_{h\to 0} \dfrac{f(5)f(h) - f(5)}{h}$

$\Rightarrow f'(5) = \left\{\lim_{h\to 0} \frac{f(h)-1}{h}\right\} f(5) = 3 \times 2 = 6$ [Using (ii)]

12 **(c)**

We have,

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$\Rightarrow f(x)' = \lim_{h\to 0} \dfrac{f(x) + f(h) - f(x)}{h}$

$\Rightarrow f(x)' = \lim_{h\to 0} \dfrac{f(h)}{h}$

$\Rightarrow f(x)' = \lim_{h\to 0} \dfrac{h\, g\,(h)}{h} \lim_{h\to 0} g(h) = g(0)$ [$\because g$ is conti. at $x = 0$]

13 **(b)**

The domain of $f(x)$ is $[2, \infty)$

We have,

$$f(x) = \sqrt{\frac{\left(\sqrt{2x-4}\right)^2}{2} + 2 + 2\sqrt{2x-4}}$$

$$+\sqrt{\frac{\left(\sqrt{2x-4}\right)^2}{2}+2-2\sqrt{2x-4}}$$

$$\Rightarrow f(x)=\frac{1}{\sqrt{2}}\sqrt{\left(\sqrt{2x-4}\right)^2+4\sqrt{2x-4}+4}$$

$$+\frac{1}{\sqrt{2}}\sqrt{\left(\sqrt{2x-4}\right)^2-4\sqrt{2x-4}+4}$$

$$\Rightarrow f(x)=\frac{1}{\sqrt{2}}\left|\sqrt{2x-4}+2\right|+\frac{1}{\sqrt{2}}\left|\sqrt{2x-4}-2\right|$$

$$\Rightarrow f(x)=\begin{cases}\frac{1}{\sqrt{2}}\times 4, & \text{if } \sqrt{2x-4}<2\\ \sqrt{2}\cdot\sqrt{2x-4}, & \text{if } \sqrt{2x-4}\geq 2\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}2\sqrt{2}, & \text{if } x\in[2,4)\\ 2\sqrt{x-2}, & \text{if } x\in[4,\infty)\end{cases}$$

Hence, $f'(x)=\begin{cases}0 & \text{if } x\in[2,4)\\ \frac{1}{\sqrt{x-2}} & \text{if } x\in(4,\infty)\end{cases}$

14 **(c)**

We have,

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{x^2\sin\left(\frac{1}{x}\right)}{x}=\lim_{x\to 0}x\sin\frac{1}{x}=0$$

So, $f(x)$ is differentiable at $x=0$ such that $f'(0)=0$

For $x\neq 0$, we have

$$f'(x)=2x\sin\left(\frac{1}{x}\right)+x^2\cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow f'(x)=2x\sin\frac{1}{x}-\cos\frac{1}{x}$$

$$\Rightarrow \lim_{x\to 0}f'(x)=\lim_{x\to 0}2x\sin\frac{1}{x}-\lim_{x\to 0}\cos\left(\frac{1}{x}\right)=0-\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$$

Since $\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$ does not exist

$\therefore \lim_{x\to 0}f'(x)$ does not exist

Hence, $f'(x)$ is not continuous at $x=0$

15 **(c)**

We have,

$$f(x)=\begin{cases}\frac{x}{\sqrt{x^2}}, & x\neq 0\\ 0, & x=0\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}\frac{x}{|x|}, & x\neq 0\\ 0, & x=0\end{cases}=\begin{cases}1, & x>0\\ -1, & x<0\\ 0, & x=0\end{cases}$$

CLearly, $f(x)$ is not continuous at $x=0$

17 **(c)**

Given, $\lim_{x\to 0}\left[(1+3x)^{\frac{1}{x}}\right]=k$

$\therefore \quad e^3 = k$

18 **(b)**

For $x > 2$, we have

$$f(x) = \int_0^x \{5 + |1 - t|\} dt$$

$$\Rightarrow f(x) = \int_0^1 (5 + (1 - t) dt + \int_1^x (5 - (1 - t) \, dt$$

$$\Rightarrow f(x) = \int_0^1 (6 - t) dt + \int_1^x (4 + t) dt$$

$$\Rightarrow f(x) = \left[6t - \frac{t^2}{2}\right]_0^1 + \left[4t + \frac{t^2}{2}\right]_1^x$$

$$\Rightarrow f(x) = 1 + 4x + \frac{x^2}{2}$$

Thus, we have

$$f(x) = \begin{cases} 5x + 1, & \text{if } x \leq 2 \\ \frac{x^2}{2} + 4x + 1, & \text{if } x > 2 \end{cases}$$

Clearly, $f(x)$ is everywhere continuous and differentiable except possibly at $x = 2$

Now,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2} 5x + 1 = 11$$

and,

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2} \left(\frac{x^2}{2} + 4x + 1\right) = 11$$

$$\therefore \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

So, $f(x)$ is continuous at $x = 2$

Also, we have (LHD at $x = 2$) $= \lim_{x \to 2^-} f'(x) = \lim_{x \to 2} 5 = 5$

19 **(b)**

The given function is clearly continuous at all points except possibly at $x = \pm 1$

For $f(x)$ to be continuous at $x = 1$, we must have

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$$\Rightarrow \lim_{x \to 1} ax^2 + b = \lim_{x \to 1} \frac{1}{|x|}$$

$\Rightarrow a + b = 1 \qquad$ ...(i)

Clearly, $f(x)$ is differentiable for all x, except possibly at $x = \pm 1$. As $f(x)$ is an even function, so we need to check its differentiability at $x = 1$ only

For $f(x)$ to be differentiable at $x = 1$, we must have

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow \lim_{x \to 1} \frac{ax^2 + b - 1}{x - 1} = \lim_{x \to 1} \frac{\frac{1}{|x|} - 1}{x - 1}$$

$\Rightarrow \lim_{x\to 1} \frac{ax^2 - a}{x - 1} = \lim_{x\to 1} \frac{\frac{1}{x} - 1}{x - 1}$ $\quad [\because a + b = 1 \;\therefore b - 1 = -a]$

$\Rightarrow \lim_{x\to 1} a(x + 1) = \lim_{x\to 1} \frac{-1}{x}$

$\Rightarrow 2a = -1 \Rightarrow a = -1/2$

Putting $a = -1/2$ in (i), we get $b = 3/2$

20 **(c)**

At no point, function is continuous

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	C	C	A	A	D	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	B	C	C	C	C	B	B	C

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

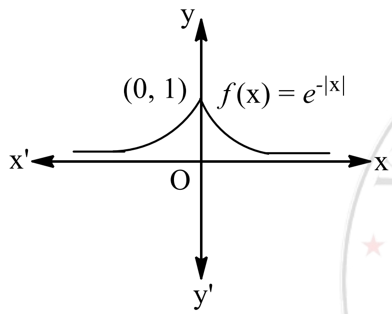
SOLUTIONS

SUBJECT : MATHS
DPP NO. : 7

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(a)**

It is clear from the figure that $f(x)$ is continuous everywhere and not differentiable at $x = 0$ due to sharp edge



2 **(c)**

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}} \times \frac{\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}}{\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}} \times \frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}$$

$\Rightarrow \lim_{x \to 0} f(x)$

$$= \lim_{x \to 0} \frac{-2ax(\sqrt{a+x} + \sqrt{a-x})}{2x\,(\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2})}$$

$$= \frac{-a(2\sqrt{a})}{(a+a)} = -\sqrt{a}$$

3 **(b)**

Given, $f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ k & x = 0 \end{cases}$

