SESSION : 2025-26

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :1

Topic :-INTEGRALS

1 **(a)**

We have,

$$f(x)=\int_0^x |x-2|\,dx \Rightarrow f'(x)=|x-2|$$

Clearly, $f'(x)$ is everywhere continuous and differentiable except at $x=2$

2 **(b)**

If $f(t)$ is an odd function, then $\int_0^x f(t)dt$ is an even function

3 **(c)**

$$\int \frac{\sec x \operatorname{cosec} x}{2\cot x - \sec x \operatorname{cosec} x}\,dx$$

$$=\int \frac{\frac{1}{\cos x \sin x}}{\frac{2\cos x}{\sin x}-\frac{1}{\sin x \cos x}}\,dx$$

$$=\int \frac{dx}{2\cos^2 x - 1}$$

$$=\int \frac{dx}{\cos 2x}=\int \sec 2x\ \ dx$$

$$=\frac{1}{2}\log|\sec 2x+\tan 2x|+c$$

4 **(d)**

Let $I=\int_0^\pi \frac{\theta \sin\theta}{1+\cos^2\theta}d\theta$ ...(i)

$$\Rightarrow I=\int_0^\pi \frac{(\pi-\theta)\sin\theta}{1+\cos^2\theta}d\theta \ \text{... (ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I=\int_0^\pi \frac{\pi \sin\theta}{1+\cos^2\theta}\,d\theta$$

Put $\cos\theta = t \Rightarrow -\sin\theta d\theta = dt$

$$\therefore 2I=-\pi\int_1^{-1}\frac{1}{1+t^2}dt=2\pi\int_0^1\frac{1}{1+t^2}dt$$

$$=2\pi[\tan^{-1} t\quad]_0^1=2\pi.\frac{\pi}{4}$$

$\Rightarrow \; I = \dfrac{\pi^2}{4}$

5 **(d)**

Let $f(\theta) = \left[\int_0^{\sin^2\theta} \sin^{-1}\sqrt{\phi}\; d\phi + \int_0^{\cos^2\theta} \cos^{-1}\sqrt{\phi}\; d\phi\right]$

$f'(\theta) = \left(\dfrac{d}{d\theta}\sin^2\theta\right)\left[\sin^{-1}\sqrt{\sin^2\theta}\right] + \left(\dfrac{d}{d\theta}\cos^2\theta\right)\left[\cos^{-1}\sqrt{\cos^2\theta}\right]$

$(2\sin\theta\cos\theta)\theta - (2\sin\theta\cos\theta)\theta = 0$

$\therefore \; f(\theta) = \text{constant} = a \quad [\text{say}]$

$\therefore \; f\left(\dfrac{\pi}{4}\right) = a$

$\Rightarrow \displaystyle\int_0^{1/2} (\sin^{-1}\sqrt{\phi} + \cos^{-1}\sqrt{\phi}) d\phi = a$

$\Rightarrow \; \dfrac{\pi}{2}[\phi]_0^{1/2} = a \; \Rightarrow \; \dfrac{\pi}{4} = a$

6 **(c)**

Let $I = \int_0^{\pi/2} \frac{dx}{1+\tan^3 x} = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x+\cos^3 x} dx$...(i)

$\Rightarrow I = \displaystyle\int_0^{\pi/2} \frac{\cos^3\left(\frac{\pi}{2}-x\right)}{\sin^3\left(\frac{\pi}{2}-x\right)+\cos^3\left(\frac{\pi}{2}-x\right)} dx$

$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x} dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$2I = \displaystyle\int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} dx$

$= \displaystyle\int_0^{\pi/2} 1 dx = \frac{\pi}{2}$

$\Rightarrow I = \dfrac{\pi}{4}$

7 **(c)**

Since, $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \int_0^1 \frac{x}{\sqrt{x}} dx$

Because in $x \epsilon (0,1), x > \sin x$

$I < \displaystyle\int_0^1 \sqrt{x}\, dx = \frac{2}{3}\left[x^{3/2}\right]_0^1 \Rightarrow I < \frac{2}{3}$

For $x \epsilon (0,1), \frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$

And $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int_0^1 x^{-\frac{1}{2}} dx = 2 \Rightarrow J < 2$

8 **(c)**

$$\int_{-1}^{1} |x| dx + \int_{1}^{3} |x-2| dx$$

$$= 2\int_0^1 x\, dx + \int_1^2 (2-x)dx + \int_2^3 (x-2)dx$$

$$= 2 \times \frac{1}{2} + \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^3$$

$$= 1 + \frac{1}{2} + \frac{1}{2} = 2$$

9 **(a)**

Putting $x^n + 1 = t$ and $n\, x^{n-1} dx = dt$, we get

$$I = \int \frac{1}{x\,(x^n+1)} dx = \frac{1}{n}\int \frac{1}{t(t-1)} dt = \frac{1}{n}\int \left(\frac{1}{t-1} - \frac{1}{t}\right) dt$$

$$\Rightarrow I = \frac{1}{n}\log\left(\frac{t-1}{t}\right) + C = \frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right) + C$$

10 **(d)**

We have,

$$I = \int_0^{1/2} |\sin \pi\, x|\, dx = \frac{1}{\pi}\int_0^{\pi/2} |\sin t|\, dt, \text{where } t = \pi x$$

$$\Rightarrow I = \frac{1}{\pi}\int_0^{\pi/2} \sin t\, dt = \frac{1}{\pi}$$

12 **(c)**

We know that,

$$\left|\int_a^b f(x)g(x)dx\right| \leq \sqrt{\int_a^b f^2(x)\,dx}.\sqrt{\int_a^b g^2(x)\,dx}$$

Putting $f(x) = 1$ and $g(x) = \sqrt{1+x^3}$, we get

$$I\int_0^1 \sqrt{1+x^3}\, dx \leq \sqrt{\int_0^1 1 + x^3\, dx} = \sqrt{1+\frac{1}{4}} = \frac{\sqrt{5}}{2}$$

$\Rightarrow I < 2$ and $I < \frac{\sqrt{7}}{2}$

13 **(b)**

We have,

$$I = \frac{1}{c}\int_{ac}^{bc} f\left(\frac{x}{c}\right) dx$$

Putting $\frac{x}{c} = t$ and $dx = c\, dt$, we get

$$I = \frac{1}{c}\int_a^b f(t)\,c\,dt = \int_a^b f(t)\,dt = \int_a^b f(x)\,dx$$

14 **(c)**

$$\int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx = \int e^x \frac{(1+x^2-2x)}{(1+x^2)^2}dx$$

$$= \int e^x\left(\frac{1}{1+x^2} - \frac{2x}{(1+x^2)^2}\right)dx$$

$$= e^x.\frac{1}{1+x^2} + \int \frac{2xe^x}{(1+x^2)^2}dx - \int \frac{2xe^x}{(1+x^2)^2}dx$$

$$= \frac{e^x}{1+x^2} + c$$

15 **(a)**

We have,

$$\frac{d}{dx}\big(f(x)\big) = \frac{e^{\sin x}}{x} \Rightarrow \int \frac{e^{\sin x}}{x}dx = F(x)$$

$$\therefore \int_1^4 \frac{3}{x}e^{\sin x^3}dx = \int_1^4 \frac{e^{\sin x^3}}{x^3}.3x^2dx = \int_1^{64}\frac{e^{\sin x^3}}{x^3}d(x^3)$$

$$= F(64) - F(1)$$

Hence, $k = 64$

17 **(a)**

We have,

$$I = \int \frac{\log(x+1) - \log x}{x(x+1)}dx = \int \frac{\log(1+1/x)}{(x^2+x)}dx$$

$$\Rightarrow I = \int \frac{\log(1+1/x)}{(x^2(1+1/x)}dx = -\int \log\left(1+\frac{1}{x}\right)d\left\{\log\left(1+\frac{1}{x}\right)\right\}$$

$$\Rightarrow I = -\frac{1}{2}\left\{\log\left(1+\frac{1}{x}\right)\right\}^2 + C$$

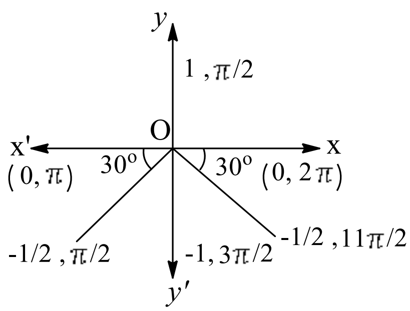
$$\Rightarrow I = -\frac{1}{2}[\log(x+1) - \log x]^2 + C$$

$$\Rightarrow I = -\frac{1}{2}[\{\log(x+1)\}^2 + (\log x)^2 - 2\log(x+1)\cdot\log x] + C$$

$$\Rightarrow I = -\frac{1}{2}\{\log(x+1)\}^2 - \frac{1}{2}(\log x)^2 + \log(x+1)\cdot\log x + C$$

18 **(a)**

It is a question of greatest integer function. We have, subdivide the interval π to 2πas under keeping in view that we have to evaluate $[2\sin x]$



We know that, $\sin\frac{\pi}{6}=\frac{1}{2}$

$\sin\left(\pi+\frac{\pi}{6}\right)=\sin\frac{7\pi}{6}=-\frac{1}{2}$

$\sin\frac{11\pi}{6}=\sin\left(2\pi-\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$

$\sin\frac{9\pi}{6}=\sin\frac{3\pi}{6}=-1$

Hence, we divide the intervalπto 2π as $\left(\pi,\frac{7\pi}{6}\right),\left(\frac{7\pi}{6},\frac{11\pi}{6}\right),\left(\frac{11\pi}{6},2\pi\right)$

$\sin x=\left(0,-\frac{1}{2}\right),\left(-1,-\frac{1}{2}\right),\left(0,-\frac{1}{2}\right)$

$2\sin x=(0,-1),(-2,-1),(0,-1)$

$[2\sin x]=-1$

$\therefore\ I=I_1+I_2+I_3$

$=\int-1\,dx+\int-2\,dx\int-1\,dx$

Between proper limits

$=-\frac{\pi}{6}-2\left(\frac{4\pi}{6}\right)-\frac{\pi}{6}=-\frac{10\pi}{6}=-\frac{5\pi}{3}$

19 **(c)**

$\int_0^1\frac{d}{dx}\left[\sin^{-1}\left(\frac{2x'}{1+x^2}\right)\right]dx$

$=\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]_0^1=\sin^{-1}(1)-\sin^{-1}(0)=\frac{\pi}{2}$

20 **(c)**

Let $F(x)=e^x$

Also, $f(x)+g(x)=x^2\ \Rightarrow\ g(x)=x^2-e^x$

Now, $\int_0^1 f(x)g(x)dx=\int_0^1 e^x(x^2-e^x)dx$

$=\int_0^1 x^2e^2\,dx-\int_0^1 e^{2x}dx$

$=[x^2e^x-\int 2xe^xdx\quad]_0^1-\frac{1}{2}[e^{2x}]_0^1$

$=[x^2e^x-2xe^x+2e^x]_0^1-\frac{1}{2}(e^2-1)$

$$= [(1-2+2)e^1 - (0-0+2)e^0] - \frac{1}{2}e^2 + \frac{1}{2}$$

$$= e - \frac{e^2}{2} - \frac{3}{2}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	C	D	D	C	C	C	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	C	A	A	A	A	C	C

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DAILY PRACTICE PROBLEMS

CLASS : XIIth
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SOLUTIONS

SUBJECT : MATHS
DPP NO. :2

Topic :-INTEGRALS

1 **(b)**

We have,

$$I=\int_0^{\pi}\log(1+\cos x)\,dx$$

$$\Rightarrow I=\int_0^{\pi}\log\left(2\cos^2\frac{x}{2}\right)dx$$

$$\Rightarrow I=\int_0^{\pi}\left(\log 2+2\log\cos\frac{x}{2}\right)dx$$

$$\Rightarrow I=\pi\log 2+2\int_0^{\pi}\log\cos\frac{x}{2}\,dx$$

$$\Rightarrow I=\pi\log 2+2\times 2\int_0^{\pi/2}\log\cos t\,dt,\text{ where } t=\frac{x}{2}\text{ and } dx=2dt$$

$$\Rightarrow I=\pi\log 2+4\times-\frac{\pi}{2}\log 2=-\pi\log 2$$

2 **(a)**

Putting $\tan^{-1}x=t$, we have

$$I=\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$$

$$\Rightarrow I=\int e^t(\tan t+\sec^2 t)dt=e^t\tan t+C=x\,e^{\tan^{-1}x}+C$$

3 **(c)**

Given,

$$I_1=\int_a^{\pi-a}x\,f(\sin x)\,dx$$

and$I_2=\int_a^{\pi-a}f(\sin x)\,dx$

Now, $I_1=\int_a^{\pi-a}x\,f(\sin x)\,dx$

$$=\int_a^{\pi-a}(\pi-x)f[\sin(\pi-x)]\,dx$$

$$= \int_{a}^{\pi-a} (\pi - x) f(\sin x)\, dx$$

$$= \int_{a}^{\pi-a} \pi\, f(\sin x) dx - I_1$$

$$\Rightarrow 2I_1 = \pi I_2 \Rightarrow I_2 = \frac{2}{\pi} I_1$$

4 **(b)**

$$\int \cos^{-1}\left(\frac{1}{x}\right) dx = \int \sec^{-1} x \cdot 1\, dx$$

$$= \sec^{-1} x \int dx - \int \left[\frac{d}{dx} \sec^{-1} x \int dx\right] dx$$

$$= x \sec^{-1} x - \int \frac{1}{x\sqrt{x^2 - 1}} x\, dx$$

$$= x \sec^{-1} x - \int \frac{1}{\sqrt{x^2 - 1}}\, dx$$

$$= x \sec^{-1} x - \cosh^{-1} x + c$$

5 **(d)**

Let $I = \int_0^1 \frac{dx}{x+\sqrt{1-x^2}}$

Put $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

$I = \int_0^{\pi/2} \frac{\cos\theta\, d\theta}{\sin\theta + \cos\theta}$...(i)

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - \theta\right) d\theta}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)}$$

$= \int_0^{\pi/2} \frac{\sin\theta}{\cos\theta + \sin\theta}\, d\theta$...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} 1\, d\theta \;\Rightarrow I = \frac{\pi}{4}$$

6 **(b)**

Let

$$I = \int_0^{\sqrt{n}} [x^2] dx$$

$$\Rightarrow I = \sum_{r=1}^{n} \int_{\sqrt{r-1}}^{\sqrt{r}} [x^2]\, dx$$

$$\Rightarrow I = \sum_{r=1}^{n} \int_{\sqrt{r-1}}^{\sqrt{r}} (r-1)\, dx$$

$$\Rightarrow I = \sum_{r=1}^{n} (r-1)\left(\sqrt{r} - \sqrt{r-1}\right)$$

$$\Rightarrow I = \left(\sqrt{2} - \sqrt{1}\right) + 2\left(\sqrt{3} - \sqrt{2}\right) + 3\left(\sqrt{4} - \sqrt{3}\right)$$

$+\cdots+(n-1)\left(\sqrt{n}-\sqrt{n-1}\right)$

$\Rightarrow I=n\sqrt{n}-\left(\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n}\right)=n\sqrt{n}-\sum_{r=1}^{n}\sqrt{r}$

