SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :1

Topic :-LIMITS & DERIVATIVES

1 **(c)**

We have, $\lim_{\theta \to \frac{\pi}{2}} \frac{\frac{\pi}{2}-\theta}{\cot \theta} = \lim_{\theta \to \frac{\pi}{2}} \frac{-1}{-\operatorname{cosec}^2 \theta}$

$= \lim_{\theta \to \frac{\pi}{2}} \sin^2 \theta = 1$

2 **(a)**

$$\lim_{x \to 0} \frac{a^x - b^x}{e^x - 1} = \lim_{x \to 0} \frac{a^x - b^x}{x} . \frac{x}{a^x - 1}$$

$$= \left[\lim_{x \to 0} \left(\frac{a^x - 1}{x}\right) - \lim_{x \to 0} \left(\frac{b^x - 1}{x}\right)\right] . \lim_{x \to 0} \frac{x}{e^x - 1}$$

$$= (\log_e a - \log_e b) . \lim_{x \to 0} \frac{1}{\frac{e^x - 1}{x}}$$

$$= \log_e \left(\frac{a}{b}\right)$$

3 **(b)**

$$\lim_{x \to -\infty} \frac{2x - 1}{\sqrt{x^2 + 2x + 1}} = \lim_{y \to \infty} \frac{-2x - \frac{1}{y}}{\sqrt{1 - \frac{2}{y} + \frac{1}{y^2}}}$$

[put $x = -y \;\therefore\; x \to -\infty \; ie, y \to \infty$]

$$= -\frac{2}{1} = -2$$

4 **(a)**

$$\lim_{x \to 2} = \frac{\sqrt{1 + \sqrt{2 + x}} - \sqrt{3}}{x - 2}$$

$$= \lim_{x \to 2} \frac{(1 + \sqrt{2 + x} - 3)}{(x - 2)(\sqrt{1 + \sqrt{2 + x}} + \sqrt{3})}$$

$$= \lim_{x \to 2} \frac{x - 2}{(x - 2)\left(\sqrt{1 + \sqrt{2 + x}} + \sqrt{3}\right)(\sqrt{2 + x} + 2)}$$

$$= \lim_{x \to 2} \frac{1}{(\sqrt{1 + \sqrt{2 + x}} + \sqrt{3})(\sqrt{2 + x} + 2)}$$

$$= \frac{1}{(\sqrt{1 + 2} + \sqrt{3})(\sqrt{2 + 2} + 2)} = \frac{1}{8\sqrt{3}}$$

5 **(d)**

We have,

$$\lim_{x\to 0}\left\{\frac{1}{x\sqrt[3]{8+x}}-\frac{1}{2x}\right\} \quad [\infty-\infty \text{ form}]$$

$$=\lim_{x\to 0}\frac{1}{2x}\left\{\left(1+\frac{x}{8}\right)^{-1/3}-1\right\}$$

$$=\frac{1}{16}\lim_{x\to 0}\frac{\left(1+\frac{x}{8}\right)^{-1/3}-1^{-1/3}}{\left(1+\frac{x}{8}\right)-1}$$

$$=\frac{1}{16}\lim_{y\to 1}\frac{y^{-1/3}-1^{-1/3}}{y-10}, \text{where } y=1+\frac{x}{8}$$

$$=\frac{1}{16}\times\frac{-1}{3}(1)^{-1/3-1}=-\frac{1}{48}$$

6 **(a)**

We have,

$$\lim_{x\to 0}\frac{a^x-b^x}{x}$$

$$=\lim_{x\to 0}\left\{\left(\frac{a^x-1}{x}\right)-\left(\frac{b^x-1}{x}\right)\right\}=\log(a)-\log(b)=\log\left(\frac{a}{b}\right)$$