LHL= $\lim_{x \to 0^-} f(x)$

$$= \lim_{h \to 0} \frac{1 - \cos 4(0-h)}{8(0-h)^2}$$

$$= \lim_{h \to 0} \frac{1 - \sin 4h}{8h^2}$$

$= \lim_{h \to 0} \frac{4 \sin 4h}{16h} = 1$ [by L 'Hospital's rule]

Since, $f(x)$ is continuous at $x = 0$

$\therefore \quad f(0) =$LHL $\Rightarrow \quad k = 1$

4 **(d)**

Given, $f(x) = |x-1| + |x-2| + \cos x$

Since, $|x-1|, |x-2|$ and $\cos x$ are continuous in $[0, 4]$

$\therefore$ $f(x)$ being sum of continuous functions is also continuous

5 **(c)**

If function $f(x)$ is continuous at $x = 0$, then

$f(0) = \lim_{x\to 0} f(x)$

$\therefore \quad f(0) = k = \lim_{x\to 0} x \sin\frac{1}{x}$

$\Rightarrow k = 0 \qquad \left[\because -1 \le \sin\frac{1}{x} \le 1\right]$

6 **(b)**

We have,

$h(x) = \{f(x)\}^2 + \{g(x)\}^2$

$\Rightarrow h'(x) = 2f(x)2f'(x) + 2\, g(x)\, g'(x)$

Now,

$f'(x) = g(x)$ and $f''(x) = -f(x)$

$\Rightarrow f''(x) = g'(x)$ and $f''(x) = -f(x)$

$\Rightarrow -f(x) = g'(x)$

Thus, we have

$f'(x) = g(x)$ and $g'(x) = -f(x)$

$\therefore h'(x) = -2\, g(x)g'(x) + 2\, g\,(x)g'(x) = 0$, for all x

$\Rightarrow h(x) =$ Constant for all x

But, $h(5) = 11$. Hence, $h(x) = 11$ for all x

7 **(a)**

$f(x) = |x|^3 = \begin{cases} 0, & x = 0 \\ x^3, & x > 0 \\ -x^3, & x < 0 \end{cases}$

Now, $Rf'(0) = \lim_{h\to 0} \frac{h^3-0}{h} = 0$

And $Lf'(0) = \lim_{h\to 0} \frac{-h^3-0}{-h} = 0$

$\because \quad Rf'(0) = Lf'(0) = 0$

$\therefore \quad f'(0) = 0$

8 **(b)**

We have,

(LHL at $x = 0$) $= \lim_{n\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$\Rightarrow$ (LHL at $x = 0$) $= \lim_{n\to 0} \sin^{-1}(\cos\ (-h)) = \lim_{h\to 0} \sin^{-1}(\cosh h)$

$\Rightarrow$ (LHL at $x = 0$) $= \sin^{-1} 1 = \pi/2$

(RHL at $x = 0$) $= \lim_{x\to 0^+} f(x)$

$\Rightarrow$ (RHL at $x = 0$) $= \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \sin^{-1}(\cos h)$

$\Rightarrow$ (RHL at $x = 0$) $= \sin^{-1}(1) = \pi/2$

and, $f(0) = \sin^{-1}(\cos 0) = \sin^{-1}(1) = \pi/2$

$\therefore$ (LHL at $x=0$) = (RHL at $x=0$) = $f(0)$

So, $f(x)$ is continuous at $x=0$

Now,

$$f'(x)=\frac{-\sin x}{\sqrt{1-\cos^2 x}}=\frac{\sin x}{|\sin x|}=\begin{cases}\dfrac{-\sin x}{-\sin x}=1, x<0\\ \dfrac{-\sin x}{\sin x}=-1, x>0\end{cases}$$

$\therefore$ (LHD at $x=0$) = 1 and (RHD at $x=0$) = -1

Hence, $f(x)$ is not differentiable at $x=0$

9 **(d)**

For any $x\neq 1,2$, we find that $f(x)$ is the quotient of two polynomials and a polynomial is everywhere continuous. Therefore, $f(x)$ is continuous for all $x\neq 1,2$

Continuity at $x=1$:

We have,

$$\lim_{x\to 1^-} f(x)=\lim_{h\to 0} f(1-h)$$

$$\Rightarrow \lim_{x\to 1^-} f(x)=\lim_{h\to 0}\frac{(1-h-2)(1-h+2)(1-h+1)(1-h-1)}{|(1-h-1)(1-h-2)|}$$

$$\Rightarrow \lim_{x\to 1^-} f(x)=\lim_{h\to 0}\frac{(3-h)(2-h)(-1-h)(-h)}{|(-h)(-1-h)|}$$

$$\Rightarrow \lim_{x\to 1^-} f(x)=\lim_{h\to 0}\frac{(3-h)(2-h)h(h+1)}{h(h+1)}=6$$

and,

$$\lim_{x\to 1^+} f(x)=\lim_{h\to 0} f(1+h)$$

$$\lim_{x\to 1^+} f(x)=\lim_{h\to 0}\frac{(1+h-2)(1+h+2)(1+h+1)(1+h-1)}{|(1+h-1)(1+h-2)|}$$

$$\lim_{x\to 1^+} f(x)=\lim_{h\to 0}\frac{(h-1)(3+h)(2+h)(h)}{|h(h-1)|}$$

$$\lim_{x\to 1^+} f(x)=-\lim_{h\to 0}\frac{(h-1)(3+h)(2+h)h}{h(1-h)}=-6$$

$$\therefore \lim_{x\to 1^-} f(x)\neq \lim_{x\to 1^+} f(x)$$

So, $f(x)$ is not continuous at $x=1$

Similarly, $f(x)$ is not continuous at $x=2$

10 **(b)**

Let $f(x)=\frac{g(x)}{h(x)}=\frac{x}{1+|x|}$

It is clear that $g(x)=x$ and $h(x)=1+|x|$ are differentiable on $(-\infty,\infty)$ and $(-\infty,0)\cup(0,\infty)$ respectively

Thus, $f(x)$ is differentiable on $(-\infty,0)\cup(0,\infty)$.Now, we have to check the differentiable at $x=0$

$$\therefore \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{\dfrac{x}{1+|x|}-0}{x}=\lim_{x\to 0}\frac{1}{1+|x|}=1$$

Hence, $f(x)$ is differntaible on $(-\infty,\infty)$

11 **(b)**

At $x=0$,

LHL= $\lim_{h\to 0}\frac{1}{1-e^{-1/(0-h)}}=\lim_{h\to 0}\frac{1}{1-e^{1/h}}=0$

RHL= $\lim_{h\to 0}\frac{1}{1-e^{-1/(0+h)}}=\lim_{h\to 0}\frac{1}{1-e^{-1/h}}=1$

$\therefore$ FUnction is not continuous at $x=0$

12 **(a)**

We have,

$fog=I$

$\Rightarrow fog(x)=x$ for all x

$\Rightarrow f'(g(x))g'(x)=1$ for all x

$$\Rightarrow f'(g(a))=\frac{1}{g'(a)}=\frac{1}{2}\Rightarrow f'(b)=\frac{1}{2}\quad [\because f(a)=b]$$

13 **(a)**

Since, $\lim_{x\to 0} f(x)=f(0)$

$$\Rightarrow \lim_{x\to 0}\frac{\sin \pi x}{5x}=k$$

$$\Rightarrow (1)\frac{\pi}{5}=k \quad\Rightarrow\quad k=\frac{\pi}{5}\qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

14 **(d)**

Given, $f(x)=[x], x\in(-3.5,100)$

As we know greatest integer is discontinuous on integer values.