7 **(d)**

Let $I=\int\frac{1}{x}(\log_{ex}e)dx=\int\frac{1}{x(1+\log_e x)}dx$

Put $\log_e x=t\Rightarrow\frac{1}{x}dx=dt$

$\therefore\ I=\int\frac{dt}{(1+t)}=\log_e(1+t)+c$

$=\log_e(1+\log_e x)+c$

8 **(a)**

$I_1-I_2=\int_0^{\pi/2}(\cos\theta-\sin 2\theta)f(\sin\theta+\cos^2\theta)d\theta$

Put $\sin\theta+\cos^2\theta=t$

$\Rightarrow(\cos\theta-\sin 2\theta)d\theta=dt$

Then, $I_1-I_2=\int_1^1 f(t)dt=0$

$\therefore\ I_1=I_2$

9 **(b)**

$(1) I=\int_{-\pi/2}^{\pi/2}\sqrt{\cos x-\cos^3 x}\,dx$

$=\int_{-\pi/2}^{\pi/2}\sqrt{\cos x}\sin x\,dx$

$=2\int_0^{\pi/2}\sqrt{\cos x}\sin x\,dx$

$=-\frac{4}{3}[\cos^{3/2}x]_0^{\pi/2}=\frac{4}{3}$

$(2) I=\int_0^1|x-1|dx+\int_1^4|x-1|dx+\int_0^3|x-3|dx+\int_3^4|x-3|dx$

$=\int_0^1-(x-1)dx+\int_1^4(x-1)dx+\int_0^3-(x-3)dx+\int_3^4(x-3)dx$

$=10$

10 **(a)**

Let $I=\int_{-\pi/4}^{\pi/4}\sin^{-4}x\,dx=\int_{-\pi/4}^{\pi/4}\operatorname{cosec}^{-4}x\,dx$

$$=\int_{-\pi/4}^{\pi/4}(1+\cot^2 x)\operatorname{cosec}^2 x\,dx$$

Put $\cot x=t$

$\Rightarrow\ -\operatorname{cosec}^2 x\,dx=dt$

$\therefore\ I=-\int_{-1}^{1}(1+t^2)\,dt=-2\int_0^1(1+t^2)\,dt$

$= -2\left[t + \frac{t^3}{3}\right]_0^1 = -2\left[1 + \frac{1}{3}\right] = -\frac{8}{3}$

11 **(d)**

We have,

$\lim_{x\to\infty} \frac{\left(\int_0^x e^x\,dx\right)^2}{\int_0^x e^{2x^2}\,dx} = \lim_{x\to\infty} \frac{(e^x - 1)^2}{\int_0^x e^{2x^2}\,dx}$

$= \lim_{x\to\infty} \frac{2(e^x-1)e^x}{\int_0^x e^{2x^2}}$ [Using L'Hospital's rule]

$= 2\lim_{x\to\infty} \frac{e^x - 1}{e^{2x^2-x}}$

$= 2\lim_{x\to\infty} \frac{e^x}{e^{2x^2-x}(4x-1)}$ [Using L'Hospital's rule]

$= 2\lim_{x\to\infty} \frac{1}{e^{2x^2-2x}(4x-1)} = 0$

12 **(a)**

Given, $\int_{\sin x}^1 t^2\ f(t)dt = 1 - \sin x$

Now, $\frac{d}{dx}\int_{\sin x}^1 t^2\ f(t)dt = \frac{d}{dx}(1 - \sin x)$

$\Rightarrow [1^2 f(1)].(0) - (\sin^2 x).f(\sin x).\cos x = -\cos x$

[by Leibnitz formula]

$\Rightarrow$ Put $\sin x = 1/\sqrt{3}$

$\therefore\ f\left(\frac{1}{\sqrt{3}}\right) = \left(\sqrt{3}\right)^2 = 3$

13 **(b)**

We have,

$f(a-x) + f(a+x) = 0$

$\Rightarrow f(2a-x) + f(x) = 0$ [On replacing $x$by $x-a$]

$\Rightarrow f(2a-x) = -f(x)$

$\because \int_0^{2a} f(x)dx = \int_0^a \{f(x) + f(2a-x)\}\,dx$

$\therefore \int_0^{2a} f(x)\,dx = \int_0^a \{f(x) - f(x)\}\,dx = 0$

14 **(a)**

$\int \frac{dx}{\sin(x-a)\sin(x-b)}$

$= \frac{1}{\sin(a-b)}\int \frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)}\ dx$

$= \frac{1}{\sin(a-b)}$

$\int \frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\ dx$

$$= \frac{1}{\sin(a-b)}\left[\int \cot(x-a)dx - \int \cot(x-b)dx\right]$$
$$= \frac{1}{\sin(a-b)}[\log\sin(x-a) - \log\sin(x-b)] + c$$
$$= \frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + c$$

15 **(b)**

Putting $2 + x = t^2$, we get

$$I = \int \sqrt{\frac{5-x}{2+x}}dx = 2\int \sqrt{7-t^2}\ dt$$
$$\Rightarrow I = t\sqrt{7-t^2} + 7\sin^{-1}\frac{t}{\sqrt{7}} + C$$
$$\Rightarrow I = \sqrt{x+2}\sqrt{5-x} + 7\sin^{-1}\frac{\sqrt{x+2}}{7} + C$$

16 **(a)**

We have,

$$I = \int \frac{2x^2+3}{(x^2-1)(x^2+4)}dx = \int \frac{1}{x^2-1}dx + \int \frac{1}{x^2+4}dx$$
$$\Rightarrow I = \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$
$$\Rightarrow I = -\frac{1}{2}\log\left(\frac{x+1}{x-1}\right) + \frac{1}{2}\tan^{-1}x + C$$
$$\therefore a = -1/2 \text{and} b = 1/2$$

17 **(c)**

$$\int e^x\frac{(x-1)}{x^2}\ dx$$
$$= \int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)\ dx$$
$$= \frac{e^x}{x} + c$$

18 **(d)**

We have,

$$I = \int_0^{3\alpha} \operatorname{cosec}(x-\alpha)\operatorname{cosec}(x-2\alpha)dx$$
$$\Rightarrow I = \frac{1}{\sin\alpha}\int_0^{3\alpha} \frac{\sin\alpha}{\sin(x-\alpha)\sin(x-2\alpha)}dx$$
$$\Rightarrow I = \frac{1}{\sin\alpha}\int_0^{3\alpha} \frac{\sin\{(x-\alpha)-(x-2\alpha)\}}{\sin(x-\alpha)\sin(x-2\alpha)}dx$$
$$\Rightarrow I = \frac{1}{\sin\alpha}\int_0^{3\alpha} \cot(x-2\alpha) - \cot(x-\alpha)\ dx$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\left[\log\frac{\sin(x-2\alpha)}{\sin(x-\alpha)}\right]_0^{3\alpha}$$
$$\Rightarrow I = \frac{1}{\sin\alpha}\left[\log\frac{\sin\alpha}{\sin 2\alpha} - \log\frac{\sin 2\alpha}{\sin\alpha}\right]$$
$$\Rightarrow I = \frac{1}{\sin\alpha}\left[\log\left(\frac{\sin\alpha}{2\sin\alpha\cos\alpha}\right)\right] = \frac{2}{\sin\alpha}\log\left(\frac{1}{2}\sec\alpha\right)$$
$$\Rightarrow I = 2\operatorname{cosec}\alpha\log\left(\frac{1}{2}\sec\alpha\right)$$

19 **(a)**

$$\int_0^3 \frac{3x+1}{x^2+9}\,dx = \frac{3}{2}\int_0^3 \frac{2x}{x^2+9}\,dx + \int_0^3 \frac{1}{x^2+9}\,dx$$
$$= \frac{3}{2}[\log(x^2+9)]_0^3 + \frac{1}{3}\left[\tan^{-1}\frac{x}{3}\right]_0^3$$
$$= \frac{3}{2}[\log 18 - \log 9] + \frac{1}{3}[\tan^{-1}(1) - \tan^{-1}(0)]$$
$$= \frac{3}{2}[\log 2] + \frac{\pi}{12}$$
$$= \log(2\sqrt{2}) + \frac{\pi}{12}$$

20 **(a)**

Let $I = \int \frac{dx}{\sqrt{(1-x)(x-2)}}$

$$= \int \frac{dx}{\sqrt{-x^2+3x-2)}} = \int \frac{dx}{\sqrt{\frac{1}{4} - \left(x-\frac{3}{2}\right)^2}}$$
$$= \sin^{-1}\left(\frac{\left(x-\frac{3}{2}\right)}{\frac{1}{2}}\right) + c = \sin^{-1}(2x-3) + c$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	C	B	D	B	D	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	B	A	B	A	C	D	A	A

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DAILY PRACTICE PROBLEMS

CLASS : XIIth
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SOLUTIONS

SUBJECT : MATHS
DPP NO. :3

Topic :-INTEGRALS

1 **(d)**

Let $g_n(x) = 1 + x^2 + x^4 + \ldots + x^{2n} = \frac{x^{2n+2}-1}{x^2-1}$

$\therefore h_n(x) = g_n'(x) = \dfrac{2x(nx^{2n+2} - (n+1)x^{2n} + 1)}{(x^2-1)^2}$

Now, $f(x) = \lim\limits_{n\to\infty} h_n(x) = \frac{2x}{(x^2-1)^2}$

As $0 < x < 1$

Thus, $\int f(x)dx = \int \frac{2x}{(x^2-1)^2} dx$

$= -\dfrac{1}{x^2-1} = \dfrac{1}{1-x^2}$

2 **(c)**

$$\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$$

$$= \int \frac{dx}{\sin x \frac{\sqrt{2}}{\sqrt{2}} - \cos x \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{2}}$$

$$= \int \frac{dx}{\sqrt{2}(\sin x \sin\frac{\pi}{4} - \cos x \cos\frac{\pi}{4} + 1)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos\left(x + \frac{\pi}{4}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos 2\left(\frac{x}{2} + \frac{\pi}{8}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{2\sin^2\left(\frac{x}{2} + \frac{\pi}{8}\right)}$$

$$= \frac{1}{2\sqrt{2}} \int \operatorname{cosec}^2\left(\frac{x}{2} + \frac{\pi}{8}\right) dx$$

$$= -\frac{1}{2\sqrt{2}} \cdot \frac{-\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)}{\frac{1}{2}} + c = \frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$$

3 **(a)**

Let $I = \int_0^{\pi/2} \frac{\cos x}{1+\sin x}\ dx$

$= [\log(1+\sin x)]_0^{\pi/2}$

$= \log 2 - \log 1 = \log 2$

4 **(b)**

Let $I = \int_0^1 \frac{2\sin^{-1}\frac{x}{2}}{x} dx$

Put $\sin^{-1}\frac{x}{2} = \text{t} \Rightarrow \frac{x}{2} = \sin t$

$\Rightarrow x = 2\sin I$

$\Rightarrow dx = 2\cos t\ dt$

$$\therefore I = \int_0^{\frac{\pi}{6}} \frac{2t}{2\sin t}.2\cos t\ dt$$

$$= \int_0^{\frac{\pi}{6}} \frac{2t}{\tan t} dt$$

$$= \int_0^{\frac{\pi}{6}} \frac{2x}{\tan x} dx$$

5 **(a)**

Given ,$\int_2^e \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right) dx = a + \frac{b}{\log 2}$

Put $\log x = z \Rightarrow x = e^z \Rightarrow dx = e^z dz$

$$\therefore \int_2^e \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right) dx = \int_{\log 2}^1 \left(\frac{1}{z} - \frac{1}{z^2}\right) e^z\ dz$$

$$= \int_{\log 2}^1 e^2 \left(\frac{1}{z} + d\left(\frac{1}{z}\right)\right) dz$$

$$= \left[e^2.\frac{1}{z}\right]_{\log 2}^1 = e - \frac{2}{\log 2}$$

$\therefore\ a = e$ and $b = -2$

7 **(c)**

Let $I = \int_0^1 \frac{x\,dx}{[x+\sqrt{1-x^2}]\sqrt{1-x^2}}$

Put $x = \sin\theta \Rightarrow dx = \cos\theta d\theta$

$$\therefore \quad I = \int_0^{\pi/2} \frac{\sin\theta\cos\theta d\theta}{(\sin\theta + \cos\theta).\cos\theta}$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin\theta}{\sin\theta + \cos\theta} d\theta \ldots \text{(i)}$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)} d\theta$$

$\Rightarrow I = \int_0^{\pi/2} \frac{\cos\theta}{\sin\theta + \cos\theta} d\theta \ldots \text{(ii)}$

On addingEqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \left(\frac{\sin\theta + \cos\theta}{\sin\theta + \cos\theta}\right) d\theta$$

$$= \int_0^{\pi/2} 1 d\theta = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

8 **(d)**

Let $I = \int_0^1 \cot^{-1}(1 - x + x^2)dx$

$$= \int_0^1 \tan^{-1}\left(\frac{1}{x^2 - x + 1}\right) dx$$

$$= \int_0^1 \tan^{-1}\left(\frac{x - (x-1)}{1 + x(x-1)}\right) dx$$

$$= \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1}(x-1)dx$$

$$= \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1}(1 - x - 1)dx$$

$$= 2\int_0^1 \tan^{-1} x\, dx = 2\left[x \tan^{-1} x - \int \frac{x}{1+x^2} dx\right]_0^1$$

$$= 2\left[x \tan^{-1} x - \frac{1}{2}\log(1 + x^2)\right]_0^1$$

$$= 2\left[\left(1 \tan^{-1} 1 - \frac{1}{2}\log 2\right) - \left(0 - \frac{1}{2}\log 1\right)\right]$$

$$= \frac{\pi}{2} - \log 2$$

9 **(a)**

Let $I = \int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}}$

Put $x = \tan^2\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

$dx = 2\tan\theta \sec^2\theta\, d\theta$

$$\therefore I = \int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \frac{2\tan\theta\sec^2\theta}{(\tan^2\theta - 3)\sqrt{\tan^2\theta + 1}} d\theta$$

$$= \int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \frac{2\tan\theta\sec^2\theta}{(\sec^2\theta - 1 - 3)\sec\theta} d\theta$$

$$= \int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \frac{2\tan\theta\sec^2\theta}{(\sec^2\theta - 4)\sec\theta} d\theta$$

$$= \int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \frac{2\tan\theta\sec\theta}{(\sec^2\theta - 4)} d\theta$$

$$= \left[\frac{1}{2}\log\frac{(\sec\theta - 2)}{(\sec\theta + 2)}\right]_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \left[\because \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)\right]$$

$$= \frac{1}{2}\left[\log\left(\frac{\sec\tan^{-1}\sqrt{15}-2}{\sec\tan^{-1}\sqrt{15}+2}\right)-\log\left(\frac{\sec\tan^{-1}\sqrt{8}-2}{\sec\tan^{-1}\sqrt{8}+2}\right)\right]$$
$$= \frac{1}{2}\left[\log\left(\frac{\sec\sec^{-1}4-2}{\sec\sec^{-1}4+2}\right)-\log\left(\frac{\sec\sec^{-1}3-2}{\sec\sec^{-1}3+2}\right)\right]$$
$$= \frac{1}{2}\left[\log\frac{2}{6}-\log\frac{1}{5}=\frac{1}{2}\log\frac{5}{3}\right]$$