7 **(b)**

We have,

$$\lim_{x\to 1}\frac{\sum_{r=1}^{n}x^r-n}{x-1}=\lim_{x\to 1}\frac{x-1}{x-1}+\frac{x^2-1^2}{x-1}+\frac{x^3-1^3}{x-1}+\cdots+\frac{x^n-1^n}{x-1}$$

$$=1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

8 **(b)**

We have,

$$\lim_{x\to 1}(\log_4 5x)^{\log_x 5}=\lim_{x\to 1}(\log_5 5+\log_5 x)^{\log_x 5}$$

$$=\lim_{x\to 1}(1+\log_5 x)^{\frac{1}{\log_5 x}}=e^{\lim_{x\to 1}\log_5 x.\frac{1}{\log_x 5}}=e^1=e$$

9 **(a)**

We have,

$$\lim_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x-x}=\lim_{x\to 0}\frac{e^x\{e^{\tan x-x}-1\}}{\tan x-x}$$

$$=\lim_{x\to 0}e^x\times\lim_{x\to 0}\frac{e^{\tan x-x}-1}{\tan x-x}e^0\times 1=1$$

10 **(a)**

We have,

$$\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$$

$$=\lim_{x\to\infty}\left(1+\frac{2x-1}{x^2-4x+2}\right)^x=e^{\lim_{x\to\infty}\frac{(2x-1)x}{x^2-4x+2}}=e^2$$

11 **(c)**

$$\lim_{x\to 0}\frac{\log(x+a)-\log a}{x}+k\lim_{x\to e}\frac{\log x-1}{x-e}=1$$

Using L' Hospital's rule

$$\lim_{x\to 0}\frac{\frac{1}{x+a}}{1}+k\lim_{x\to 0}\frac{\frac{1}{x}}{1}=1$$

$$\Rightarrow \quad \frac{1}{a}+\frac{k}{e}=1$$

$$\Rightarrow \quad k=e\left(1-\frac{1}{a}\right)$$

12 **(b)**

We have,

$$\lim_{x\to\infty}\frac{\sin x}{x}=\lim_{y\to 0}y\sin\left(\frac{1}{y}\right)=0$$

13 **(a)**

$$\lim_{x\to 0}x\log\sin x=\lim_{x\to 0}\frac{\log\sin x}{1/x}\left(\frac{0}{0}\text{form}\right)$$

$$=\lim_{x\to 0}\frac{\frac{1}{\sin x}\cos x}{\frac{-1}{x^2}}=\lim_{x\to 0}-\frac{x^2}{\tan x} \qquad \text{[by L'Hospital's rule]}$$

$$=\lim_{x\to 0}\frac{-2x}{\sec^2 x} \qquad \text{[by L'Hospital's rule]}$$

$$=0$$

14 **(c)**

$$\lim_{x\to 0}\frac{d}{dx}\int\left(\frac{1-\cos x}{x^2}\right)dx=\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x/2}{4.x^2/4}$$

$$=\frac{1}{2}\lim_{x\to 0}\left(\frac{\sin x/2}{x/2}\right)^2=\frac{1}{2}$$

15 **(a)**

We have,

$$\lim_{x\to 0}\frac{\left(\int_y^a e^{\sin^2 t}dt-\int_{x+y}^a e^{\sin^2 t}\,dt\right)}{x}$$

$$=\lim_{x\to 0}\frac{\left(\int_y^a e^{\sin^2 t}dt+\int_a^{x+y} e^{\sin^2 t}\,dt\right)}{x}$$

$$=\lim_{x\to 0}\frac{\int_y^{x+y} e^{\sin^2 t\,dt}}{x}$$

$$=\lim_{x\to 0}\frac{\frac{d}{dx}(x+y)e^{\sin^2(x+y)}-0}{1} \qquad \text{[Using L' Hospital's Rule]}$$

$$=\lim_{x\to 0}1.e^{\sin^2(x+y)}=e^{\sin^2 y}$$

16 **(c)**

We have,

$$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}\left[\text{Form}\frac{0}{0}\right]$$
$$=\lim_{x\to 0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2x} \quad \text{[By L' Hospital's Rule]}$$
$$=\lim_{x\to 0}\frac{2f'(x)-3f'(2x)+2f'(4x)}{x}\left[\text{Form}\frac{0}{0}\right]$$
$$=\lim_{x\to 0}\frac{2f''(x)-6f''(2x)+8f''(4x)}{1} \quad \text{[By L' Hospital's Rule]}$$
$$=f''(0)-6f''(0)+8f''(0)=3f''(0)=3\times 4=12$$