In given interval, the integer values are

$(-3,-2,-1,0,\dots,99)$

$\therefore$ Total numbers of integers are 103.

15 **(a)**

LHL= $\lim_{h\to 0} f(0-h)$

$=\lim_{h\to 0}\frac{e^{-1/h}-1}{e^{-1/h}+1}=-1\quad \left[\because \lim_{h\to 0}\frac{1}{e^{1/h}}=0\right]$

RHL= $\lim_{h\to 0} f(0+h)=\lim_{h\to 0}\frac{e^{1/h}-1}{e^{1/h}+1}$

$$=\lim_{h\to 0}\frac{1-\frac{1}{e^{1/h}}}{1+\frac{1}{e^{1/h}}}=1$$

$\therefore$ LHL$\neq$RHL

So, limit does not exist at $x=0$

16 **(d)**

We have,

$$f(x)=x^4+\frac{x^4}{1+x^4}+\frac{x^4}{(1+x^4)}+\cdots$$

$$\Rightarrow f(x)=\frac{x^4}{1-\frac{1}{1+x^4}}=1+x^4, \text{if } x\neq 0$$

Clearly, $f(x)=0$ at $x=0$

Thus, we have

$$f(x)=\begin{cases}1+x^4, & x\neq 0\\ 0, & x=0\end{cases}$$

Clearly, $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=1\neq f(0)$

So, $f(x)$ is neither continuous nor differentiable at $x=0$

17 **(a)**

We have,

$$f(x)=\begin{cases}1+x, & 0\le x\le 2\\ 3-x, & 2<x\le 3\end{cases}$$

$\therefore g(x)=fof(x)$

$\Rightarrow f(x)=f\big(f(x)\big)$

$$\Rightarrow g(x)=\begin{cases}f(1+x), & 0\le x\le 2\\ f(3-x), & 2<x\le 3\end{cases}$$

$$\Rightarrow g(x)=\begin{cases}1+(1+x), & 0\le x\le 1\\ 3-(1+x), & 1<x\le 2\\ 1+(3-x), & 2<x\le 3\end{cases}$$

$$\Rightarrow g(x)=\begin{cases}2+x, & 0\le x\le 1\\ 2-x, & 1<x\le 2\\ 4-x, & 2<x\le 3\end{cases}$$

Clearly, $g(x)$ is continuous in $(0,1)\cup(1,2)\cup(2,3)$ except possibly at $x=0,1,2$ and 3

We observe that

$\lim_{x\to 0^+} g(x)=\lim_{x\to 0^+}(2+x)=2=g(0)$

and $\lim_{x\to 3^-} g(x)=\lim_{x\to 3^-} 4-x=1=g(3)$

Therefore, $g(x)$ is right continuous at $x=0$ and left continuous at $x=3$

At $x=1$, we have

$\lim_{x\to 1^-} g(x)=\lim_{x\to 1^-} 2+x=3$

and, $\lim_{x\to 1^+} g(x)=\lim_{x\to 1^+} 2-x=1$

$\therefore \lim_{x\to 1^+} g(x)\neq \lim_{x\to 1^-} g(x)$

So, $g(x)$ is not continuous at $x=1$

At $x=2$, we have

$\lim_{x\to 2^-} g(x)=\lim_{x\to 2^-}(2-x)=0$

and,

$\lim_{x\to 2^+} g(x)=\lim_{x\to 2^+}(4-x)=0$

$\therefore \lim_{x\to 2^-} g(x)\neq \lim_{x\to 2^+} g(x)$

So, $g(x)$ is not continuous at $x=2$

Hence, the set of points of discontinuity of $g(x)$ is $\{1,2\}$

18 **(b)**

Since $g(x)$ is the inverse of function $f(x)$

$\therefore gof(x)=I(x)$, for all x

Now, $gof(x)=I(x)$, for all x

$\Rightarrow gof(x)=x$, for all x

$\Rightarrow (gof)'(x)=1$, for all x

$\Rightarrow g'(f(x))f'(x) = 1$, for all x [Using Chain Rule]

$\Rightarrow g'(f(x)) = \frac{1}{f'(x)}$, for all x

$\Rightarrow g'(f(c)) = \frac{1}{f'(c)}$ [Putting $x = c$]

19 **(d)**

Given, $f(x) = \begin{cases} x^p \cos\left(\frac{1}{x}\right), x \neq 0 \\ 0, \quad x = 0 \end{cases}$

Since, $f(x)$ is differentiable at $x = 0$, therefore it is continuous at $x = 0$

$\therefore \lim_{x \to 0} f(x) = f(0) = 0$

$\Rightarrow \lim_{x \to 0} x^p \cos\left(\frac{1}{x}\right) = 0 \quad \Rightarrow \quad p > 0$

As $f(x)$ is differentiable at $x = 0$

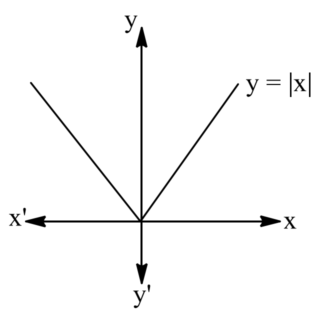
$\therefore \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$ exists finitely

$\Rightarrow \lim_{x \to 0} \frac{x^p \cos\frac{1}{x} - 0}{x}$ exists finitely

$\Rightarrow \lim_{x \to 0} x^{p-1} \cos\frac{1}{x} - 0$ exists finitely

$\Rightarrow \quad p - 1 > 0 \quad \Rightarrow \quad p > 1$

20 **(a)**



It is clear from the graph that $f(x)$ is continuous everywhere and also differentiable everywhere except at $x = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	D	C	B	A	B	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	A	D	A	D	A	B	D	A

SESSION : 2025-26

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 8

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(c)**

We know that the function

$\phi(x) = (x-a)^2 \sin\left(\frac{1}{x-a}\right)$

Is continuous and differentiable at $x = a$ whereas the function $\Psi(x) = |x - a|$ is everywhere continuous but not differentiable at $x = a$

Therefore, $f(x)$ is not differentiable at $x = 1$

2 **(d)**

$\lim_{x\to 0} \frac{2^x - 2^{-x}}{x} = \lim_{x\to 0} 2^x \log 2 + 2^{-x} \log 2$

[by L' Hospital's rule]

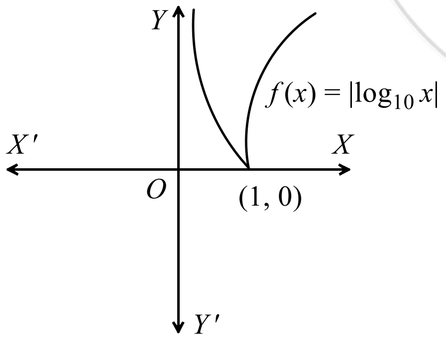
$= \log 4$

Since, the function is continuous at $x = 0$

$\therefore \quad f(0) = \lim_{x\to 0} f(x) \quad \Rightarrow \quad f(0) = \log 4$

3 **(a)**

As is evident from the graph of $f(x)$ that it is continuous but not differentiable at $x = 1$