10 **(c)**

We have,

$$\Rightarrow I=\int\{1+2\tan x\,(\tan x+\sec x)\}^{1/2}dx$$
$$\Rightarrow I=\int\{1+\tan^2 x+\tan^2 x+2\tan x\sec x\}^{1/2}\,dx$$
$$\Rightarrow I=\int(\sec^2 x+\tan^2 x+2\tan x\sec x)^{1/2}dx$$
$$\Rightarrow I=\int(\tan x+\sec x)dx$$
$$\Rightarrow I=\log\sec x+\log(\sec x+\tan x)+C$$
$$\Rightarrow I=\log\sec x\,(\sec x+\tan x)+C$$

11 **(a)**

We have,

$$I_1=\int_0^\infty\frac{1}{1+x^4}dx \text{ and} I_2=\int_0^\infty\frac{x^2}{1+x^4}dx$$

Putting $x=\frac{1}{t}$in I_1, we get

$$I_1=\int_\infty^0\frac{t^4}{1+t^4}\times-\frac{1}{t^2}dt=\int_0^\infty\frac{t^2}{t+t^4}dt=I_2\Rightarrow\frac{I_1}{I_2}=1$$

12 **(c)**

Let $I=\int\frac{\sin x}{3+4\cos^2 x}dx$

Put $\cos x=t\ \Rightarrow\ -\sin x\,dx=dt$

$$\therefore I=\int\frac{-dt}{3+4t^2}=\frac{-1}{4}\int\frac{dt}{t^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$
$$\Rightarrow I=-\frac{1}{4\cdot\frac{\sqrt{3}}{2}}\cdot\tan^{-1}\frac{t}{\left(\frac{\sqrt{3}}{2}\right)}+c$$
$$=-\frac{1}{2\sqrt{3}}\tan^{-1}\frac{2t}{\sqrt{3}}+c$$
$$\Rightarrow I=\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+c$$

13 **(c)**

Let $f(x)=e^{\cos^2 x\cdot}\cos^3(2n+1)x$

Then, $f(\pi-x)=e^{\cos^2(\pi-x)\cdot}\cos^3[(2n+1)\pi-(2n+1)x]$

$= -e^{\cos^2 x}.\cos^3(2n+1)x$

$\Rightarrow f(\pi - x) = -f(x)$

$\therefore f(x)$is an odd function.

Hence, $\int_0^{\pi} e^{\cos^2 x} \quad .\cos^3(2n+1)x\,dx = 0$

14 **(b)**

Given ,$\frac{d}{dx}\{f(x)\} = \frac{1}{1+x^2}$

On integrating both sides, we get

$f(x) = \tan^{-1} x$

$$\therefore \frac{d}{dx} f(x^3) = \frac{d}{dx} (\tan^{-1} x^3)$$

$$= \frac{1}{1+(x^3)^2}.3x^2$$

$$= \frac{3x^2}{1+x^6}$$

15 **(d)**

Let $I = \int_0^{\pi} [\cot x] dx$ ... (i)

$$\Rightarrow I = \int_0^{\pi} [\cot(\pi - x)] dx = \int_0^{\pi} [-\cot x] dx \quad \text{... (ii)}$$

On addingEqs. (i) and (ii),

$$2I = \int_0^{\pi} [\cot x] dx + \int_0^{\pi} [-\cot x] dx$$

$$= \int_0^{\pi} (-1) dx$$

$[\because [x] + [-x] = -,\text{if} x \notin z\ and [x] + [-x] = 0,\text{if} x \in z]$

$= [-x]_0^{\pi} = -\pi$

$$\therefore I = -\frac{\pi}{2}$$

16 **(c)**

Let $I = \int_{-3}^{2} (|x+1| + |x+2| + |x-1| \quad) dx$

Again, let $f(x) = |x+1| + |x+2| + |x-1|$

$$= \begin{cases} -(x+1)-(x+2)-(x-1), & -3 < x \le -2 \\ -(x+1)+x+2-(x-1), & -2 < x \le -1 \\ 1+x+x+2-(x-1) & -1 < x \le 0 \\ 1+x+x+2-(x-1) & 0 \le x < 1 \\ 1+x+x+2+x-1 & 1 \le x < 2 \end{cases}$$

$$= \begin{cases} -3x-2, & -3 < x \le -2 \\ -x+2, & -2 < x \le -1 \\ x+4, & -1 \le x < 1 \\ 3x+2, & 1 \le x < 2 \end{cases}$$

$$\therefore I=\int_{-3}^{-2}(-3-2)dx+\int_{-2}^{-1}(-x+2)dx$$
$$+\int_{-1}^{1}(x+4)dx+\int_{1}^{2}(3x+2)dx$$
$$=\left[-\frac{3x^2}{2}-2x\right]_{-3}^{-2}+\left[-\frac{x^2}{2}+2x\right]_{-2}^{-1}$$
$$+\left[\frac{x^2}{2}+4x\right]_{-1}^{1}+\left[\frac{3x^2}{2}+2x\right]_{1}^{2}$$
$$=\left[-6+4-\left(-\frac{27}{2}+6\right)\right]+\left[-\frac{1}{2}-2-(-2-4)\right]$$
$$+\left[\frac{1}{2}+4-\left(\frac{1}{2}-4\right)\right]\left[6+4-\left(\frac{3}{2}+2\right)\right]$$
$$=\frac{11}{2}+\frac{7}{2}+8+\frac{13}{2}$$
$$=\frac{31}{2}+8=\frac{47}{2}$$

Alternate

Let $I=\int_{-3}^{2}\{|x+1|+|x+2|+|x-1|\}dx$
$$=\int_{-3}^{-1}|x+1|dx+\int_{-1}^{2}|x+1|dx+\int_{-3}^{-2}|x+2|dx$$
$$+\int_{-2}^{2}|x+2|dx+\int_{-3}^{1}|x-1|dx$$
$$+\int_{1}^{2}|x-1|dx$$
$$=-\int_{-3}^{-1}(x+1)dx+\int_{-1}^{2}(x+1)dx-\int_{-3}^{-2}(x+2)dx$$
$$+\int_{-2}^{2}(x+2)dx-\int_{-3}^{1}(x-1)dx+\int_{1}^{2}(x-1)dx$$
$$=-\left(\frac{x^2}{2}+x\right)_{-3}^{-1}+\left(\frac{x^2}{2}+x\right)_{-1}^{2}-\left(\frac{x^2}{2}+2x\right)_{-3}^{-2}$$
$$+\left(\frac{x^2}{2}+2x\right)_{-2}^{2}-\left(\frac{x^2}{2}-x\right)_{-3}^{1}-\left(\frac{x^2}{2}-x\right)_{1}^{2}$$
$$=\frac{47}{2}$$

17 **(b)**

$$\int_{0}^{3}|x^3+x^2+3x|dx=\int_{0}^{3}(x^3+x^2+3x)dx$$

$= \left[\frac{x^4}{4} + \frac{x^3}{3} + \frac{3x^2}{2}\right]_0^3$

$= \frac{81}{4} + \frac{27}{3} + \frac{27}{2} = \frac{171}{4}$

18 **(b)**

$\int \frac{dx}{\sin x \cos x} = \int \frac{\cos x}{\sin x \cos^2 x} dx$

$= \int \frac{1}{\tan x} \sec^2 x \, dx = \log|\tan x| + c$

19 **(b)**

Let $I = \int_0^{2n\pi} \left\{|\sin x| - \frac{1}{2}|\sin x|\right\} dx$

$= \int_0^{2n\pi} \frac{1}{2}|\sin x| dx$

$= \frac{1}{2}\left[\int_0^{2\pi} |\sin x| dx + \int_{2\pi}^{4\pi} |\sin x| \, dx + \ldots\right.$

$\left. + \int_{2(n-1)\pi}^{2n\pi} |\sin x\} dx \quad \right]$

Now, $I_1 = \int_0^{2\pi} |\sin x| dx$

$= \int_0^{\pi} \sin x \, dx - \int_{\pi}^{2\pi} \sin x \, dx$

$= [-\cos x]_0^{\pi} + [\cos x \quad]_{\pi}^{2\pi} = 4$

$\therefore \; I = \frac{1}{2}[4 + 4 + 4 + \ldots + n \text{ times}]$

$= \frac{1}{2}(4n) = 2n$

$I = \int_0^{2n\pi} \frac{1}{2}|\sin x| \, dx$

Since $|\sin x|$ is periodic with period π.

$\therefore \; I = 2n \int_0^{\pi} \frac{|\sin x| dx}{2}$

$I = n \int_0^{\pi} \sin x \, dx$

$= [-\cos x \quad]_0^{\pi}$

$\Rightarrow I = 2n$

Alternate

$I = \int_0^{2n\pi} \frac{1}{2}|\sin x| dx$

Since, $|\sin x|$is periodic with period π.

$$\therefore I + 2n\int_0^{\pi} \frac{|\sin x|}{2} dx = n\int_0^{\pi} \sin x \, dx = n[-\cos x \quad]_0^{\pi}$$

$\Rightarrow \quad I = 2n$

20 **(d)**

Given, $\int_a^b x^3 \, dx = 0$ and $\int_a^b x^2 \, dx = \frac{2}{3}$

If we take $a = -1$and $b = 1$, then it will satisfy the given integration.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	A	B	A	A	C	D	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	C	B	D	C	B	B	B	D

SESSION : 2025-26

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth

DATE :

SOLUTIONS

SUBJECT : MATHS

DPP NO. :4

Topic :-INTEGRALS

1 **(b)**

Let

$$I = \int_0^{\pi/2} \operatorname{cosec}(x - \pi/3)\operatorname{cosec}(x - \pi/6)dx$$

$$\Rightarrow I = 2\int_0^{\pi/2} \frac{\sin[(x-\pi/6)-(x-\pi/3)]}{\sin(x-\pi/6)\sin(x-\pi/3)}dx$$

$$\Rightarrow I = 2\int_0^{\pi/2} [\cot(x-\pi/3) - \cot(x-\pi/6)]\,dx$$

$$\Rightarrow I = 2\left[\log\sin\left(x-\frac{\pi}{3}\right) - \log\sin\left(x-\frac{\pi}{6}\right)\right]_0^{\pi/2}$$

$$\Rightarrow I = 2\left[\log\left(\frac{\sin(x-\pi/3)}{\sin(x-\pi/6)}\right)\right]_0^{\pi/2}$$

$$\Rightarrow I = 2\left[\log\left(\frac{1/2}{\sqrt{3}/2}\right) - \log\left(\frac{\sqrt{3}/2}{1/2}\right)\right]$$

$$\Rightarrow I = 2[-\log\sqrt{3} - \log\sqrt{3}] = -4\log\sqrt{3} = -2\log 3$$

2 **(c)**

Primitive function means indefinite integral.

$\therefore$ Primitive function of $f(x)$

$$I = \int \frac{\sqrt{(a^2-x^2)}}{x^4}dx \qquad \text{(say)}$$

$$= \int \frac{x\sqrt{\frac{a^2}{x^2}-1}}{x\cdot x^3}dx$$

$$= \int \frac{1}{x^3}\cdot\sqrt{\left(\frac{a^2}{x^2}-1\right)}dx$$

Put $\frac{a^2}{x^2} - 1 = t^2$

$$\Rightarrow -\frac{2a^2}{x^3}dx = 2t\,dt$$

$$\Rightarrow \frac{1}{x^3}dx = -\frac{1}{a^2}t\,dt$$

Then, $I = -\frac{1}{a^2}\int t^2 dt$

$$= -\frac{1}{3a^2}t^3 + c$$

$$= -\frac{1}{3a^2}\left(\frac{a^2}{x^2} - 1\right)^{3/2} + c$$

$$= -\frac{(a^2 - x^2)^{3/2}}{3a^2x^3} + c$$

3 **(c)**

We have,

$$(x) = \begin{cases} x, \text{for } x < 1 \\ x - 1 \text{ for } x \geq 1' \end{cases}$$

$$\Rightarrow x^2 f(x) = \begin{cases} x^3, & \text{for } x < 1 \\ x^3 - x^2 & \text{for } x \geq 1 \end{cases}$$

$$\Rightarrow \int_0^2 x^2 f(x)\,dx = \int_0^1 x^3 dx + \int_1^2 x^3 - x^2\,dx = \frac{5}{3}$$

4 **(d)**

$$\int \frac{x^2 + x - 6}{(x-2)(x-1)}dx = \int \frac{x+3}{x-1}dx$$

$$= \int 1\,dx + \int \frac{4}{(x-1)}dx$$

$$= x + 4\log(x-1) + c$$

5 **(b)**

We have,

$$I = \int \frac{x^3}{(1+x^2)^{1/3}}dx = \frac{1}{2}\int \frac{x^2}{(1+x^2)^{1/3}} \cdot 2x\,dx$$

$$\Rightarrow I = \frac{1}{2}\int \frac{(1+x^2) - 1}{(1+x^2)^{1/3}}d(1+x^2)$$

$$\Rightarrow I = \frac{1}{2}\int \left\{(1+x^2)^{2/3} - (1+x^2)^{-1/3}\right\} d(1+x^2)$$

$$\Rightarrow I = \frac{1}{2}\left\{\frac{3}{5}(1+x^2)^{5/3} - \frac{3}{2}(1+x^2)^{2/3}\right\} + C$$

$$\Rightarrow I = \frac{1}{2}(1+x^2)^{2/3}\left\{\frac{3}{5}(1+x^2) - \frac{2}{3}\right\} + C$$

$$\Rightarrow I = \frac{1}{20}(1+x^2)^{2/3}(6x^2 - 9) + C = \frac{3}{20}(1+x^2)^{2/3}(2x^2 - 3) + C$$

6 **(b)**

We have,

$$\int_{-\pi/4}^{\pi/4} \left\{a|\sin x| + \frac{b\sin x}{1+\cos x} + c\right\} dx = 0$$

$$\Rightarrow a\int_{-\pi/4}^{\pi/4} |\sin x|\, dx + b\int_{-\pi/4}^{\pi/4} \frac{\sin x}{1+\cos x} dx + c\int_{-\pi/4}^{\pi/4} dx = 0$$

$$\Rightarrow 2a\int_{0}^{\pi/4} |\sin x|\, dx + b \times 0 + 2c\int_{0}^{\pi/4} dx = 0$$

$$\Rightarrow 2a\int_{0}^{\pi/4} \sin x\ dx + 2c \times \frac{\pi}{4} = 0$$

$$\Rightarrow -2a\left(\cos\frac{\pi}{4} - \cos 0\right) + \frac{\pi c}{2} = 0$$

$$\Rightarrow -2a\left(\frac{1}{\sqrt{2}} - 1\right) + \frac{\pi c}{2} = 0 \Rightarrow a\left(2-\sqrt{2}\right) + \frac{\pi c}{2} = 0$$