17 **(d)**

Given, $\lim_{x\to 0}\frac{\{(a-n)nx-\tan x\}\sin nx}{x^2}=0$

$$\Rightarrow \quad \lim_{x\to 0}\left((a-n)n-\frac{\tan x}{x}\right).\frac{\sin nx}{x}=0$$
$$\Rightarrow \quad [\{a-n\}n-1]n=0$$
$$\Rightarrow \quad a=n+\frac{1}{n}$$

18 **(c)**

We have,

$$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$$
$$\Rightarrow \lim_{x\to 0}\frac{x\left\{1+a\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots\right)\right\}-b\left\{x-\frac{x^3}{3!}+\frac{x^5}{5!}\dots\right\}}{x^3}=1$$
$$\Rightarrow \lim_{x\to 0}\frac{(1+a-b)+x^2\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^4\left(\frac{a}{4!}-\frac{b}{5!}\right)+\cdots}{x^2}=1 \quad \dots\text{(i)}$$

If $1+a-b\neq 0$, then LHS $\to\infty$ as $x\to 0$ which RHS $=1$

$\therefore 1+a-b=0$

From (i), we have

$$\lim_{x\to 0}\frac{x^2\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^4\left(\frac{a}{4!}-\frac{b}{5!}\right)+\cdots}{x^2}=1$$
$$\therefore \frac{b}{3!}-\frac{a}{2!}=1\Rightarrow b-3a=6$$

Solving $1+a-b=0$ and $b-3a=6$, we get $a=-\frac{5}{2}, b=-\frac{3}{2}$

19 **(a)**

$$\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x=\lim_{x\to\infty}\left(1+\frac{2x-1}{x^2-4x+2}\right)^x$$
$$=e^{\lim_{x\to\infty}\left(\frac{x(2x-1)}{x^2-4x+2}\right)}=e^2$$

20 **(d)**

$$\lim_{x\to 0}\frac{(1-\cos 2x)}{x^2}$$
$$=\lim_{x\to 0}\frac{2\sin^2 x}{x^2}$$

$$= 2\lim_{x\to 0}\left(\frac{\sin x}{x^2}\right)^2 = 2$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	B	A	D	A	B	B	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	A	C	A	C	D	C	A	D

SESSION : 2025-26

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth

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Solutions

SUBJECT : MATHS

DPP NO. :2

Topic :-LIMITS & DERIVATIVES

1 **(b)**

We have,

$$\lim_{x\to 0}\frac{1+\sin x-\cos x+\log(1-x)}{x^3}$$

$$=\lim_{x\to 0}\frac{1+\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\right)-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}\dots\right)+\left(-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}\right)}{x^3}$$

$$=\lim_{x\to 0}\left(-\frac{1}{3!}-\frac{1}{3}\right)+x^2\left(\frac{1}{5!}-\frac{1}{5}\right)+\cdots=-\frac{1}{6}-\frac{1}{3}=-\frac{1}{2}$$

2 **(b)**

$$\lim_{x\to 0}\left\{\frac{1+\tan x}{1+\sin x}\right\}^{\operatorname{cosec} x}$$

$$=\lim_{x\to 0}\frac{\left[\left(1+\frac{\sin x}{\cos x}\right)^{\frac{\cos x}{\sin x}}\right]^{1/\cos x}}{(1+\sin x)^{1/\sin x}}$$

$$=\frac{e^{\lim_{x\to 0}\frac{1}{\cos x}}}{e}$$

$$=\frac{e}{e}=1$$

3 **(b)**

$$\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{4\sin x\cos x+\cos x}{4\sin x\cos x-3\cos x}$$