Now,

$f''(1^+) = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x-1}$

$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$

$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{\log_{10}(1+h) - 0}{h}$

$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{\log(1+h)}{h.\log_e 10} = \frac{1}{\log_e 10} = \log_{10} e$

$$f''(1^-) = \lim_{x\to1^+}\frac{f(x)-f(1)}{x-1}$$
$$\Rightarrow f''(1^-) = \lim_{h\to0}\frac{f(1-h)-f(1)}{h}$$
$$\Rightarrow f''(1^-) = \lim_{h\to0}\frac{\log_{10}(1-h)}{h} = \lim_{h\to0}\frac{\log_e(1-h)}{h\log_e 10} = -\log_{10} e$$

4 **(a)**

We have,

$$f'(x) = \lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$
$$\Rightarrow f'(x) = \lim_{h\to0}\frac{f(x)+f(h)-f(x)}{h} \qquad [\because f(x+y) = f(x)+f(y)]$$
$$\Rightarrow f'(x) = \lim_{h\to0}\frac{f(h)}{h}$$
$$\Rightarrow f'(x) = \lim_{h\to0}\frac{\sin h\ g(h)}{h} = \lim_{h\to0}\frac{\sin h}{h}\lim_{h\to0} g(h) = g(0) = k$$

5 **(a)**

We have,

$$f(x) = |x| + |x-1| = \begin{cases} -2x+1, & x<0 \\ 1, & 0\le x<1 \\ 2x-1, & 1\le x \end{cases}$$

Clearly, $\lim_{x\to1^-} f(x) = \lim_{x\to1} 1 = 1$, $\lim_{x\to1^+} f(x) = \lim_{x\to1}(2x-1) = 1$

and, $f(1) = 2\times1-1 = 1$

$\therefore \lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = f(1)$

So, $f(x)$ is continuous at $x = 1$

Now, $\lim_{x\to1^-}\frac{f(x)-f(1)}{x-1} = \lim_{h\to0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to0}\frac{1-1}{-h} = 0$

and,

$$\lim_{x\to1^+}\frac{f(x)-f(1)}{x-1} = \lim_{h\to0}\frac{f(1+h)-f(1)}{h}$$
$$\Rightarrow \lim_{x\to1^+}\frac{f(x)-f(1)}{x-1} = \lim_{h\to0}\frac{2(1+h)-1-1}{h} = 2$$

$\therefore$ (LHD at $x = 1$) $\neq$ (RHD at $x = 1$)

So, $f(x)$ is not differentiable at $x = 1$

6 **(d)**

The given function is differentiable at all points except possibly at $x = 0$

Now,

(RHD at $x = 0$)

$$= \lim_{h\to0}\frac{f(0+h)-f(0)}{h}$$
$$= \lim_{h\to0}\frac{\sqrt{h+1}-1}{h^{3/2}}$$
$$= \lim_{h\to0}\frac{h}{h^{3/2}(\sqrt{h+1}+1)} = \lim_{h\to0}\frac{1}{\sqrt{h}(\sqrt{h+1}+1)} \to \infty$$

So, the function is not differentiable at $x = 0$

Hence, the required set is $R-\{0\}$

7 **(a)**

We have,

$f(x)\,f(y)=f(x)+f(y)+f(xy)-2$

$\Rightarrow f(x).f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)+f(1)-2$

$\Rightarrow f(x).f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)\quad\left[\begin{array}{c}\because f(1)=2\text{ (Putting } x=y=1\\ \text{in the given relation)}\end{array}\right]$

$\Rightarrow f(x)=x^n+1$

$\Rightarrow f(2)=2^n+1$

$\Rightarrow 5=2^n+1\quad[\because f(2)=5\text{ (given)}]$

$\Rightarrow n=2$

$\therefore f(x)=x^2+1\Rightarrow f(3)=10$

8 **(b)**

We have,

$f(x)=\frac{1}{2}x-1$, for $0\le x\le\pi$

$\therefore \{f(x)\}=\begin{cases}-1, \text{for } 0\le x<2\\ 0, \text{for } 2\le x\le\pi\end{cases}$

$\Rightarrow \tan[f(x)]=\begin{cases}\tan(-1)=-\tan(1)\,, 0\le x<2\\ \tan 0=0, \quad 2\le x\le\pi\end{cases}$

It is evident from the definition of $\tan[f(x)]$ that

$\lim\limits_{x\to 2^-}\tan[f(x)]=-\tan 1$ and, $\lim\limits_{x\to 2^+}\tan[f(x)]=0$

So, $\tan[f(x)]$ is not continuous at $x=2$

Now,

$f(x)=\frac{1}{2}x-1\Rightarrow f(x)=\frac{x-2}{2}\Rightarrow\frac{1}{f(x)}=\frac{2}{x-2}$

Clearly, $f(x)$ is not continuous at $x=2$

So, $\tan[f(x)]$ and $\tan\left[\frac{1}{f(x)}\right]$ both are discontinuous at $x=2$

9 **(c)**

$\lim\limits_{x\to 0}(1+x)^{\cot x}=\lim\limits_{x\to 0}\left\{(1+x)^{\frac{1}{x}}\right\}^{x\cot x}$

$=\lim\limits_{x\to 0}\ e^{x\cot x}=e$

Since $f(x)$ is continuous at $x=0$

$\therefore\quad f(0)=\lim\limits_{x\to 0}f(x)=e$

10 **(b)**

LHL$=\lim\limits_{h\to 0}\ f\left(\frac{\pi}{4}-h\right)$

$$=\lim_{h\to 0}\ \frac{\tan\left(\frac{\pi}{4}-h\right)-\cot\left(\frac{\pi}{4}-h\right)}{\frac{\pi}{4}-h-\frac{\pi}{4}}$$

$$=\lim_{h\to 0}\ \frac{-\sec^2\left(\frac{\pi}{4}-h\right)-\mathrm{cosec}^2\left(\frac{\pi}{4}-h\right)}{-1}=4$$

[by L 'Hospital's rule]

Since, $f(x)$ is continuous at $x = \frac{\pi}{4}$, then LHL= $f\left(\frac{\pi}{4}\right)$

$\therefore \quad a = 4$

11 **(a)**

If $-1 \le x < 0$, then

$$f(x) = \int_{-1}^{x} |t|\, dt = \int_{-1}^{x} -t\, dt = -\frac{1}{2}(x^2 - 1)$$

If $x \ge 0$, then

$$f(x) = \int_{-1}^{0} -t\, dt + \int_{-1}^{x} -t\, dt = \frac{1}{2}(x^2 + 1)$$

$$\therefore f(x) = \begin{cases} -\frac{1}{2}(x^2 - 2), & -1 \le x < 0 \\ \frac{1}{2}(x^2 + 1), & 0 \le x \end{cases}$$

It can be easily seen that $f(x)$ is continuous at $x = 0$

So, it is continuous for all $x > -1$

Also, $Rf'(0) = 0 = Lf'(0)$

So, $f(x)$ is differentiable at $x = 0$

$$\therefore f'(x) = \begin{cases} -x, & -1 < x = 0 \\ 0, & x = 0 \\ x, & x > 0 \end{cases}$$

Clearly, $f'(x)$ is continuous at $x = 0$

Consequently, it is continuous for all $x > -1$ i.e. for $x + 1 > 0$

Hence, f and f' are continuous for $x + 1 > 0$

12 **(c)**

We have,

$$f(x) = \lim_{n\to\infty} \frac{x^{-n} - x^n}{x^{-n} + x^n}$$

$$\Rightarrow f(x) = \lim_{n\to\infty} \frac{1 - x^{2n}}{1 + x^{2n}}$$

$$\Rightarrow f(x) = \begin{cases} \frac{1-0}{1+0} = 1, & \text{if} -1 < x < 1 \\ \frac{1-1}{1+1} = 0, & \text{if } x = \pm 1 \\ \frac{0-1}{0+1} = -1, & \text{if } |x| > 1 \end{cases}$$