7 **(c)**

Let $I = \int_2^4 \{|x-2| + |x-3|\}dx$

$$= \int_2^3 \{(x-2) + (3-x)\}dx + \int_3^4 \{(x-2) + (x-3)\}dx$$

$$= \int_2^3 dx + \int_3^4 (2x-5)dx$$

$= [x]_2^3 + [x^2 - 5x]_3^4$

$= 3 - 2 + [16 - 20 - (9 - 15)]$

$= 1 + 2 = 3$

8 **(a)**

$g(x) = \int_0^x \cos^4 t\, dt$ (given)

$$g(x+\pi) = \int_0^{\pi+x} \cos^4 t\, dt$$

$$= \int_0^{\pi} \cos^4 t\, dt + \int_{\pi}^{\pi+x} \cos^4 t\, dt = I_1 + I_2$$

$\Rightarrow I_1 = g(\pi)$

$$I_2 = \int_{\pi}^{\pi+x} \cos^4 t\, dt$$

Put $t = \pi + y \Rightarrow dt = dy$

$$I_2 = \int_0^x \cos^4(y+\pi)dy = \int_0^x [\cos(\pi+y)]^4 dy$$

$$= \int_0^x (-\cos y)^4\, dy = \int_0^x \cos^4 y\, dy = g(x)$$

$\therefore\ g(x+\pi) = g(x) + g(\pi)$

9 **(b)**

We have,

$$\Rightarrow I = \int \cos^3 x\, e^{\log(\sin x)} dx = \int \cos^3 x \sin x \ dx$$

$$\Rightarrow I = -\int \cos^3 x \ d(\cos x) = -\frac{\cos^4 x}{4} + C$$

10 **(a)**

Put $x + 1 = t^2 \implies dx = 2t\, dt$

At $x = 8, t = 3$ and $x = 15, t = 4$

$$\therefore I = \int_3^4 \frac{2t\, dt}{(t^2 - 1 - 3)t}$$

$$= \int_3^4 \frac{2dt}{(t^2 - 4)} = 2.\frac{1}{4}\left[\log \frac{t-2}{t+2}\right]_3^4$$

$$= \frac{1}{2}\left[\log \frac{1}{3} - \log \frac{1}{5}\right] = \frac{1}{2}\log \frac{5}{3}$$

11 **(a)**

Using $\sin^2 x = \frac{1-\cos x}{2}$and $\cos^2 x = \frac{1+\cos 2x}{2}$,

We get

$$I = \int \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx = \int \frac{(1-\cos 2x)^2}{2(1+\cos^2 2x)} dx$$

$$\Rightarrow I = \frac{1}{2}\int 1 - \frac{2\cos 2x}{2 - \sin^2 2x} dx$$

$$\Rightarrow I = \frac{1}{2}\left\{x - \frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+\sin 2x}{\sqrt{2}-\sin 2x}\right|\right\} + C$$

12 **(c)**

We have,

$$f(x) = \begin{cases} \int_{-1}^{x} -t\, dt, & -1 \le x \le 0 \\ \int_{-1}^{0} -tdt + \int_{0}^{x} tdt, & x \ge 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{2}(1 - x^2), & -1 \le x \le 0 \\ \frac{1}{2}(1 + x^2), & x \ge 0 \end{cases}$$

13 **(b)**

Since $\sqrt{1+x^2} > x^2$ for all $x \in [1, 2]$. Therefore,

$\frac{1}{\sqrt{1+x^2}} < \frac{1}{x}$for all $x \in [1, 2]$

$$\Rightarrow \int_1^2 \frac{1}{\sqrt{1+x^2}} dx < \int_1^2 \frac{1}{x} dx \Rightarrow I_1 < I_2$$

14 **(a)**

Let $f(x) = (1 - x^2) \sin x \cos^2 x$

$f(-x) = [1 - (-x)^2][\sin(-x)] \cos^2(-x)$

$= -(1 - x^2) \sin x \cos^2 x = -f(x)$

$\Rightarrow f(x)$is an odd function.

$\therefore \int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x \, dx = 0$

15 **(c)**

We have,

$$I_n = \int_0^{\pi/2} x^n \sin x \; dx$$

I II

$$\Rightarrow I_n = [-x^n \cos x]_0^{\pi/2} + n \int_0^{\pi/2} x^{n-1} \cos x \, dx$$

I II

$$\Rightarrow I_n = n[x^{n-1} \sin x]_0^{\pi/2} - n(n-1) \int_0^{\pi/2} x^{n-2} \sin x \, dx$$

$$\Rightarrow I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$$

Putting $n = 4$, we get

$$I_4 + 12I_2 = 4\left(\frac{\pi}{2}\right)^3$$

16 **(b)**

We have,

$$\int_0^2 x[x] \, dx = \int_0^1 x \times 0 \, dx + \int_1^2 x \, dx = \frac{3}{2}$$

17 **(b)**

$$\int_{-1}^{0} \frac{dx}{x^2 + 2x + 2} = \int_{-1}^{0} \frac{dx}{(x+1)^2 + 1}$$

$= [\tan^{-1}(x+1)]_{-1}^{0}$

$= [\tan^{-1} 1 - \tan^{-1} 0] = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

18 **(d)**

Let $I = \int_{-\pi/4}^{\pi/4} x^3 \sin^4 x \, dx$

Since, $x^3 \sin^4 x$ is an odd function

$\therefore \; I = 0$

19 **(c)**

$$I_1 = \int_{1-k}^{k} x f\{x(1-x)\} dx$$

$$= \int_{1-k}^{k} (1-x) f[(1-x)\{1-(1-x)\}]dx (\text{put} x = 1-x)$$

$$= \int_{1-k}^{k} (1-x) f\{x(1-x)\}dx$$

$$= \int_{1-k}^{k} f\{x(1-x)\}dx - \int_{1-k}^{k} x f\{x(1-x)\}dx$$

$$= I_2 - I_1$$

$$\therefore 2I_1 = I_2$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$$

20 **(a)**

We have,

$$I_n = \int_0^{\pi/4} \tan^n x \, dx$$

$$\therefore I_{n+1} + I_{n-1} = \int_0^{\pi/4} (\tan^{n+1} x + \tan^{x-1} x) \, dx$$

$$\Rightarrow I_{n+1} + I_{n-1} = \int_0^{\pi/4} (\tan^{n-1} x \sec^2 x) \, dx$$

$$\Rightarrow I_{n+1} + I_{n-1} = \left[\frac{\tan^n x}{n}\right]_0^{\pi/4} = \frac{1}{n}$$

$$\Rightarrow n(I_{n+1} + I_{n-1}) = 1$$

$$\Rightarrow \lim_{n\to\infty} (I_{n+1} + I_{n-1}) = 1$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	C	D	B	B	C	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	B	A	C	B	B	D	C	A

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :5

Topic :-INTEGRALS

1 **(c)**

$$\int_1^e \frac{1}{x} dx = [\log x]_1^e = \log e - \log 1 = 1$$

2 **(b)**

We have,

$$\lim_{n\to\infty} \frac{1^{99} + 2^{99} + \cdots + n^{99}}{n^{100}} = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{r^{99}}{n^{100}} = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \left(\frac{r}{n}\right)^{99}$$

$$= \int_0^1 x^{99} dx = \left[\frac{x^{100}}{100}\right]_0^1 = \frac{1}{100} - 0 = \frac{1}{100}$$

3 **(a)**

We have,

$$I = \int \frac{\cos 2x}{\cos x} dx = \int 2\cos x - \sec x \, dx$$

$$\Rightarrow I = 2\sin x - \log(\sec x + \tan x) + C$$

$$\Rightarrow I = 2\sin x + \log(\sec x - \tan x) + C$$

4 **(c)**

We have,

$$I = \int f'(ax+b)\{f(ax+b)\}^n \; dx$$

$$\Rightarrow I = \frac{1}{a} \int \{f(ax+b)\}^n d\{f(ax+b)\} = \frac{1}{a} \times \frac{\{(ax+b)\}^{n+1}}{n+1} + C$$

5 **(c)**

$$\because 2^{x^3} < 2^{x^2}, 0 < x < 1 \;\; \text{and} 2^{x^3} > 2^{x^2}, \qquad x > 1$$

$\therefore I_4 > I_3 \text{and} I_2 < I_1$

6 **(c)**

Let $I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} \, dx$

$$= \int \frac{\sqrt{\sin x}}{\sqrt{\cos x} \sin x \cos x} \, dx$$

$$= \int \frac{1}{\sqrt{\sin x} \cos^{3/2} x} \, dx$$

$$= \int \frac{1}{\sqrt{\tan x}\cos^2 x}\, dx$$

$$= \int \frac{\sec^2 x}{\sqrt{\tan x}}\, dx$$

Put $\tan x = t \Rightarrow \sec^2 x\ dx = dt$

$$\therefore\ I = \int \frac{1}{\sqrt{t}}\, dt = 2\sqrt{t} = 2\sqrt{\tan x}$$

7 **(b)**

Let $I = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$

$$= \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{2n} \frac{r}{n\sqrt{1+(r/n)^2}}$$

$$= \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{2n} \frac{r/n}{\sqrt{1+(r/n)^2}}$$

Put $\frac{r}{n} = x, \frac{1}{n} dx, \lim_{n\to\infty}\quad \sum_{r=1}^{2n} = \int_0^2$

$$\therefore I = \int_0^2 \frac{x}{\sqrt{1+x^2}} dx = [\sqrt{1+x^2}\quad]_0^2 = \sqrt{5}-1$$

8 **(a)**

$$\because I = \int_0^\infty \frac{x\log x}{(1+x^2)^2} dx$$

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$$\therefore I = \int_0^{\pi/2} \frac{\tan\theta \log(\tan\theta)}{\sec^4\theta} \sec^2\theta\, d\theta$$

$$= \int_0^{\pi/2} \frac{\sin\theta\log(\tan\theta)}{\cos\theta\cdot\frac{1}{\cos^2\theta}} d\theta$$

$$= \int_0^{\pi/2} \sin\theta\cos\theta\log(\tan\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \sin 2\theta\log(\tan\theta)\, d\theta = 0\left(\because \int_0^{\pi/2} \sin\ 2\theta\log\tan\theta\, d\theta = 0\right)$$

9 **(a)**

$$\int_{-1}^{1} [x[1+\sin\pi x]+1]dx$$

$$= \int_{-1}^{0} [x[1+\sin\pi x]+1]dx + \int_0^1 [x[1+\sin\pi x]+1]dx$$

Now, $-1 < x < 0\ \Rightarrow\ [1+\sin\pi x] = 0$

$0 < x < 1\ \Rightarrow\ [1+\sin\pi x] = 1$

$\Rightarrow\ [x[1+\sin\pi x]+1] = 1$

$$\therefore\ \int_{-1}^{1} [x[1+\sin\pi x]+1]dx = 2$$

10 **(d)**

We have,

$$I = \int_0^{\infty} e^{-ax^2}\, dx$$

$$\Rightarrow I = \frac{1}{\sqrt{a}} \int_0^{\infty} e^{-(\sqrt{a}\,x)^2}\, d(\sqrt{a}x)$$

$$\Rightarrow I = \frac{1}{\sqrt{a}} \times \frac{\sqrt{\pi}}{2} \Rightarrow I = \frac{1}{2}\frac{\sqrt{\pi}}{a}\left[\because \int_0^{\infty} e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}\right]$$

11 **(d)**

Let $I = \int \frac{2x}{\sqrt{1-4^x}}\, dx \cdot$ Then,

$$I = \frac{1}{\log 2} \int \frac{1}{1-(2^x)^2}\, d(2^x) = \frac{1}{\log 2} \sin^{-1}(2^x) + C$$

$$\therefore K = \frac{1}{\log 2}$$

12 **(d)**

We have,

$$I = \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2\, dx$$

$$\Rightarrow I = \int_{-\pi}^{\pi} (\cos^2 ax - \sin^2 bx - 2\cos ax \sin bx)\, dx$$

$$\Rightarrow I = \int_{-\pi}^{\pi} (\cos^2 ax - \sin^2 bx)\, dx - 2\int_{-\pi}^{\pi} \cos ax \sin bx\, dx$$

$$\Rightarrow I = 2\int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 bx)\, dx - 0$$

$$\Rightarrow I = 2\int_0^{\pi} \frac{1+\cos 2ax}{2} + \frac{1-\cos 2bx}{2}\, dx$$

$$\Rightarrow I = \int_0^{\pi} 2 + \cos 2ax - \cos 2bx\, dx = 2\pi$$

13 **(d)**

Let $I = \int \frac{mx^{m+2n-1} - nx^{n-1}}{x^{2m+2n} + 2x^{m+n} + 1}\, dx$

$$= \int \frac{mx^{m+2n-1} - nx^{n-1}}{(1+x^{m+n})^2}\, dx$$

$$\therefore I = \int \frac{mx^{m-1} - \frac{n}{x^{n+1}}}{\left(x^m + \frac{1}{x^n}\right)^2}\, dx$$

Let $x^m + \frac{1}{x^n} = t$

$\Rightarrow mx^{m-1} - \frac{n}{x^{n+1}} dx = dt$

$\therefore I = \int \frac{1}{t^2} dt = -\frac{1}{t} + c$

$= \frac{-1}{\left(x^m + \frac{1}{x^n}\right)} + c = \frac{-x^n}{x^{m+n} + 1} + c$

14 **(b)**

We have,

$$I = \int_0^{16\pi/3} |\sin x| dx = \int_0^{5\pi} |\sin x| dx + \int_{5\pi}^{16\pi/3} |\sin x| dx$$

$$\Rightarrow I = 5\int_0^{\pi} |\sin x|\, dx + \int_0^{\pi/3} |\sin x|\, dx \quad [\because |\sin x| \text{is periodic with period } \pi]$$

$$\Rightarrow I = 5\int_0^{\pi} \sin x\, dx + \int_0^{\pi/3} \sin x\, dx = 5 \times 2 + \left(-\frac{1}{2} + 1\right) = \frac{21}{2}$$

15 **(d)**

Let $I = \int_{-\pi/2}^{\pi/2} \{f(x) + f(-x)\}\{g(x) - g(-x)\}\ dx$

Again, let

$h(x) = \{f(x) + f(-x)\}\{g(x) - g(-x)\}$

$\Rightarrow h(-x) = \{f(-x) + f(x)\}\{g(-x) - g(x)\}$

$\Rightarrow h(-x) = -h(x)$

Hence, $h(x)$ is an odd function

$\therefore\ I = 0$

16 **(a)**

We have,

$$I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx = \int \frac{\sqrt{\tan x}}{\tan x} \sec^2 x\, dx$$

$$\Rightarrow I = \int (\tan x)^{-1/2}\, d(\tan x) = 2\sqrt{\tan x} + C$$

18 **(c)**

We have,

$$\int_{-\pi/3}^{\pi/3} \left\{\frac{a}{3}|\tan x| + \frac{6 \tan x}{1 + \sec x} + c\right\} dx = 0$$

$$\Rightarrow \frac{a}{3}\int_{-\pi/3}^{\pi/3}|\tan x|\,dx + b\int_{-\pi/3}^{\pi/3}\frac{\tan x}{1+\sec x}dx + c\int_{-\pi/3}^{\pi/3}dx = 0$$

$$\Rightarrow 2a\int_{0}^{\pi/3}\tan x\,dx + b\times 0 + c\left(\frac{2\pi}{3}\right) = 0$$

$$\Rightarrow \frac{2a}{3}[\log_e|\sec x|]_0^{\pi/3} + \frac{2\pi}{3}c = 0$$

$$\Rightarrow \frac{2a}{3}\ln 2 + \frac{2\pi}{3}c = 0 \Rightarrow c = -\frac{a}{\pi}\ln 2$$