[by L'Hospital's rule]

$$=\lim_{x\to\frac{\pi}{6}}\frac{\cos x\,(4\sin x+1)}{\cos x\,(4\sin x-3)}$$

$$=\frac{4\sin\frac{\pi}{6}+1}{4\sin\frac{\pi}{6}-3}=-3$$

4 **(a)**

$$\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}=\lim_{x\to 0}\left(1+\frac{2x^2}{1+3x^2}\right)^{1/x^2}$$

$= e^{\lim_{x\to 0}\frac{1}{x^2}\left(\frac{2x^2}{1+3x^2}\right)} = e^2$

5 **(a)**

We have,

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} x = 0$ and, $\lim_{x\to 0^+} f(x) = \lim_{x\to 0} x^2 = 0$

Hence $\lim_{x\to 0} f(x) = 0$

6 **(a)**

If $x \in Q$, then $n!\,\pi\,x$ will be an integral multiple of π for large values of n. Therefore, $\cos(n!\,\pi x)$ will be either 1 or -1 and so $\cos^{2\,m}(n!\,\pi x) = 1$

$\therefore \lim_{m\to\infty}\lim_{n\to\infty}[1+\cos^{2m}(n!\ \pi\ x)] = 1+1 = 2$

If $x \notin Q, n!\,\pi\,x$ will not be an integral multiple of π and so $\cos(n!\,\pi x)$ will lie between -1 and 1

Thus, $\lim_{m\to\infty}\cos^{2m}(n!\,\pi x) = 0$

$\Rightarrow \lim_{m\to\infty}\lim_{n\to\infty}[1+\cos^{2m}(n!\,\pi x)] = 1+0 = 1$

7 **(c)**

We have,

$\lim_{x\to 1}(1+\cos\pi\,x)\cot^2\pi\,x$

$$= \lim_{x\to 1}\frac{(1+\cos\pi x)(\cos^2\pi x)}{(1-\cos^2\pi x)} = \lim_{x\to 1}\frac{\cos^2\pi x}{1-\cos\pi x} = \frac{1}{2}$$

8 **(c)**

$$\lim_{x\to 0}\frac{(e^{kx}-1)\sin kx}{x^2} = 4$$

$$\Rightarrow \lim_{x\to 0}\frac{e^{kx}-1}{kx}\times k\times\frac{\sin kx}{kx}\times k = 4$$

$$\Rightarrow k^2 = 4 \Rightarrow k = \pm 2$$

9 **(b)**

$$\lim_{x\to 0}\frac{\log(1+x^3)}{\sin^3 x} = \lim_{x\to 0}\frac{\left(\frac{x^3}{1}-\frac{(x^3)^2}{2}+\frac{(x^3)^3}{3}-\ldots\infty\right)}{\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots\infty\right)^3}$$

$$= \lim_{x\to 0}\frac{\left(1-\frac{x^3}{2}+\frac{(x^3)^2}{3}-\ldots\infty\right)}{\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\ldots\infty\right)^3} = 1$$

10 **(c)**

$l_1 = \lim_{x\to 2^+}(x+[x])$

$= \lim_{h\to 0} 2+h+[2+h] = 4$

$l_2 = \lim_{x\to 2^-}(2x-[x])$

$= \lim_{h\to 0}\{2(2-h)-[2-h]\}$

$= \lim_{h\to 0}\{2(2-h)-1\} = 3$

$$l_3 = \lim_{x\to\frac{\pi}{2}}\frac{\cos x}{x-\frac{\pi}{2}} = \lim_{x\to\frac{\pi}{2}} -\sin x = -1$$