Clearly, $f(x)$ is discontinuous at $x = \pm 1$

13 **(b)**

Clearly, $\log|x|$ is discontinuous at $x = 0$

$f(x) = \frac{1}{\log|x|}$ is not defined at $x = \pm 1$

Hence, $f(x)$ is discontinuous at $x = 0, 1, -1$

14 **(a)**

For continuity, $\lim_{x\to 0} f(x) = k$

$$\Rightarrow \quad \lim_{x\to 0} \frac{\sin 3x}{\sin x} = k \quad \Rightarrow \quad \lim_{x\to 0} \frac{\sin 3x}{3x} . \frac{3x}{\sin 3x} = k$$

$\Rightarrow \quad 3 = k$

15 **(b)**

Since, the function $f(x)$ is continuous

$\therefore \quad f(0)$ =RHL $f(x)$ =LHL$f(x)$

Now, RHL $f(X) = \lim_{h\to 0} \frac{\log(1+0+h)+\log(1-0-h)}{0+h}$

$$= \lim_{h\to 0} \frac{\log(1+h) + \log(1-h)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{1+h} - \frac{1}{1-h}}{1} = 0$$

[by L 'Hospital's rule]

$\therefore \quad f(0)$ =RHL $f(x) = 0$

16 **(d)**

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & x < 4 \\ a + b, & x = 4 \\ \frac{x-4}{|x-4|} + b, & x > 4 \end{cases} = \begin{cases} -1 + a, x < 4 \\ a + b \\ 1 + b, x > 4 \end{cases}$$

LHL= $\lim_{x\to 4^{+}} f(x) = a - 1$

RHL= $\lim_{x\to 4^{+}} f(x) = 1 + b$

Since, LHL=RHL= $f(4)$

$\Rightarrow \quad a - 1 = a + b = b + 1$

$a = 1$ and $b = -1$

17 **(d)**

We have,

$$f(x) = \begin{cases} \frac{-1}{x-1}, & 0 < x < 1 \\ \frac{1-1}{x-1} = 0, & 1 < x < 2 \\ 0, & x = 1 \end{cases}$$

Clearly, $\lim_{x\to 1^-} f(x) \to -\infty$ and $\lim_{x\to 1^+} f(x) = 0$

So, $f(x)$ is not continuous at $x = 1$ and hence it is not differentiable at $x = 1$

18 **(d)**

$$\lim_{x\to\frac{\pi}{4}} f(x) = \lim_{x\to\frac{\pi}{4}} \frac{1 - \sqrt{2}\sin x}{\pi - 4x}$$

$= \lim_{x\to\frac{\pi}{4}} \frac{-\sqrt{2}\cos x}{4} = \frac{1}{4}$ [by L 'Hospital's rule]

Since, $f(x)$ is continuous at $x=\frac{\pi}{4}$

$\therefore \lim_{x\to\frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right) \quad \Rightarrow \quad \frac{1}{4} = a$

19 **(d)**

LHL= $\lim_{x\to 1^-} f(x) = \lim_{h\to 0} 1-h+a = 1+a$

RHL= $\lim_{x\to 1^+} f(x) = \lim_{h\to 0} 3-(1+h)^2 = 2$

For $f(x)$ to be continuous, LHL=RHL

$\Rightarrow 1+a=2 \quad \Rightarrow \quad a=1$

20 **(b)**

LHL= $\lim_{h\to 0} \frac{\cos 3(0-h)-\cos(0-h)}{(0-h)^2}$

$= \lim_{h\to 0} \frac{\cos 3h - \cos h}{h^2}$

$= \lim_{h\to 0} \frac{-3\sin 3h + \sin h}{2h}$

$= \lim_{h\to 0} \frac{-9\cos 3h + \cos h}{2} = \frac{-9+1}{2} = -4$

$\because \lim_{x\to 0^-} f(x) = f(0) \quad \Rightarrow \quad \lambda = -4$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	D	A	A	A	D	A	B	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	B	A	B	D	D	D	D	B

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 9

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(c)**

LHL$= \lim_{x\to a^-} \frac{x^3-a^3}{x-a} = \lim_{h\to 0} \frac{(a-h)^3-a^3}{a-h-a}$

$= \lim_{h\to 0} \frac{(a-h-a)\{(a-h)^2+a^3+a(a-h)\}}{-h} = 3a^2$

Since, $f(x)$ is continuous at $x=a$

$\therefore$ LHL $= f(a)$

$\Rightarrow$ $3a^2 = b$

3 **(a)**

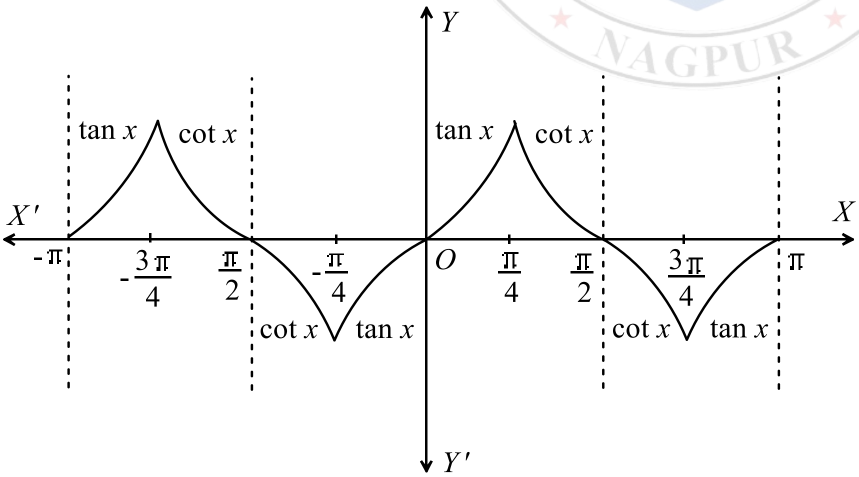
We have,

$$f(x) = \begin{cases} \tan x, & 0 \le x \le \pi/4 \\ \cot x, & -\pi/4 \le x \le \pi/2 \\ \tan x, & \pi/2 < x \le 3\pi/4 \\ \cot x, & 3\pi/4 \le x < \pi \end{cases}$$

Since $\tan x$ and $\cot x$ are periodic functions with period π. So, $f(x)$ is also periodic with period π

It is evident from the graph that $f(x)$ is not continuous at $x = \pi/2$. Since $f(x)$ is periodic with period π. So, it is not continuous at $x = 0, \pm\pi/2, \pm\pi, \neq 3\pi/2$

Also, $f(x)$ is not differentiable $x = \pi/4, 3\pi/4, 5\pi/4$ etc



4 **(c)**

We have,

$f(x) = \{|x| - |x-1\}^2$

$$\Rightarrow f(x) = \begin{cases} (-x+x-1)^2, & \text{if } x < 0 \\ (x+x-1)^2, & \text{if } 0 \le x < 1 \\ (x-x+1)^2, & \text{if } x \ge 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 1, & \text{if } x < 0 \\ (2\,x-1)^2, & \text{if } 0 < x < 1 \\ 1, & \text{if } x \ge 1 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 0, & \text{if } x < 0 \text{ or if } x > 1 \\ 4(2\,x-1), & \text{if } 0 < x < 1 \end{cases}$$