19 **(d)**

Given, $f'(2) = \tan\frac{\pi}{4} = 1, f'(4) = \tan\frac{\pi}{3} = \sqrt{3}$

$$\therefore \int_2^4 f'(x)f''(x)dx = \left[\frac{1}{2}[f'(x)\quad]^2\right]_2^4$$

$$= \frac{1}{2}[f'(4)\quad]^2 - \frac{1}{2}[f'(2)]^2$$

$$= \frac{3}{2} - \frac{1}{2} = 1$$

20 **(a)**

Let $I = \int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

$$= \int \frac{dx}{\cos^3 x\sqrt{4\sin x\cos x}} = \frac{1}{2}\int\frac{dx}{\cos^{7/2}x\sin^{1/2}x}$$

$$= \frac{1}{2}\int\frac{\sec^4 x}{\sqrt{\tan x}}dx = \frac{1}{2}\int\frac{(1+\tan^2 x)\sec^2 x}{\sqrt{\tan x}}dx$$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$\therefore I = \frac{1}{2}\int\frac{1+t^2}{\sqrt{t}}dt$$

$$= \frac{1}{2}\int t^{-1/2}dt + \frac{1}{2}\int t^{3/2}dt = t^{1/2} + \frac{t^{5/2}}{5} + c$$

$$= \sqrt{\tan x} + \frac{1}{5}\tan^{5/2}x + c$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	A	C	C	C	B	A	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	D	D	B	D	A	B	C	D	A

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :6

Topic :-INTEGRALS

1 **(b)**

On putting, $x = \tan\theta \Rightarrow \quad dx = sce^2\theta\, d\theta$, we get

$$f(x) = \int \frac{\tan^2\theta . \sec^2\theta}{\sec^2\theta(1+\sec\theta)} d\theta$$

$$= \int \frac{\sec^2\theta - 1}{1+\sec\theta} d\theta = \int (\sec\theta - 1)\, d\theta$$

$$= \log(\sec\theta + \tan\theta) - \theta + c$$

$$\Rightarrow \quad f(x) = \log(\sqrt{1+x^2} + x) - \tan^{-1} x + c$$

At $x = 0,\ f(0) = \log(1+0) - 0 + c \ \Rightarrow \ c = 0$

$\therefore \ f(x) = \log(\sqrt{1+x^2} + x) - \tan^{-1} x$

At $x = 1,\ f(1) = \log(1+\sqrt{2}) - \frac{\pi}{4}$

2 **(b)**

We have,

$I = \int_0^1 \frac{1}{(1+x^2)^{3/2}} dx = \int_0^{\pi/4} \cos\theta$, where $x = \tan\theta$

$$\Rightarrow I = \frac{1}{\sqrt{2}}$$

43 **(a)**

We have, $I(m,n) = I = \int_0^1 t^m (1+t)^n dt$

$$\Rightarrow \ I(m,n) = \left[(1+t)^n \cdot \frac{t^{m+1}}{m+1}\right]_0^1 - \frac{n}{m+1}\int_0^1 (1+t)^{n-1} t^{m+1} dt$$

$$\Rightarrow \ I(m,n) = \frac{2^n}{m+1} - \frac{n}{m+1} I(m+1, n-1)$$

4 **(b)**

Let $I = \int_0^1 \frac{\tan^{-1} x}{1+x^2}\, dx$

Put $\tan^{-1} x = t \Rightarrow \frac{1}{1+x^2} dx = dt$

$$\therefore \ I = \int_0^{\pi/4} t\, dt = \left[\frac{t^2}{2}\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{4}\right)^4 - 0^2\right] = \frac{\pi^2}{32}$$

5 **(b)**

Let $I=\int_3^5 \frac{x^2}{x^2-4}\,dx=\int_3^5\left(\frac{x^2-4}{x^2-4}+\frac{4}{x^2-4}\right)dx$

$=\int_3^5\left(1+\frac{4}{x^2-4}\right)dx$

$=\left[x+\frac{4}{2\times 2}\log_e\left(\frac{x-2}{x+2}\right)\right]_3^5$

$=\left[5+\log_e\left(\frac{5-2}{5+2}\right)-3-\log_e\left(\frac{3-2}{3+2}\right)\right]$

$=2+\log_e\left(\frac{3}{7}\right)-\log_e\left(\frac{1}{5}\right)$

$=2+\log_e\left(\frac{3}{7}\times\frac{5}{1}\right)=2+\log_e\left(\frac{15}{7}\right)$

6 **(b)**

$\int\sqrt{x^2+a^2}\,dx=\int\sqrt{x^2+a^2}\cdot 1\,dx$

$=\sqrt{x^2+a^2}\int 1dx-\int\left[\frac{d}{dx}\left(\sqrt{x^2+a^2}\right)\int 1dx\right]dx$

$=x\sqrt{x^2+a^2}-\int\left[\frac{2x}{2\sqrt{x^2+a^2}}x\right]dx$

$=x\sqrt{x^2+a^2}-\int\left[\frac{x^2+a^2-a^2}{\sqrt{x^2+a^2}}\right]dx$

$=x\sqrt{x^2+a^2}-\int\left[\sqrt{x^2+a^2}-\frac{a^2}{\sqrt{x^2+a^2}}\right]dx$

$=x\sqrt{x^2+a^2}-\int\sqrt{x^2+a^2}dx+a^2\int\frac{dx}{\sqrt{x^2+a^2}}$

$=x\sqrt{x^2+a^2}-I+a^2\log[x+\sqrt{x^2+a^2}]+c$

$\Rightarrow 2I=x\sqrt{x^2+a^2}+a^2\log[x+\sqrt{x^2+a^2}]+c$

$\Rightarrow I=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left[x\sqrt{x^2+a^2}\right]+c$

7 **(b)**

$I_n+I_{n+2}=\int_0^{\pi/4}\tan^n x(1+\tan^2 x)dx=\int_0^{\pi/4}\tan^n x\sec^2 x\,dx$

Put $\tan x=t\Rightarrow\sec^2 xdx=dt$

$\therefore I_n+I_{n+2}=\int_0^1 t^n\,dt=\frac{1}{n+1}$

$\Rightarrow\lim_{n\to\infty}n[I_n+I_{n+2}]=\lim_{n\to\infty}\frac{n}{n+1}=\lim_{n\to\infty}\frac{1}{\left(1+\frac{1}{n}\right)}=1$

8 **(b)**

Put $x+1=t^2$

$\Rightarrow\ dx=2t\,dt$

$\Rightarrow\ x^2 + 1 = (t^2 - 1)^2 + 1$

$= t^4 - 2t^2 + 2$

$\therefore$ Given integral

$= \int (t^4 - 2t^2 + 2)t.2t\,dt$

$= 2\int (t^6 - 2t^4 + 2t^2)dt$

$= 2\left[\frac{t^7}{7} - 2\frac{t^5}{5} + 2\frac{t^3}{3}\right] + c$

$= 2\left[\frac{(x+1)^{\frac{7}{2}}}{7} - 2\frac{(x+1)^{\frac{5}{2}}}{5} + 2\frac{(x+1)^{\frac{3}{2}}}{3}\right] + c$

9 **(c)**

$\int \frac{f'(x)}{f(x)\log[f(x)]}dx = \log[\log f(x)] + c$

10 **(d)**

We have, $0 < x < 1$

$\therefore \frac{1}{2}x^2 < x^2 < x$

$\Rightarrow -x^2 > -x$

$\Rightarrow e^{-x^2} > e^{-x}$

$\Rightarrow e^{-x^2}\cos^2 x > e^{-x}\cos^2 x$

$\Rightarrow \int_0^1 e^{-x^2}\cos^2 x\,dx > \int_0^1 e^{-x}\cos^2 x\ dx$

$\Rightarrow I_2 > I_1$...(i)

Also, $\cos^2 x \leq 1$

$\Rightarrow e^{-x^2}\cos^2 x < e^{-x^2} < e^{-(1/2)x^2}\left[\because -\frac{1}{2}x^2 > -x^2 > -x\right]$

$\Rightarrow \int_0^1 e^{-x^2}\cos^2 x\,dx < \int_0^1 e^{-x^2}\,dx < \int_0^1 e^{-(1/2)x^2}\,dx$

$\Rightarrow I_2 < I_3 < I_4$...(ii)

From (i) and (ii), we get $I_1 < I_2 < I_3 < I_4$

Hence, I_4 is the greatest integral

11 **(c)**

We have,

$$I=\int \frac{1}{(1+x^2)\sqrt{1-x^2}}dx$$
$\Rightarrow I=\int \frac{-t\,dt}{(t^2+1)\sqrt{t^2-1}}$, where $x=\frac{1}{t}, dx=-\frac{1}{t^2}dt$
$\Rightarrow I=\int \frac{-u\,du}{(u^2+2)\sqrt{u^2}}$, where $t^2-1=u^2$
$$\Rightarrow I=-\int \frac{1}{u^2+\left(\sqrt{2}\right)^2}du=-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)+K$$
$$\Rightarrow I=-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right)+K$$
$$\Rightarrow I=-\frac{1}{\sqrt{2}}\left\{\frac{\pi}{2}-\cot^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right)\right\}+K\ [\because \tan^{-1}x+\cot^{-1}x=\pi/2]$$
$$\Rightarrow I=-\frac{1}{\sqrt{2}}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right)+\left(K-\frac{\pi}{2\sqrt{2}}\right)$$
$$\Rightarrow I=-\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)+C, \text{where } C=K-\frac{\pi}{2\sqrt{2}}$$
12 **(c)**
$$I=\int_{1/n}^{(an-1)/n}\frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx$$
$=\int_{1/n}^{a-(1/n)}\frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx$ ...(i)
$$=\int_{1/n}^{a-(1/n)}\frac{\sqrt{\frac{1}{n}+a-\frac{1}{n}-x dx}}{\sqrt{a-\left(\frac{1}{n}+a-\frac{1}{n}-x\right)}+\sqrt{\left(\frac{1}{n}+a-\frac{1}{n}-x\right)}}$$
$\Rightarrow I=\int_{\frac{1}{n}}^{a-\frac{1}{n}}\frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$ ...(ii)
On adding Eqs.(i) and (ii), we get
$$2I=\int_{1/n}^{a-(1/n)}1\,dx=[x]_{\frac{1}{n}}^{a-\frac{1}{n}}$$
$$\Rightarrow 2I=a-\frac{1}{n}-\frac{1}{n}=\frac{na-2}{n}$$
$$\Rightarrow I=\frac{na-2}{2n}$$
13 **(b)**
Let $I=\int e^x(x^5+5x^4+1)dx$
$$=\int e^x x^5 dx+5\int e^x x^4 dx+\int e^x\,dx$$
$$=x^5e^x-\int 5x^4e^x dx+5\int e^x x^4 dx+e^x$$
$$=x^5e^x+e^x+c$$
14 **(a)**
$\cos x\log\left(\frac{1+x}{1-x}\right)$is an odd function

$\therefore \int_{-1/2}^{1/2} \cos x \log\left(\frac{1+x}{1-x}\right) dx = 0$

$\therefore k = 0$

15 **(b)**

Let $I = \int \sin\sqrt{3}\ dx$

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\therefore I = \int 2t \sin t\, dt$

$= 2[-t\cos t + \int \cos t\, dt]$

$= 2[-t\cos t + \sin t] + c$

$= 2\left[-\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x}\right] + c$

16 **(d)**

$f'(x) = (x^2-1)(2x) - (x-1) = (x-1)(2x^2+2x-1)$

Which is positive for $x > 1$. Hence, f increases in [1, 2]

Hence, global maximum of f is $f(2) = 4$

17 **(b)**

$$\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \frac{\left(\frac{r}{n}\right)}{\sqrt{1+\left(\frac{r}{n}\right)^2}} = \frac{1}{2}\int_0^1 \frac{2x}{\sqrt{1+x^2}} dx$$

$$= \frac{1}{2}\left[\frac{\sqrt{1+x^2}}{1/2}\right]_0^1 = \sqrt{2} - 1$$

18 **(d)**

We have,

$$I = \int_0^{2\pi} \cos^{99} x\, dx,$$

$$\Rightarrow I = 2\int_0^{\pi} \cos^{99} x\, dx \quad [\because \cos^{99}(2\pi - x) = \cos^{99} x]$$

But, $\int_0^{\pi} \cos^{99} x\, dx = 0 \quad [\because \cos^{99}(\pi - x) = -\cos^{99} x]$

$\therefore I = 2 \times 0 = 0$

19 **(b)**

Let $I = \int_{-2}^{2}(ax^3 + bx + c)dx$

In the given integral, ax^3 and bx are odd functions. Hence, it depends only on the value of c.

20 **(b)**

Let $I = \int_0^{\pi} x\, f(\sin x) dx$

$$\Rightarrow I = \int_0^{\pi} (\pi - x) f\{\sin(\pi - x)\} dx$$

$$= \pi \int_0^{\pi} f(\sin x)dx - x\int_0^{\pi} f(\sin x)dx$$

$$\Rightarrow I = \pi \int_0^{\pi} f(\sin x)dx - I$$

$$\Rightarrow 2I = 2\pi \int_0^{\pi/2} f(\sin x)dx$$

$$\Rightarrow I = \pi \int_0^{\pi/2} f(\sin x)dx$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	A	B	B	B	B	B	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	B	A	B	D	B	D	B	B

SESSION : 2025-26

DPP
DAILY PRACTICE PRODUCTS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :7

Topic :-INTEGRALS

1 **(a)**

Let $I=\int \frac{\log(x+1)-\log x}{x(x+1)}dx=\int \frac{\log\left(1+\frac{1}{x}\right)}{x(x+1)}dx$

$=\int \frac{\log\left(1+\frac{1}{x}\right)}{x^2\left(1+\frac{1}{x}\right)}dx$

Put $1+\frac{1}{x}=t \quad \Rightarrow \quad -\frac{1}{x^2}\,dx=dt$

$\therefore\ I=-\int \frac{\log t}{t}dt=-\frac{1}{2}(\log t)^2+c$

$=-\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2+c$

$=-\frac{1}{2}[\log(x+1)-\log x]^2+c$

$=-\frac{1}{2}\{\log(x+1)\}^2-\frac{1}{2}(\log x)^2+\log(x+1).\log x+c$

2 **(c)**

Let $I=\int_0^1 x\left|x-\frac{1}{2}\right|dx$

$=-\int_0^{1/2} x\left(x-\frac{1}{2}\right)dx+\int_{1/2}^1 x\left(x-\frac{1}{2}\right)dx$

$=\left[\frac{x^2}{4}-\frac{x^3}{3}\right]_0^{1/2}+\left[\frac{x^3}{3}-\frac{x^2}{4}\right]_{1/2}^1$

$=\left(\frac{1}{16}-\frac{1}{24}\right)+\left(\frac{1}{3}-\frac{1}{4}-\frac{1}{24}+\frac{1}{16}\right)=\frac{1}{8}$

3 **(a)**

Let

$I=\int \frac{a^{x/2}}{\sqrt{a^{-x}-a^x}}dx=\int \frac{a^x}{\sqrt{1-(a^x)^2}}dx$

$\Rightarrow I=\frac{1}{\log a}\int \frac{1}{\sqrt{1^2-(a^x)^2}}d(a^x)=\frac{1}{\log a}\sin^{-1}(a^x)+C$

4 **(c)**

Let $I=\int_{\log 2}^{x} \frac{e^u}{e^u(e^u-1)^{1/2}}du$

Put $e^u - 1 = t^2 \Rightarrow e^u du = 2t\, dt$

$\therefore I = \int_1^{\sqrt{e^x-1}} \frac{2t}{(t^2+1)t} dt$

$= 2\int_1^{\sqrt{e^x-1}} \frac{dt}{(1+t^2)}$

$= 2[\tan^{-1} t \quad]_1^{\sqrt{e^x-1}}$

$= 2\left[\tan^{-1}\sqrt{e^x-1} - \frac{\pi}{4}\right] = \frac{\pi}{6}$ [given]

$\Rightarrow \tan^{-1}\sqrt{e^x-1} = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3}$

$\Rightarrow \sqrt{e^x-1} = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

$\Rightarrow \quad e^x = 3 + 1 = 4$

5 **(c)**

We have,

$I = \int \{f(x)g''(x) - f''(x)g(x)\}dx$

$\Rightarrow I = \int f(x)g''(x)dx - \int f''(x)g(x)dx$

$\Rightarrow I = \left\{f(x)g'(x) - \int f'(x)g'(x)dx\right\} - \left\{g(x)f'(x) - \int g'(x)f'(x)dx\right\}$

$\Rightarrow I = f(x)g'(x) - f'(x)g(x)$

6 **(d)**

Let $I = \int_{-\pi/4}^{\pi/4} x^3 \sin^4 x\, dx$

Since, $x^3 \sin^4 x$ is an odd function.