5 **(b)**

We have,

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$\Rightarrow f'(x_0) = \lim_{x \to x_0} \frac{(x - x_0)\phi(x) - 0}{(x - x_0)}$$

$$\Rightarrow f'(x_0) = \lim_{x \to x_0} \phi(x) = \phi(x_0) \qquad [\because \phi(x) \text{ is continuous at } x = x_0]$$

6 **(b)**

Since, $\lim\limits_{x \to 2^+} f(x) = f(2) = k$

$$\Rightarrow \quad k = \lim_{h \to 0} f(2+h)$$

$$\Rightarrow \quad k = \lim_{h \to 0} \left[(2+h)^2 + e^{\frac{1}{2-(2+h)}}\right]^{-1}$$

$$\Rightarrow \quad \lim_{h \to 0} \left[4 + h^2 + 4h + e^{-1/h}\right]^{-1} = \frac{1}{4}$$

7 **(c)**

For $f(x)$ to be continuous at $x = \pi/2$, we must have

$$\lim_{x \to \pi/2} f(x) = f(\pi/2)$$

$$\Rightarrow \lim_{x \to \pi/2} \frac{1 - \sin x}{(\pi - 2x)^2} \cdot \frac{\log \sin x}{\log(1 + \pi^2 - 4\pi x + 4x^2)} = k$$

$$\Rightarrow \lim_{h \to 0} \frac{1 - \cos h}{4h^2} \times \frac{\log \cos h}{\log(1 + 4h^2)} = k$$

$$\Rightarrow \lim_{h \to 0} \frac{1 - \cos h}{4h^2} \times \frac{\log\{1 + \cos h - 1\}}{\cos h - 1} \times \frac{4h^2}{\log(1 + 4h^2)} \times \frac{\cos h - 1}{4h^2} = k$$

$$\Rightarrow -\lim_{h \to 0} \left(\frac{1 - \cos h}{4h^2}\right)^2 \frac{\log\big(1 + (\cos h - 1)\big)}{\cos h - 1} \times \frac{4\,h^2}{\log(1 + 4h^2)} = k$$

$$\Rightarrow -\lim_{h \to 0} \left(\frac{\sin^2 h/2}{2\,h^2}\right)^2 \frac{\log\big(1 + (\cos h - 1)\big)}{\cos h - 1} \times \frac{4h^2}{\log(1 + 4h^2)} = k$$

$$\Rightarrow -\frac{1}{64} \lim_{h \to 0} \left(\frac{\sin h/2}{h/2}\right)^4 \frac{\log\big(1 + (\cos h - 1)\big)}{\cos h - 1} \times \frac{4h^2}{\log(1 + 4h^2)} = k$$

$$\Rightarrow -\frac{1}{64} = k$$

8 **(c)**

LHL$= \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sin 5(0-h)}{(0-h)^2+2(0-h)}$

$$= -\lim_{h\to 0} \frac{\frac{\sin 5h}{5h}}{\frac{1}{5}(h-2)} = \frac{5}{2}$$

Since, it is continuous at $x=0$, therefore LHL$= f(0)$

$\Rightarrow \frac{5}{2} = k + \frac{1}{2} \quad \Rightarrow \quad k = 2$

9 **(a)**

Since $f(x)$ is continuous at $x=0$

$\therefore \lim_{x\to 0} f(x) = f(0) = 0$

$\Rightarrow \lim_{x\to 0} x^n \sin\left(\frac{1}{x}\right) = 0 \Rightarrow n > 0$

$f(x)$ is differentiable at $x=0$, if

$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$ exists finitely

$\Rightarrow \lim_{x\to 0} \frac{x^n \sin\frac{1}{x} - 0}{x}$ exists finitely

$\Rightarrow \lim_{x\to 0} x^{n-1} \sin\left(\frac{1}{x}\right)$ exists finitely

$\Rightarrow n-1 > 0 \Rightarrow n > 1$

If $n \le 1$, then $\lim_{x\to 0} x^{n-1}\sin\left(\frac{1}{x}\right)$ does not exist and hence $f(x)$ is not differentiable at $x=0$

Hence $f(x)$ is continuous but not differentiable at $x=0$ for $0 < n \le 1$ i.e. $n \in (0,1]$

10 **(b)**

Clearly, $f(x)$ is not differentiable at $x=3$

Now, $\lim_{h\to 3^-} f(x) = \lim_{h\to 0} f(3-h)$

$= \lim_{h\to 0} |3-h-3|$

$= 0$

$\lim_{h\to 3^+} f(x) = \lim_{h\to 0} f(3+h)$

$= \lim_{h\to 0} |3+h-3| = 0$

and $f(3) = |3-3| = 0$

$\therefore$ $f(x)$ is continuous at $x=3$

11 **(a)**

It can easily be seen from the graphs of $f(x)$ and that both are continuous at $x=0$

Also, $f(x)$ is not differentiable at $x=0$ whereas $g(x)$ is differentiable at $x=0$

12 **(c)**

We have,

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{-\sin(a+1)h - \sin h}{-h}$$

$$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\left\{\frac{\sin(a+1)h}{h} + \frac{\sin h}{h}\right\}$$

$$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = (a+1)+1 = a+2$$

and, $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{h\to 0} \frac{\sqrt{h+bh^2}-\sqrt{h}}{b\, h^{3/2}}$

$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{h\to 0} \frac{h+bh^2-h}{bh^{3/2}(\sqrt{h+bh^2}-\sqrt{h})} = \lim_{h\to 0} \frac{1}{\sqrt{1+bh}+1} = \frac{1}{2}$

Since, $f(x)$ is continuous at $x=0$. Therefore,

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\Rightarrow a+2 = \frac{1}{2} = c \Rightarrow c = \frac{1}{2}, a = -\frac{3}{2}$ and $b \in R - \{0\}$

13 **(c)**

For $f(x)$ to be continuous at $x=0$, we must have

$\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{(9^x-1)(4^x-1)}{\sqrt{2}-\sqrt{2\cos^2 x/2}} = k$

$\Rightarrow \lim_{x\to 0} \frac{(9^x-1)(4^x-1)}{\sqrt{2}.\,2\sin^2 x/4} = k$

$\Rightarrow \lim_{x\to 0} \frac{16 \times \left(\frac{9^x-1}{x}\right)\left(\frac{4^x-1}{x}\right)}{2\sqrt{2}\left(\frac{\sin x/2}{x/4}\right)^2} = k$

$\Rightarrow \frac{16}{2\sqrt{2}} \log 9 \,.\, \log 4 = k = 4\sqrt{2} \log 9 \,.\, \log 4 = 16\sqrt{2} \log 3 \log 2$

14 **(b)**

Given, $f(x) = [\tan^2 x]$

Now, $\lim_{x\to 0} f(x) = \lim_{x\to 0} [\tan^2 x] = 0$

And $f(0) = [\tan^2 0] = 0$

Hence, $f(x)$ is continuous at $x=0$

15 **(b)**

Let, $f(x) = x$

Which is continuous at $x=0$

Also, $f(x+y) = f(x) + f(y)$

$\Rightarrow \quad f(0+0) = f(0) + f(0)$

$= 0+0$

$\Rightarrow f(0) = 0$

$f(1+0) = f(1) + f(0)$

$\Rightarrow \quad f(1) = 1+0$

$\Rightarrow \quad f(1) = 1$

As, it satisfies it.