$\therefore I = 0$

7 **(a)**

Let $I = \int_0^{\pi/2} \sin^8 x\ dx = \frac{7.5.3.1}{8.6.4.2}.\frac{\pi}{2}$ [by Walli's formula]

$= \frac{105\pi}{32(4.3.2.1)}$

$= \frac{105\pi}{32.4!}$

8 **(b)**

We have,

$$I = \int_{-\pi/2}^{\pi/2} \frac{|x|}{8\cos^2 2x + 1} dx$$

$$= 2\int_0^{\pi/2} \frac{x}{8\cos^2 2x + 1} dx$$

$= 2I_1, \text{where} I_1 = \int_0^{\pi/2} \frac{x}{8\cos^2 2x + 1} dx$

Now,

$I_1 = \int_0^{\pi/2} \frac{x}{8\cos^2 2x+1} dx$ ...(i)

$\Rightarrow I_1 = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{8\cos^2(\pi - 2x) + 1} dx$

$\Rightarrow I_1 = \int_0^{\pi/2} \frac{\frac{\pi}{2}-x}{8\cos^2 2x+1} dx$ ...(ii)

Adding (i) and (ii), we get

$2I_1 = \frac{\pi}{2} \int_0^{\pi/2} \frac{x}{8\cos^2 2x + 1} dx$

$\Rightarrow I_1 = \frac{\pi}{4} \int_0^{\pi/2} \frac{\sec^2 2x}{9 + \tan^2 2x} dx = 2 \times \frac{\pi}{4} \int_0^{\pi/2} \frac{\sec^2 2x}{9 + \tan^2 2x} dx$

$\Rightarrow I_1 = \frac{\pi}{4} \int_0^{\pi/4} \frac{2\sec^2 2x}{9 + \tan^2 2x} dx$

$\Rightarrow I_1 = \frac{\pi}{4} \int_0^{\pi/4} \frac{1}{3^2 + \tan^2 2x} d(\tan 2x)$

$\Rightarrow I_1 = \frac{\pi}{4} \times \frac{1}{3} \left[\tan^{-1}\left(\frac{\tan 2x}{3}\right)\right]_0^{\pi/4} = \frac{\pi}{12} \times \frac{\pi}{2} = \frac{\pi^2}{24}$

Hence, $I = 2I_1 = \frac{\pi^2}{12}$

9 **(b)**

$\because f(x)\cos x = \frac{1}{2} \cdot 2f(x)f'(x)$

Then, $f'(x) = \cos x$

$\therefore f(x) = \sin x + c$

10 **(d)**

On putting $y^2 = f(x) = \frac{x+2}{2x+3}$, we have

$x = \frac{3y^2-2}{1-2y^2} \text{and} dx = -\frac{2y}{(1-2y^2)^2} dy$

$\therefore \int (f(x))^{1/2} \frac{dx}{x} = -\int y \frac{2y}{(1-2y^2)^2} \cdot \frac{1-2y^2}{3y^2-2} dy$

$= 2\int \frac{y^2}{(2y^2-1)(3y^2-2)} dy$

$= -2\int \left[\frac{1}{2y^2-1} - \frac{2}{3y^2-2}\right] dy$

$$= \frac{1}{\sqrt{2}} \log \left| \frac{1+\sqrt{2y}}{1-\sqrt{2y}} \right| - \sqrt{\frac{2}{3}} \log \left| \frac{\sqrt{3y}+\sqrt{2}}{\sqrt{3y}-\sqrt{2}} \right| + c$$

Thus, $g(x) = \log|x|$ and $h(x) = \log|x|$

11 **(c)**

Let $5^{5^{5^x}} = t \because$ Then, $5^{5^{5^x}} 5^{5^x} 5^x (\log 5)^3 dx = dt$,

$$\therefore I = \int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x \, dx = \frac{1}{(\log 5)^3} \int 1 \cdot dt = \frac{5^{5^{5^x}}}{(\log 5)^3} + C$$

12 **(a)**

$$f(t) = \int_{-t}^{t} \frac{e^{-[x]}}{2} dx = 2 \int_0^t \frac{e^{-x}}{2} dx$$

$$= -[e^{-x} \quad]_0^t = -e^{-t} + 1$$

Now, $\lim\limits_{t \to \infty} f(t) = -\lim\limits_{t \to \infty} e^{-t} + 1 = 1$

13 **(b)**

We have,

$$I = \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$$

$$\Rightarrow I = \log(\sin x + \cos x) + C$$

14 **(c)**

The primitive of the given function is

$$\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\theta)}} = \int \frac{dx}{\sqrt{\sin^3 x (\sin x \cos \theta + \cos x \sin \theta)}}$$

$$= \int \frac{\operatorname{cosec}^2 x \, dx}{\sqrt{\cos \theta + \cot x \sin \theta}}$$

Put $\cot x = t \Rightarrow -\operatorname{cosec}^2 x \, dx = dt$

$$= -\frac{1}{\sqrt{\sin \theta}} \int \frac{dt}{\sqrt{\cot \theta + 1}}$$

$$= -\frac{2}{\sqrt{\sin \theta}} (\cot \theta + t)^{1/2} + c$$

$$= -\frac{2}{\sin \theta} (\cos \theta + t \sin \theta)^{1/2} + c$$

$$= \frac{-2 \operatorname{cosec} \theta (\sin x \cos \theta + \sin \theta \cos x)^{1/2}}{\sqrt{\sin x}} + c$$

$$= -2 \operatorname{cosec} \theta \left(\frac{\sin(\theta + x)}{\sin x} \right)^{1/2} + c$$

15 **(b)**

We have,

$$I = \int \frac{x^4 - 1}{x^2 \sqrt{x^4 + x^2 + 1}} dx = \int \frac{(x^2-1)(x^2+1)}{x^2 \sqrt{x^4 + x^2 + 1}} dx$$

$$\Rightarrow I=\int\frac{\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x^2}\right)}{\sqrt{x^2+\frac{1}{x^2}+1}}dx=\int\frac{\left(x+\frac{1}{x}\right)}{\sqrt{\left(x+\frac{1}{x}\right)^2-1^2}}d\left(x+\frac{1}{x}\right)$$

$$\Rightarrow I=\frac{1}{2}\int\frac{2\left(x+\frac{1}{x}\right)}{\sqrt{\left(x+\frac{1}{x}\right)^2-1^2}}d\left(x+\frac{1}{x}\right)$$

$$\Rightarrow I=\frac{1}{2}\int\frac{1}{\sqrt{\left(x+\frac{1}{x}\right)^2-1^2}}d\left\{\left(x+\frac{1}{x}\right)^2-1^2\right\}$$

$$\Rightarrow I=\sqrt{\left(x+\frac{1}{x}\right)^2-1^2+C}=\sqrt{x^2+\frac{1}{x^2}+1+C}$$

$$\Rightarrow I=\sqrt{\frac{x^4+x^2+1}{x}+C}$$

16 **(b)**

$$\int 32x^3(\log x)^2\ dx$$

$$=32\{(\log x)^2\frac{x^4}{4}-\int 2\log x\cdot\frac{1}{x}\cdot\frac{x^4}{4}dx\}$$

$$=8x^4(\log x)^2-16\int x^3\log x\,dx$$

$$=8x^4(\log x)^2-16\,\{\log x\cdot\frac{x^4}{4}-\int\frac{1}{x}\cdot\frac{x^4}{4}dx\ \}$$

$$=8x^4(\log x)^2-4x^4\log x+4\int x^3dx$$

$$=8x^4(\log x)^2-4x^4+\log x+x^4+c$$

$$=x^4[\ 8(\log x)^2-4\log x+1\ ]+c$$

17 **(a)**

Let $I=\int\frac{e^x(1+\sin x)}{1+\cos x}dx$

$$=\int\frac{1}{2}e^x\sec^2\frac{x}{2}dx+\int e^x\tan\frac{x}{2}dx$$

$$=\frac{1}{2}\left[2e^x\tan\frac{x}{2}-\int 2e^x\tan\frac{x}{2}dx\right]+\int e^x\tan\frac{x}{2}dx$$

$$=e^x\tan\frac{x}{2}+c$$

18 **(a)**

We have,

$$I=\int\log\left(\sqrt{1-x}+\sqrt{1+x}\right)\cdot 1\,dx$$

I II

$$\Rightarrow I=x\log\left(\sqrt{1-x}+\sqrt{1+x}\right)-\int\frac{1}{2}\frac{1}{\sqrt{1-x^2}}\left(\frac{\sqrt{1-x^2}}{x}-\frac{1}{x}\right)\cdot x\,dx$$

$\Rightarrow I = x\log(\sqrt{1-x}+\sqrt{1+x}) - \frac{1}{2}\int \left(1-\frac{1}{\sqrt{1-x^2}}\right)dx$

$\Rightarrow I = x\log(\sqrt{1-x}+\sqrt{1+x}) - \frac{x}{2} + \frac{1}{2}\sin^{-1}x + c$

Hence, $f(x) = \log(\sqrt{1-x}+\sqrt{1+x})$, $A = -\frac{1}{2}$ and $B = \frac{1}{2}$

19 **(a)**

Let $I = \int_0^{\pi/4} \log(1+\tan x)dx$ ....(i)

$\Rightarrow I = \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$

$= \int_0^{\pi/4} \log\left[1+\frac{1-\tan x}{1+\tan x}\right]dx$

$= \int_0^{\pi/4} \log 2dx - \int_0^{\pi/4} \log(1+\tan x)dx$

$\Rightarrow I = \log 2[x]_0^{\pi/4} - I$

$\Rightarrow 2I = \frac{\pi}{4}\log_e 2 \Rightarrow I = \frac{\pi}{8}\log_e 2$

20 **(a)**

$f(x) = |x|\sin^3 x$is an odd function

$\therefore \int_{-2}^{2} |x|\sin^3 x\, dx = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	A	C	C	D	A	B	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	B	C	B	B	A	A	A	A

SESSION : 2025-26

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth | **SOLUTIONS** | **SUBJECT : MATHS**

DATE : | | **DPP NO. :8**

Topic :-INTEGRALS

1 **(c)**

Since, $e^{x-[x]}$is a periodic function with period 1.

$$\therefore \int_0^{1000} e^{x-[x]} dx = 1000 \int_0^1 e^x dx = 1000[e^x]_0^1$$

$= 1000(e-1)$

2 **(b)**

Put $\tan^{-1} x = t \Rightarrow \frac{1}{(1+x^2)} dx = dt$

$$\therefore \int \frac{(\tan^{-1} x)^3}{(1+x^2)} dx = \int t^3\, dt = \frac{t^4}{4} + c = \frac{(\tan^{-1} x)^4}{4} + c$$

3 **(c)**

Let $I = \int_0^{\pi} \sin^3 \theta d\theta$

$\left[\begin{array}{c} \because \sin\theta > 0 \\ \text{for } 0 < \theta < \pi \end{array}\right]$

$$= \int_0^{\pi} \sin\theta(1-\cos^2\theta) d\theta$$

Put$\cos\theta = t \Rightarrow -\sin\theta d\theta = dt$

$$\therefore I = \int_{-1}^{1} (1-t^2) dt = \left[t - \frac{t^3}{3}\right]_{-1}^{1}$$

$= 2 - \frac{2}{3}$

$= \frac{4}{3}$

4 **(a)**

We have, $d[f(x)] = e^{\tan x} \sec^2 x\, dx$

On integrating w. r. tx, we get

$f(x) = e^{\tan x} + c$

5 **(c)**

We have,

$$I = \int_1^2 \{f(g(x))\}^{-1} f'(g(x)) g'(x) dx$$

$$\Rightarrow I = \int_1^2 \frac{1}{fog(x)} f'(g(x))g'(x)dx$$

$$\Rightarrow I = \int_1^2 \frac{1}{fog(x)} d(fog(x)) = [\log(fog\ (x))]_1^2$$

$$\Rightarrow I = \log\{f(g(2))\} - \log\{f(g(1))\} = 0\ \ [\because g(1) = g(2)]$$

6 **(d)**

Given, $f'(x) \geq 2$

On integrating both sides w. r. t. x, we get

$f(x) \geq 2x + c$

$f(1) \geq 2 \times 1 + c$

$\Rightarrow c \leq 4 - 2 \Rightarrow c \leq 2$

$\Rightarrow f(x) \geq 2x + 2$

$\therefore\ f(4) \geq 2 \times 4 + 2 = 10$

7 **(c)**

Let $I = \int \frac{f'(x)}{f(x) \log f(x)} dx$

Put $\log f(x) = t \ \Rightarrow\ \frac{f'(x)}{f(x)} dx = dt$

$$\therefore\ I = \int \frac{1}{t}\ dt = \log t + c$$

$= \log \log f(x) + c$

8 **(a)**

Let $I = \frac{2}{3} \int_0^{\pi/2} \frac{\sqrt{\cos\theta}}{(\sqrt{\sin\theta} + \sqrt{\cos\theta})} d\theta$...(i)

And $I = \frac{2}{3} \int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2 - \theta)}}{\sqrt{\sin(\pi/2-\theta)} + \sqrt{\cos(\pi/2-\theta)}} d\theta$

$\Rightarrow I = \frac{2}{3} \int_0^{\pi/2} \frac{\sqrt{\sin\theta}}{\sqrt{\sin\theta} + \sqrt{\cos\theta}} d\theta$...(ii)

On addingEqs.(i) and (ii), we get

$$2I = \frac{2}{3} \int_0^{\pi/2} 1 d\theta = \frac{2}{3} [\theta]_0^{\pi/2} \Rightarrow I = \frac{1}{3} . \frac{\pi}{2} = \frac{\pi}{6}$$