Hence, $f(x)$ is continous for every values of x

16 **(c)**

Here, $gof = \begin{cases} e^{\sin x}, & x \geq 0 \\ e^{1-\cos x}, & x \leq 0 \end{cases}$

$\therefore$ LHD$= \lim_{h\to 0}\frac{gof(0-h)-gof(h)}{-h}$

$= \lim_{h\to 0}\dfrac{e^{1-\cos h}-e^{1-\cos h}}{-h} = 0$

RHD$= \lim_{h\to 0}\frac{gof(0+h)-gof(h)}{h}$

$= \lim_{h\to 0}\dfrac{e^{\sin h}-e^{\sin h}}{h} = 0$

Since, RHD=LHD=0

$\therefore$ $(gof)'(0) = 0$

17 **(b)**

We have,

$$f(x)\begin{cases} (x+1)^{2-\left(\frac{1}{x}+\frac{1}{x}\right)} = (x+1)^2, & x<0 \\ 0, & x=0 \\ (x+1)^{2-\left(\frac{1}{x}+\frac{1}{x}\right)} = (x+1)^{2-\frac{2}{x}}, & x>0 \end{cases}$$

Clearly, $f(x)$ is everywhere continuous except possibly at $x = 0$

At $x = 0$, we have

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(x+1)^2 = 1$

and, $\lim_{x\to 0^+} f(x) = \lim_{x\to 0}(x+1)^{2-\frac{2}{x}} = \lim_{x\to 0}(x+1)^{-2/x}$

$\Rightarrow \lim_{x\to 0^+} f(x) = e^{\lim_{x\to 0} -\frac{2}{x}\log(1+x)} = e^{-2}$

Clearly, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

So, $f(x)$ is not continuous at $x = 0$

18 **(b)**

Since $f(x)$ is continuous at $x = 0$. Therefore,

$\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0} f(x) = k$

$\Rightarrow \lim_{x\to 0}\dfrac{\log(1+ax)-\log(1-bx)}{x} = k$

$\Rightarrow a\lim_{x\to 0}\dfrac{\log(1+ax)}{ax} - (-b)\lim_{x\to 0}\dfrac{\log(1-bx)}{-bx} = k$

$\Rightarrow a+b = k$

19 **(c)**

Since $f(x)$ is continuous at $x = 0$

$\therefore f(0) = \lim_{x\to 0} f(x)$

$\Rightarrow f(0) = \lim_{x\to 0}\dfrac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}} \qquad \left[\text{Form}\,\dfrac{0}{0}\right]$

$\Rightarrow f(0) = \lim_{x\to 0}\dfrac{\frac{1}{3}(27-2x)^{-\frac{2}{3}}(-2)}{-\frac{3}{5}(243+5x)^{-\frac{4}{5}}(5)} = \left(-\dfrac{2}{3}\right)\left(-\dfrac{1}{3}\right)\dfrac{3^4}{3^2} = 2$

20 **(d)**

$\lim_{x\to 0} \frac{e^{2x}-1-2x}{x(e^{2x}-1)}$

$= \lim_{x\to 0} \frac{2e^{2x}-2}{(e^{2x}-1)+2xe^{2x}}$ [using L 'Hospital rule]

$= \lim_{x\to 0} \frac{4e^{2x}}{4e^{2x}+4xe^{2x}} = 1$ [using L 'Hospital's rule]

Since, $f(x)$ is continuous at $x = 0$, then

$\lim_{x\to 0} f(x) = f(0) \quad \Rightarrow \quad 1 = f(0)$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	A	C	B	B	C	C	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	C	B	B	C	B	B	C	D

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 10

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(b)**

If a function $f(x)$ is continuous at $x = a$, then it may or may not be differentiable at $x = a$

∴ Option (b) is correct

2 **(c)**

Let $f(x) = |x - 1| + |x - 3|$

$$= \begin{cases} x - 1 \quad + x - 3 \quad, x \geq 3 \\ x - 1 + 3 - x, \quad 1 \leq x < 3 \\ 1 - x \quad + 3 - x, \quad x \leq 1 \end{cases}$$

$$= \begin{cases} 2x - 4, x \geq 3 \\ 2, 1 \leq x < 3 \\ 4 - 2x, x \leq 1 \end{cases}$$

At $x = 2$, function is

$f(x) = 2$

$\Rightarrow \quad f'(x) = 0$

3 **(d)**

We have,

$$f(x) = \begin{cases} (x+1)\, e^{-\left(\frac{1}{x}+\frac{1}{x}\right)} = (x+1), \qquad x < 0 \\ (x+1)\, e^{-\left(\frac{1}{x}+\frac{1}{x}\right)} = (x+1)e^{-2/x}\,, x > 0 \end{cases}$$

Clearly, $f(x)$ is continuous for all $x \neq 0$

So, we will check its continuity at $x = 0$

We have,

(LHL at $x = 0) = \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0}(x+1) = 1$

(RHL $at\ x = 0) = \lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0}(x+1)\ e^{-2/x} = \lim\limits_{x\to 0} \dfrac{x+1}{e^{2/x}} = 0$

$\therefore \lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+ f(x)}$

So, $f(x)$ is not continuous at $x = 0$

Also, $f(x)$ assumes all values from $f(-2)$ to $f(2)$ and $f(2) = 3/e$ is the maximum value of $f(x)$

4 **(c)**

Since, it is a polynomial function, so it is continuous for every value of x except at $x = 2$

LHL$= \lim\limits_{x\to 2^-} x - 1$

$= \lim_{h\to 0} 2 - h - 1 = 1$

RHL= $\lim_{x\to 2^+} 2x - 3 = \lim_{h\to 0} 2(2+h) - 3 = 1$

And $f(2) = 2(2) - 3 = 1$

$\therefore$ LHL+RHL= $f(2)$

Hence, $f(x)$ is continuous for all real values of x

5 **(c)**

Continuity at $x = 0$

LHL= $\lim_{x\to 0^-} \frac{\tan x}{x} = \lim_{h\to 0} \frac{-tan\, h}{-h} = 1$

RHL= $\lim_{x\to 0^+} \frac{\tan x}{x} = \lim_{h\to 0} \frac{tan\, h}{h} = 1$

$\therefore$ LHL=RHL= $f(0) = 1$, it is continuous

Differentiability at $x = 0$

LHD= $\lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0} \frac{\frac{\tan(-h)}{-h}-1}{-h}$

$$= \lim_{h\to 0} \frac{+\frac{h^2}{3} + \frac{2h^4}{15} + \cdots}{-h} = 0$$

RHD= $\lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{\frac{\tan h}{h}-1}{h}$

$$= \lim_{h\to 0} \frac{\frac{h^2}{3} + \frac{2h^4}{15} + \cdots}{-h} = 0$$

$\therefore$ LHD=RHD

Hence, it is differentiable.