9 **(a)**

$$\lim_{n\to\infty} \left[\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \ldots + \frac{n}{n^2 + n^2}\right]$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{n}{n^2 + r^2}$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{1}{1 + \left(\frac{r}{n}\right)^2}$$

$$= \int_0^1 \frac{dx}{1 + x^2} = [\tan^{-1} x\ \ ]_0^1 = \frac{\pi}{4}$$

10 **(d)**
Integrand is an odd function. So, the value of the integral is zero
11 **(a)**
Let $I = \int \frac{\sin x}{\cos x(1+\cos x)} dx$
Put $\cos x = t \Rightarrow -\sin x\, dx = dt$
$\therefore\ I = \int \frac{-dt}{t(1+t)} = -\int \left[\frac{1}{t} - \frac{1}{(1+t)}\right] dt$
$= \log\left(\frac{t+1}{t}\right) + c = f(x) + c$ [given]
$\therefore\ f(x) = \log\left(\frac{t+1}{t}\right) = \log\left(\frac{1+\cos x}{\cos x}\right)$
12 **(a)**
Since $f(x)$ satisfies conditions of Rolle's theorem on [1, 2]. Therefore, $f(1) = f(2)$
Hence, $\int_1^2 f'(x)\, dx = f(2) - f(1) = 0$
13 **(a)**
$\int e^{2x}(2\sin 3x + 3\cos 3x)dx$
$= 2\int e^{2x} \sin 3x + dx + 3\int e^{2x}\cos 3x\, dx$
$= e^{2x}\sin 3x - 3\int e^{2x}\cos 3x\, dx + 3\int e^{2x}\cos 3x\, dx$
$= e^{2x}\sin 3x + c$
14 **(d)**
$\int_{-2}^{2} |[x]| dx$
$= \int_{-2}^{-1} |[x]|dx + \int_{-1}^{0} |[x]|dx + \int_{0}^{1} |[x]|dx + \int_{1}^{2} |[x]|dx$
$= \int_{-2}^{-1} 2\, dx + \int_{-1}^{0} 1\, dx + \int_{0}^{1} 0\, dx + \int_{1}^{2} 1\, dx$
$= 2[x]_{-2}^{-1} + [x]_{-1}^{0} + 0 + [x]_1^2$
$= 2(-1+2) + (0+1) + (2-1) = 2+1+1 = 4$
15 **(c)**
We have,
$$\int_0^{\pi/2} \cos^n x \sin^n x\ dx = \lambda \int_0^{\pi/2} \sin^n x\ dx$$
$$\Rightarrow \int_0^{\pi/2} (\sin 2x)^n\, dx = 2^n \lambda \int_0^{\pi/2} \sin^n x\ dx$$
$$\Rightarrow \int_0^{\pi} (\sin t)^n dt = 2^{n+1}\lambda \int_0^{\pi/2} \sin^n x\ dx$$

$$\Rightarrow 2\int_0^{\pi/2} \sin^n t\, dt = 2^{n+1}\lambda \int_0^{\pi/2} \sin^n x\ dx$$

$$\left[\because \int_0^{2a} f(x)\, dx = 2\int_0^{a} f(x)\, dx \text{ if } f(2a-x) = f(x)\right]$$

$$\Rightarrow 2 = 2^{n+1}\lambda \Rightarrow \lambda = \frac{1}{2^n}$$

16 **(a)**

Clearly, $\frac{t^2 \sin 2t}{t^2+1}$ is an odd function

$$\therefore \int_{-3}^{3} \frac{t^2 \sin 2t}{t^2+1} dt = 0$$

Also,

$$\int_0^1 \frac{1}{t^2+2t\cos\alpha+1} dt = \frac{1}{\sin\alpha}\left[\tan^{-1}\left(\frac{t+\cos\alpha}{\sin\alpha}\right)\right]_0^1 = \frac{\alpha}{2\sin\alpha}$$

Thus the given equation reduces to

$$x^2 \frac{\alpha}{2\sin\alpha} - 2 = 0 \Rightarrow x = \pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$$

17 **(a)**

Let $I = \int_{\pi/3}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

$$\int_{\pi/6}^{\pi/3} \left(\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right) dx \ \ldots \text{(i)}$$

$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}$...(ii)

$$\left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$$

On addingEqs. (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} 1\, dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{6}$$

$$\Rightarrow\ I = \frac{\pi}{12}$$

18 **(d)**

$$\int_0^2 [x^2]dx$$

$$= \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx = \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx + \int_{\sqrt{3}}^{2} [x^2]dx$$

$$= \int_0^1 0\, dx + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\, dx + \int_{\sqrt{3}}^{2} 3dx$$

$= [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^2$

$= 5 - \sqrt{3} - \sqrt{2}$

19 **(a)**

Given, $\int f(x) \sin x \cos x \, dx = \frac{1}{2(b^2-a^2)} \log\{f(x)\} + c$

On differentiating both sides, we get

$$f(x) \sin x \cos x = \frac{1}{2(b^2-a^2)} \cdot \frac{1}{f(x)} f'(x)$$

$$\Rightarrow \ 2(b^2 - a^2) \sin x \cos x = \frac{f'(x)}{[f(x)]^2}$$

$$\Rightarrow \ \int (2b^2 \sin x \cos x - 2a^2 \sin x \cos x) dx = \int \frac{f'(x)}{[f(x)]^2} dx$$

$I_1 - I_2 = \int \frac{f'(x)}{[f(x)]^2}$...(i)

Where $I_1 = \int 2b^2 \sin x \cos x \, dx$ and $I_2 = \int 2a^2 \sin x \cos x \, dx$

Now, $I_1 = 2b^2 \sin x \cos x \, dx$

Put $\cos x = t \ \Rightarrow \ -\sin x \, dx = dt$

$$\therefore \ I_1 = -\int 2b^2 t dt = -b^2 t^2 = -b^2 \cos^2 x$$

and $I_2 = \int 2a^2 \sin x \cos x \, dx$

Put $\sin x = t \ \Rightarrow \ \cos x \, dx = t$

$$\therefore \ I_2 = \int 2a^2 \, t dt = a^2 t^2 = a^2 \sin^2 x$$

$\therefore$ From Eq. (i),

$$-b^2 \cos^2 x - a^2 \sin^2 x = -\frac{1}{f(x)}$$

$$\Rightarrow \ f(x) = \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$$

20 **(c)**

Let $I = \int_0^a x f(x) dx$...(i)

$$\Rightarrow I = \int_0^a (a-x) f(a-x) dx$$

$$I = \int_0^a (a-x) f(x) dx$$

$[\because f(x) = f(a-x)$given$]$

$$\Rightarrow I = a \int_0^a f(x) dx - \int_0^a x f(x) dx \ \ldots \text{(ii)}$$

On addingEqs.(i) and (ii), we get

$$2I = a \int_0^a f(x) dx$$

$$\Rightarrow I = \frac{a}{2} \int_0^a f(x) dx$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	A	C	D	C	A	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	A	D	C	A	A	D	A	C

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :9

Topic :-INTEGRALS

1 **(a)**

We have,

$$I = \int_{1/e}^{e} |\log x|\, dx$$

$$\Rightarrow I = \int_{1/e}^{e} -\log x\, dx + \int_{1}^{e} \log x\, dx$$

$$\Rightarrow I = [x - x\log x]_{1/e}^{1} + [x\log x - x]_{1}^{e} = 2\left(\frac{e-1}{e}\right)$$

2 **(d)**

Let $I = \int_0^{\pi/8} \cos^3 4\theta d\theta$

$$= \int_0^{\pi/8} \left(\frac{3\cos 4\theta + \cos 12\theta}{4}\right) d\theta$$

$$= \frac{1}{4}\left[\frac{3\sin 4\theta}{4} + \frac{\sin 12\theta}{12}\right]_0^{\pi/8}$$

$$= \frac{1}{4}\left[\frac{3}{4}\left(\sin\frac{\pi}{2} - 0\right) + \frac{1}{12}\left(\sin\frac{3\pi}{2} - 0\right)\right]$$

$$= \frac{1}{4}\left[\frac{3}{4} + \frac{1}{12}(-1)\right] = \frac{1}{4}\left(\frac{8}{12}\right) = \frac{1}{6}$$

3 **(c)**

$$I = \int_{-1}^{1} \frac{\cos h\, x}{1+e^{2x}}\, dx = \int_{-1}^{1} \frac{e^x + e^{-x}}{2(1+e^{2x})}\, dx \left[\because \cosh x = \frac{e^x + e^{-x}}{2}\right]$$

$$= \frac{1}{2}\int_{-1}^{1} \frac{1+e^{2x}}{(1+e^{2x})e^x}\, dx = \frac{1}{2}\int_{-1}^{1} e^{-x}\, dx$$

$$= -\frac{1}{2}[e^{-x}]_{-1}^{1} = -\frac{1}{2}(e^{-1} - e^{1}) = \frac{e^2-1}{2e}$$

4 **(c)**

We have,

$$u_{10} = \int_0^{\pi/2} x^{10} \sin x\; dx$$

$$\Rightarrow u_{10} = [-x^{10}\cos x]_0^{\pi/2} + 10\int_0^{\pi/2} x^9 \cos x\, dx$$

$$\Rightarrow u_{10} = 10[x^9 \sin x]_0^{\pi/2} - 90\int_0^{\frac{\pi}{2}} x^8 \sin x\, dx = 10\left(\frac{\pi}{2}\right)^9 - 90\, u_8$$

$$\Rightarrow u_{10} + 90u_8 = 10\left(\frac{\pi}{2}\right)^9$$

5 **(c)**

Let $I = \int \frac{x^3 \sin[\tan^{-1}(x^4)]dx}{1+x^8}$

Put $x^4 = t \Rightarrow 4x^3\, dx = dt$

$$\therefore\ I = \int \frac{1}{4}.\ \frac{\sin[\tan^{-1}(t)]dt}{1+t^2}$$

Again, put $\tan^{-1} t = u \Rightarrow \frac{1}{1+t^2}dt = du$

$$\therefore\ I = \int \frac{1}{4}\sin u\, du = -\frac{1}{4}\cos u + c = -\frac{1}{4}\cos[\tan^{-1}(x^4)] + c$$

6 **(d)**

We have,

$$\sum_{n=1}^{10}\int_{-2n-1}^{-2n} \sin^{27} x\, dx + \sum_{n=1}^{10}\int_{2n}^{2n+1} \sin^{27} x\, dx$$

$$= -\sum_{n=1}^{10}\int_{2n}^{2n+1} \sin^{27} t\, dt + \sum_{n=1}^{10}\int_{2n}^{2n+1} \sin^{27} x\, dx \text{, where } x = -t$$

$$= -\sum_{n=1}^{10}\int_{2n}^{2n+1} \sin^{27} x\, dx + \sum_{n=1}^{10}\int_{2n}^{2n+1} \sin^{27} x\, dx = 0$$

7 **(d)**

$$\int \frac{1}{\sin^2 x . \cos^2 x}\, dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx$$

$$= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right) dx$$

$$= \int (\sec^2 x + \operatorname{cosec}^2 x)\, dx$$

$$= \tan x - \cot x + c$$

8 **(a)**

We have,

$$I = \int_{-1}^{1} (x - [2x])\, dx = \int_{-1}^{1} x\, dx - \int_{-1}^{1} [2x]\, dx$$

$$\Rightarrow I = 0 - \left[\int_{-1}^{-1/2} [2x]\, dx + \int_{-1/2}^{0} [2x] dx + \int_{0}^{1/2} [2x] dx + \int_{1/2}^{1} [2x]\, dx\right]$$

$$\Rightarrow I=-\left[\int_{-1}^{-1/2} -2dx+\int_{-1/2}^{0} -1dx+\int_{0}^{1/2} 0dx+\int_{1/2}^{1} 1dx\right]$$
$$\Rightarrow I=-\left[-2\left(-\frac{1}{2}+1\right)-1\left(0+\frac{1}{2}\right)+0+\left(1-\frac{1}{2}\right)\right]$$
$$\Rightarrow I=-\left[-1-\frac{1}{2}+\frac{1}{2}\right]=1$$

9 **(a)**

We know that if $f(t)$ is an odd function. Then, $\int_0^x f(t)\,dt$ is an even function.

Since the function $f(x)=\log\frac{1-x}{1+x}$ is an odd function. Therefore, $F(x)$ is an even function

11 **(c)**

We have

$$\frac{d}{dx}\left\{\int_{f(x)}^{g(x)} \phi(t)dt\right\}=g'(x).\phi(g(x))-f'(x).\phi(f(x))$$

12 **(d)**

$$\int_1^3 (x-1)(x-2)(x-3)dx$$
$$=\int_1^3 (x-1)(x^2-5x+6)dx$$
$$=\int_1^3 (x^3-6x^2+11x-6)dx$$
$$=\left[\frac{x^4}{4}-\frac{6x^3}{3}+\frac{11x^2}{2}-6x\right]_1^3$$
$$=\left[\frac{81}{4}-\frac{162}{3}+\frac{99}{2}-18-\left(\frac{1}{4}-\frac{6}{3}+\frac{11}{2}-6\right)\right]$$
$$=\left[-\frac{27}{12}+\frac{27}{12}\right]=0$$

13 **(c)**

Given, $f(x)=\begin{vmatrix}\sin x+\sin 2x+\sin 3x & \sin 2x & \sin 3x\\ 3+4\sin x & 3 & 4\sin x\\ 1+\sin x & \sin x & 1\end{vmatrix}$

Applying $C_1\to C_1-(C_2+C_3)$, we get

$$=\begin{vmatrix}\sin x & \sin 2x & \sin 3x\\ 0 & 3 & 4\sin x\\ 0 & \sin x & 1\end{vmatrix}$$
$$=\sin x(3-4\sin^2 x)=3\sin x-4\sin^3 x$$
$$f(x)=\sin 3x$$
$$\therefore\ \int_0^{\pi/2} f(x)dx=\int_0^{\pi/2}\sin 3x\,dx$$
$$=\left[-\frac{\cos 3x}{3}\right]_0^{\pi/2}=\left(-\frac{1}{3}\right)[\cos 3x]_0^{\pi/2}$$

$= \left(-\frac{1}{3}\right)\left[\cos\left(3 \times \frac{\pi}{2}\right) - \cos 0\right]$

$= \left(-\frac{1}{3}\right)(0-1) = \frac{-1}{3}(-1) = \frac{1}{3}$

14 **(c)**

We have,

$I = \int g(x)\{f(x) + f'(x)\}\, dx$

$\Rightarrow I = \int g(x)f(x)dx + \int g(x)f'(x)\, dx$

$\Rightarrow I = f(x)\left(\int g(x)dx\right) - \int \left(f'(x)\int g(x)dx\right)dx + \int g(x)f'(x)\, dx$

$\Rightarrow I = f(x)g(x) - \int g(x)g'(x)dx + \int g(x)g'(x)dx + C$

$\Rightarrow I = f(x)g(x) + C \quad \left[\because \int g(x)dx = g(x)\right]$

15 **(a)**

$\int \left(\frac{x+2}{x+4}\right)^2 e^x\, dx = \int e^x \left[\frac{x^2+4x+4}{(x+4)^2}\right] dx$