6 **(b)**

We have,

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(x - 1) = 0$

and,

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(x^3 - 1) = 0$. Also, $f(1) = 1 - 1 = 0$

So, $f(x)$ is continuous at $x = 1$

Clearly, $(f'(1)) = 3$ and $Rf'(1) = 1$

Therefore, $f(x)$ is not differentiable at $x = 1$

7 **(d)**

We have,

$$f(x) = \begin{cases} \frac{x^2 - x}{x^2 - x} = 1, & \text{if } x < 0 \text{ or } x > 1 \\ -\frac{(x^2 - x)}{x^2 - x} = -1, & \text{if } 0 < x < 1 \\ 1, & \text{if } x = 0 \\ -1, & \text{if } x = 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 1, \text{if } x \le 0 \text{ or } x > 1 \\ -1, \text{if } 0 < x \le 1 \end{cases}$$

Now,

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0} 1 = 1$ and, $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0} -1 = -1$

Clearly, $\lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x)$

So, $f(x)$ is not continuous at $x = 0$. It can be easily seen that it is not continuous at $x = 1$

8 **(b)**

We have,

$f(x) = |x-1| + |x-3|$

$$\Rightarrow f(x) = \begin{cases} -(x-1)-(x-3), & x < 1 \\ (x-1)-(x-3), & 1 \le x < 3 \\ (x-1)+(x-3), & x \ge 3 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} -2x+4, & x < 1 \\ 2, & 1 \le x < 3 \\ 2x-4, & x \ge 3 \end{cases}$$

Since, $f(x) = 2$ for $1 \le x < 3$. Therefore $f'(x) = 0$ for all $x \in (1,3)$

Hence, $f'(x) = 0$ at $x = 2$

9 **(d)**

We have,

$Lf'(0) = 0$ and $Rf'(0) = 0 + \cos 0° = 1$

$\therefore Lf'(0) \neq Rf'(0)$

Hence, $f'(x)$ does not exist at $x = 0$

10 **(c)**

Given, $g(x) = \frac{(x-1)^n}{\log\cos^m(x-1)}$; $0 < x < 2$, $m \neq 0$, n are integers and $|x-1| = \begin{cases} x-1; & x \ge 1 \\ 1-x; & x < 1 \end{cases}$

The left hand derivative of $|x-1|$ at $x = 1$ is $p = -1$

Also, $\lim\limits_{x\to 1^+} g(x) = p = -1$

$$\Rightarrow \lim_{h\to 0} \frac{(1+h-1)^n}{\log\cos^m(1+h-1)} = -1$$

$$\Rightarrow \lim_{h\to 0} \frac{h^n}{m\log\cos h} = -1$$

$$\Rightarrow \lim_{h\to 0} \frac{n.h^{n-1}}{m\frac{1}{\cos h}(-\sin h)} = -1$$

[using L 'Hospital's rule]

$$\Rightarrow \left(\frac{n}{m}\right)\lim_{h\to 0} \frac{h^{n-2}}{\left(\frac{\tan h}{h}\right)} = 1$$

$\Rightarrow n = 2$ and $\frac{n}{m} = 1$

$\Rightarrow m = n = 2$

11 **(c)**

Given, $f(x) = \frac{2x^2+7}{(x^2-1)(x+3)}$

Since, at $x = 1, -1, -3$, $f(x) = \infty$

Hence, function is discontinuous

13 **(a)**

LHL= $\lim_{x\to 1^-} f(x) = \lim_{h\to 0} \; [1-(1-h)^2] = 0$

RHL= $\lim_{x\to 1^+} f(x) = \lim_{h\to 0}\{1+(1+h)^2\} = 2$

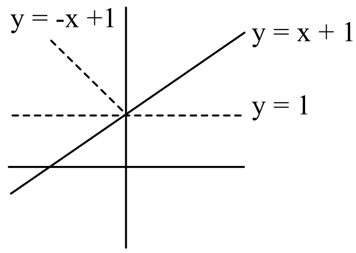
Also, $f(1) = 0$

$\Rightarrow$ RHL $\neq$ LHL $= f(1)$

Hence, $f(x)$ is not continuous at $x = 1$

14 **(c)**

It is clear from the graph that minimum $f(x)$ is



$f(x) = x + 1, \qquad \forall\, x \in R$

Hence, it is a straight line, so it is differentiable everywhere

15 **(c)**

Since, $f(x)$ is continuous at $x = \frac{\pi}{2}$

$\lim_{x\to \frac{\pi^{-1}}{2}} (mx+1) = \lim_{x\to\frac{\pi^+}{2}} (\sin x + n)$

$\Rightarrow \quad m\dfrac{\pi}{2} + 1 = \sin\dfrac{\pi}{2} + n$

$\Rightarrow \quad \frac{m\pi}{2} = n$

16 **(a)**

This function is continuous at $x = 0$, then

$$\lim_{x\to 0}\frac{\log_e(1+x^2\tan x)}{\sin x^3} = f(0)$$

$$\Rightarrow \lim_{x\to 0}\frac{\log_e\left\{1+x^2\left(x+\dfrac{x^3}{3}+\ldots\right)\right\}}{x^3-\dfrac{x^9}{3!}+\dfrac{x^{15}}{5!}-\ldots} = f(0)$$

$$\Rightarrow \lim_{x\to 0}\frac{\log_e(1+x^3)}{x^3-\dfrac{x^9}{3!}+\dfrac{x^{15}}{5!}-\ldots} = f(0)$$

[neglecting higher power of x in $x^2\tan x$]

$$\Rightarrow \lim_{x\to 0}\frac{x^3-\dfrac{x^6}{2}+\dfrac{x^9}{3}-\cdots}{\mathrm{x}^3+\dfrac{\mathrm{x}^9}{3!}+\dfrac{\mathrm{x}^{15}}{5!}-\ldots} = f(0)$$

$\Rightarrow \quad 1 = f(0)$

17 **(a)**

Given, $f(x)$ is continuous at $x = 0$

$\therefore$ Limit must exist

ie, $\lim_{x\to 0} x^p \sin\frac{1}{x} = (0)^p \sin\infty = 0$, when, $0 < p < \infty$ ...(i)

Now, RHD$= \lim_{h\to 0} \frac{h^p \sin\frac{1}{h} - 0}{h} = \lim_{h\to 0} h^{p-1} \sin\frac{1}{h}$

LHD$= \lim_{h\to 0} \frac{(-h)^p \sin\left(-\frac{1}{h}\right) - 0}{-h}$

$= \lim_{h\to 0} (-1)^p h^{p-1} \sin\frac{1}{h}$

Since, $f(x)$ is not differentiable at $x = 0$

$\therefore \quad p \leq 1$ ...(ii)

From Eqs.(i) and (iii), $0 < p \leq 1$

18 **(a)**

We have,

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin x^2}{x} = \lim_{x\to 0} \left(\frac{\sin x^2}{x^2}\right) x = 1 \times 0 = 0 = f(0)$$

So, $f(x)$ is continuous at $x = 0$. $f(x)$ is also derivable at $x = 0$, because

$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{\sin x^2}{x} = \lim_{x\to 0} \frac{\sin x^2}{x^2} = 1$ exists finitely

19 **(a)**

A function f on R into itself is continuous at a point a in R, iff for each $\in > 0$ there exist $\delta > 0$, such that

$|f(x) - f(a)| < \in \;\Rightarrow\; |x - a| < \delta$

20 **(a)**

We have,

$f(x) = x - |x - x^2|, \qquad -1 \leq x \leq 1$

$$\Rightarrow f(x) = \begin{cases} x + x - x^2, & -1 \leq x < 0 \\ x - (x - x^2), & 0 \leq x \leq 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 2x - x^2, & -1 \leq x < 0 \\ x^2, & 0 \leq x \leq 1 \end{cases}$$

Clearly, $f(x)$ is continuous at $x = 0$

Also,

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1} 2x - x^2 = -2 - 1 = -3 = f(-1)$

and,

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2 = 1 = f(1)$

So, $f(x)$ is right continuous at $x = -1$ and left continuous at $x = 1$

Hence, $f(x)$ is continuous on $[-1, 1]$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	D	C	C	B	D	B	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	A	C	C	A	A	A	A	A