$= \int e^x \left[\frac{x(x+4)}{(x+4)^2} + \frac{4}{(x+4)^2}\right] dx$

$= \int e^x \left[\frac{x}{x+4} + \frac{4}{(x+4)^2}\right] dx$

$= \int \frac{e^x\, x}{(x+4)} dx + \int \frac{4e^x}{(x+4)^2} dx$

$= e^x \left(\frac{x}{x+4}\right) - \int \frac{4e^x}{(x+4)^2} dx + \int \frac{4e^x}{(x+4)^2} dx$

$= \frac{xe^x}{(x+4)} + c$

16 **(c)**

Given, $P = \int_0^{3\pi} f(\cos^2 x)dx$ ...(i)

and$Q = \int_0^{\pi} f(\cos^2 x)\, dx$ ...(ii)

From Eq. (i)

$P = 3\int_0^{\pi} f(\cos^2 x)dx$

$\Rightarrow P = 3Q \;\Rightarrow P - 3Q = 0$

17 **(d)**

Let $g(x) = f(\cos^2 x)$. Then,

$g(x + n\pi) = f\{\cos^2(x + n\pi)\} = f(\cos^2 x) = g(x)$

$\therefore \int_0^{n\pi} g(x)\, dx = n\int_0^{\pi} g(x)\, dx$

$\Rightarrow \int_0^{3\pi} g(x)\, dx = 3\int_0^{\pi} g(x)\, dx$ [Putting $n = 3$]

$$\Rightarrow \int_0^{3\pi} f(\cos^2 x)\,dx = 3\int_0^{\pi} f(\cos^2 x)\,dx$$

$\Rightarrow I_1 = 3I_2$

18 **(c)**

We know that $x - [x]$ is a periodic function with period 1 unit. Therefore,

$$\int_0^{n[x]} (x-[x])dx = n[x]\int_0^1 (x-[x])\,dx = n[x]\int_0^1 xdx = \frac{n}{2}[x]$$

19 **(d)**

Put $t = x^2 + 1 \Rightarrow dt = 2x\,dx$

$$\int_0^2 \frac{x^3}{(x^2+1)^{3/2}}dx = \frac{1}{2}\int_1^5 \frac{(t-1)}{t^{3/2}}dt$$

$$= \frac{1}{2}\int_1^5 [t^{-\frac{1}{2}} - t^{-3/2}]dt$$

$$= \frac{1}{2}\left[2\sqrt{t} + 2\frac{1}{\sqrt{t}}\right]_1^5$$

$$= \frac{1}{2}\left[2\sqrt{5} + \frac{2}{\sqrt{5}} - 2 - 2\right]$$

$$= \left[\sqrt{5} + \frac{1}{\sqrt{5}} - 2\right] = \frac{6-2\sqrt{5}}{\sqrt{5}}$$

20 **(d)**

We have,

$$I = \int_0^{2\pi} |\cos x - \sin x|\,dx$$

$$\Rightarrow I = \sqrt{2}\int_0^{2\pi} \left|\cos\left(x+\frac{\pi}{4}\right)\right|dx$$

$$\Rightarrow I = \sqrt{2}\int_{\pi/4}^{9\pi/4} |\cos t|\,dt, \text{where } t = x + \frac{\pi}{4}$$

$$\Rightarrow I = \sqrt{2}\left\{\int_{\pi/4}^{\pi/2} \cos t\,dt + \int_{\pi/2}^{3\pi/2} (-\cos t)dt + \int_{3\pi/2}^{9\pi/4} \cos t\,dt\right\}$$

$$\Rightarrow I = \sqrt{2}\left[\left(1-\frac{1}{\sqrt{2}}\right) - (-1-1) + \left(\frac{1}{\sqrt{2}}+1\right)\right] = 4\sqrt{2}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	C	C	C	D	D	A	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	C	C	A	C	D	C	D	D

SESSION : 2025-26

DPP

DAILY PRACTICE PRODUCTS

CLASS : XIIth

DATE :

SOLUTIONS

SUBJECT : MATHS

DPP NO. :10

Topic :-INTEGRALS

1 **(c)**

Let $I=\int_{-\pi/2}^{\pi/2}(x^3+x\cos x+\tan^5 x+1)dx$

Since, x^3, $x\cos x$ and $\tan^5 x$ are odd functions,

therefore

$$I=\int_{-\pi/2}^{\pi/2}1dx=[x]_{-\pi/2}^{\pi/2}=\pi$$

2 **(a)**

$\because f'(x)=g(x)$

$$\Rightarrow\int_a^b f(x)g(x)dx=\int_a^b f(x)f'(x)dx$$

$$=\left[\frac{(f(x))^2}{2}\right]_a^b$$

$$=\frac{1}{2}[(f(b))^2-(f(a))^2]$$

3 **(c)**

We have,

$$I=\int\frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx=\int\frac{4e^{2x}+6}{9e^{2x}-4}dx$$

$$\Rightarrow I=\int\frac{4\,e^{2x}}{9\,e^{2x}-4}dx+6\int\frac{1}{9\,e^{2x}-4}dx$$

$$\Rightarrow I=\frac{2}{9}\int\frac{18\,e^{2x}}{9e^{2x}-4}dx+6\int\frac{e^{-2x}}{9-4e^{-2x}}dx$$

$$\Rightarrow I=\frac{2}{9}\log(9e^{2x}-4)+\frac{6}{8}\log(9-4e^{-2x})+C$$

$$\Rightarrow I=\frac{2}{9}\log(9\,e^{2x}-4)+\frac{3}{4}\log(9\,e^{2x}-4)-\frac{3}{4}\log e^{2x}+C$$

$$\Rightarrow I=\frac{-3}{2}x+\frac{35}{36}\log(9e^{2x}-4)+C$$

Hence, $A=-\frac{3}{2}, b=\frac{35}{36}$ and $C\in R$

4 **(b)**

Let $I=\int \frac{\cos 4x+1}{\cot x-\tan x} dx$

$$= \int \frac{2\cos^2 2x}{\frac{\cos^2 x-\sin^2 x}{\sin x \cos x}} dx$$

$$= \int \frac{2\cos^2 2x}{\cos 2x} . \sin x \cos x \, dx$$

$$= \int \cos 2x . \sin 2x \;\; dx$$

$$= \frac{1}{2}\int 2\cos 2x \sin 2x \;\; dx$$

$$= \frac{1}{2}\int \sin 4x \;\; dx = \frac{1}{2}\frac{(-\cos 4x)}{4} + c$$

$$\Rightarrow I = \frac{-\cos 4x}{8} + c = k\cos 4x + c$$

$$\therefore \;\; k = -\frac{1}{8}$$

5 **(c)**

Let $I=\int_{-2}^{4} |x+1| dx$

$$= \int_{-2}^{-1} -(x+1)dx + \int_{-1}^{4} (1+x)dx$$

$$= -\left[\frac{x^2}{2}+x\right]_{-2}^{-1} + \left[x+\frac{x^2}{2}\right]_{-1}^{4}$$

$$= -\left[\frac{1}{2}-1-(2-2)\right] + \left[4+8—\left(1+\frac{1}{2}\right)\right]$$

$$= \frac{1}{2} + \left(\frac{25}{2}\right) = 13$$

6 **(c)**

$$\int_2^3 \frac{dx}{x^2-x}$$

$$= \int_2^3 \frac{dx}{x(x-1)} = \int_2^3 \left[\frac{1}{x-1}-\frac{1}{x}\right] dx$$

$= [\log(x-1)]_2^3 - [\log x]_2^3$

$= [\log 2 - \log 1] - [\log 3 - \log 2]$

$$= 2\log 2 - \log 3 = \log\frac{4}{3}$$

7 **(d)**

Let $I=\int \frac{x^2+1}{x^4-x^2+1} dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-1} dx$

$$= \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+1}dx$$

Put $x-\frac{1}{x}=t \Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt$

$\therefore\ I = \int \frac{dt}{t^2+1} = \tan^{-1} t + c$

$= \tan^{-1}\left(x-\frac{1}{x}\right)+c$

$= \tan^{-1}\left(\frac{x^2-1}{x}\right)+c$

8 **(c)**

Let $I = \int e^x \sec x\, dx + \int e^x \sec x \tan x\, dx$

$= e^x \sec x - \int e^x \sec x \tan x\, dx + \int e^x \sec x \tan x\, dx + c$

$= e^x \sec x + c$

9 **(d)**

$f'(x) = 2xe^{-(x^2+1)^2} - 2xe^{-x^4}$

$= 2x\left(e^{-(x^2+1)^2} - e^{-x^4}\right)\left[\text{here} e^{-x^4} > e^{-(x^2+1)^2}\right]$

So, $f'(x) > 0$, when $x < 0$ $\quad [\text{as} x^4 < (x^2+1)^2]$

11 **(b)**

Given, $\int_0^a x\, dx \le a+4$

$\Rightarrow \left[\frac{x^2}{2}\right]_0^a \le a+4 \Rightarrow \frac{a^2}{2} \le a+4$

$\Rightarrow a^2 \le 2a+8 \Rightarrow a^2-2a-8 \le 0$

$\Rightarrow (a-4)(a+2) \le 0 \Rightarrow -2 \le a \le 4$

12 **(a)**

Given, $u_n = \int_0^{\pi/4} \tan^n x\, dx$

or$u_n = \int_0^{\pi/4} \tan^{n-2} x\,(\sec^2 x - 1)\, dx$

$= \int_0^{\pi/4} \tan^{n-2} x \sec^2 x\, dx - \int_0^{\pi/4} \tan^{n-2} x\, dx$

On putting $\tan x = t, \sec^2 x\, dx = dt$ in Ist integral, we get

$u_n = \int_0^1 t^{n-2}\, dt - u_{n-2}$

$\Rightarrow u_n + u_{n-2} = \left[\frac{t^{n-1}}{n-1}\right]_0^1 = \left[\frac{1^{n-1}}{n-1} - 0\right] = \frac{1}{n-1}$

13 **(b)**

Let $I = \int_0^{\pi} x \sin^4 x\ dx$...(i)

$I = \int_0^{\pi} (\pi - x) \sin^4 x\ dx$...(ii)

On adding Eqs. (i) and (ii), we get

$$2I = \pi \int_0^{\pi} \sin^4 x \ dx$$

$$= 2\pi \int_0^{\pi/2} \sin^4 x \ dx$$

$$= 2\pi . \frac{4-1}{4} . \frac{4-3}{4-2} . \frac{\pi}{2}$$

$$= 2\pi . \frac{3}{4} . \frac{1}{2} . \frac{\pi}{2} = \frac{3\pi^2}{8}$$

$$\Rightarrow \ I = \frac{3\pi^2}{16}$$

14 **(a)**

We have,

$$\int \frac{\sin x - \cos x}{\sqrt{1 - \sin 2x}} e^{\sin x} \cos x \ dx$$

$$\Rightarrow I = \int \frac{\sin x - \cos x}{\sin x - \cos x} e^{\sin x} \cos x \ dx = \int e^{\sin x} \cos x \ dx = e^{\sin x} + C$$

15 **(c)**

Let $I = \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$

Now, let $x = t^6 \Rightarrow dx = 6t^5 dt$

$$\Rightarrow I = \int \frac{6t^5}{t^3 + t^2} dt = 6 \int \frac{t^3}{t+1} dt$$

$$= 6 \int \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt$$

$$= 2t^3 - 3t^2 + 6t - 6 \log(t+1) + c$$

$$= 2\sqrt{x} - 3\left(\sqrt[3]{x}\right) + 6\left(\sqrt[6]{x}\right) - 6 \log\left(\sqrt[6]{x} + 1\right) + c$$

16 **(a)**

We have,

$$I = \int \sqrt[3]{x} \sqrt[7]{1 + \sqrt[3]{x^4}} dx = \frac{3}{4} \int \sqrt[7]{1 + x^{4/3}} \times \frac{4}{3} x^{1/3} dx$$

$$\Rightarrow I = \frac{3}{4} \int (1 + x^4)^{1/7} \, d(1 + x^{4/3})$$

$$\Rightarrow I = \frac{3}{4} \times \frac{7}{8} \times \left(1 + x^{4/3}\right)^{8/7} + C = \frac{21}{32} \left(1 + x^{4/3}\right)^{8/7} + C$$

17 **(a)**

Let $I = \int \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$

Put $x = \tan \theta \Rightarrow dx = \sec^2 \theta \, d\theta$

$$\therefore \quad I = \int \sin^{-1}(\sin 2\theta) . \sec^2 \theta \, d\theta$$

$= 2\int \theta \sec^2\theta \, d\theta = 2[\theta \tan\theta - \int \tan\theta \, d\theta]$

$= 2[\theta \tan\theta + \log\cos\theta] + c$

$= 2\left[x \tan^{-1} x + \log \frac{1}{\sqrt{1+x^2}}\right] + c$

$= 2x \tan^{-1} x - \log(1+x^2) + c$

$= f(x) - \log(1+x^2) + c$ [given]

$\therefore\ f(x) = 2x \tan^{-1} x$

18 **(b)**

Given $I_{10} = \int_0^{\pi/2} x^{10} \sin x \, dx$

$= [-x^{10} \cos x]_0^{\pi/2} + \int_0^{\pi/2} 10x^9 . \cos x dx$

$= 0 + [10x^9 \sin x]_0^{\pi/2} - \int_0^{\pi/2} 90x^8 \sin x \, dx$

$= 10\left(\frac{\pi}{2}\right)^9 - 90I_8$

$\Rightarrow I_{10} + 90I_8 = 10\left(\frac{\pi}{2}\right)^9$

19 **(d)**

Let $I = \int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2 \sin^2 x}$...(i)

and$I = \int_0^{\pi} \frac{(\pi - x)dx}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}$

$\Rightarrow\ I = \int_0^{\pi} \frac{(\pi - x)dx}{a^2\cos^2 x + b^2 \sin^2 x}$...(ii)

On adding Eqs.(i) and (ii), we get

$2I = \int_0^{\pi} \frac{(x + \pi - x)dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

$\Rightarrow\ 2I = \pi \int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

$\Rightarrow\ I = \frac{\pi}{2} \int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

$\Rightarrow\ I = 2 \cdot \frac{\pi}{2} \int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

On dividing numerator and denominator be $\cos^2 x$, we get

$I = \pi \int_0^{\pi/2} \frac{\sec^2 x \, dx}{a^2 + b^2 \tan^2 x}$

Put $b \tan x = t\ \Rightarrow b \sec^2 x \, dx = dt$

$\therefore\ \ I = \frac{\pi}{b} \int_0^{\infty} \frac{dt}{a^2 + t^2} = \frac{\pi}{b} \cdot \frac{1}{a} \left[\tan^{-1} \frac{t}{a}\right]_0^{\infty}$

$= \frac{\pi}{ab}\left[\frac{\pi}{2} - 0\right] = \frac{\pi^2}{2ab}$

20 **(a)**

$$\int_{-1}^{1}(x-[x])dx=\int_{-1}^{0}(x+1)dx+\int_{0}^{1}(x-0)dx$$

$$=\left[\frac{x^2}{2}+x\right]_{-1}^{0}+\left[\frac{x^2}{2}\right]_{0}^{1}=\frac{1}{2}+\frac{1}{2}=1$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	C	B	C	C	D	C	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	B	A	C	A	A	B	D	A