[by L'Hospital's rule]

Thus, $l_3 < l_2 < l_1$

11 **(b)**

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin(1+[x])}{[x]}$$

$$= \frac{\sin(1-1)}{-1} = 0$$

12 **(a)**

$$\lim_{x\to 0} \frac{(1-cos\ 2x)\sin 5x}{x^2 \sin 3x}$$

$$= \lim_{x\to 0} \frac{1-cos\ 2x}{x^2} \lim_{x\to 0} \frac{\sin 5x}{\sin 3x}$$

$$= \lim_{x\to 0} \frac{2\sin^2 x}{x^2} \lim_{x\to 0} \frac{\sin 5x}{5x} . \frac{3x}{\sin 3x} . \frac{5x}{3x}$$

$$= \lim_{x\to 0} 2\left(\frac{\sin x}{x}\right)^2 . \frac{5}{3} = \frac{10}{3}$$

13 **(d)**

We have,

$$\Rightarrow \lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{\sin(e^{x-2}-1)}{\log(x-1)}$$

$$\Rightarrow \lim_{x\to 2} f(x) = \lim_{x\to 2} \left\{\frac{\sin(e^{x-2}-1)}{e^{x-2}-1} . \frac{e^{x-2}-1}{x-2} . \frac{x-2}{\log\big(1+(x-2)\big)}\right\}$$

$$\Rightarrow \lim_{x\to 2} f(x) = 1 \times 1 \times 1 = 1$$

14 **(a)**

We have,

$$\lim_{n\to\infty} \frac{S_{n+1}-S_n}{\sqrt{\sum_{k=1}^{n} k}} = \lim_{n\to\infty} \frac{a_{n+1}}{\sqrt{\frac{n(n+1)}{2}}} = 0$$

15 **(c)**

$$\lim_{h\to 0} \frac{\sin\sqrt{x+h} - \sin\sqrt{x}}{h}$$

Applying L'Hospital's rule,

$$= \lim_{h\to 0} \frac{\frac{\cos\sqrt{x+h}}{2\sqrt{x+h}}}{1} = \frac{\cos\sqrt{x}}{2\sqrt{x}}$$

16 **(a)**

Let $y = \lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan x}$

$$\Rightarrow \quad \log y = \lim_{x\to\frac{\pi}{2}} \tan x \log\sin x$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\log\sin x}{\cot x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\frac{1}{\sin x}.\cos x}{-\text{cosec}^2 x}$$ [by L' Hospital's rule]

$= 0$

$\Rightarrow \quad y = e^0 = 1$

17 **(c)**

We know that

$\cos A \cos 2A \cos 4A \ldots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$

$\lim_{n\to\infty} \cos\left(\frac{x}{2}\right) \cos\left(\frac{x}{4}\right) \ldots \cos\left(\frac{x}{2^{n-1}}\right) \cos\left(\frac{x}{2^n}\right)$

$= \lim_{n\to\infty} \frac{\sin x}{2^n \sin(x/2^n)} \left[\text{put } A = \frac{x}{2^n}\right]$

$= \lim_{n\to\infty} \frac{\sin x}{x} . \frac{(x/2^n)}{\sin(x/2^n)}$

$= \frac{\sin x}{x}$

18 **(d)**

$\lim_{x\to 1} \frac{\sin(e^{x-1} - 1)}{\log x}$

$= \lim_{h\to 0} \frac{\sin(e^h - 1)}{\log(1+h)}$

$= \lim_{h\to 0} \frac{\sin(e^h - 1)}{(e^h - 1)} \times \frac{(e^h - 1)}{\log(1+h)}$

$= 1 \times \lim_{h\to 0} \frac{\left(h + \frac{h^2}{2!} + \ldots\right)}{\left(h - \frac{h^2}{2!} + \ldots \infty\right)} = 1 \times 1 = 1$

19 **(c)**

We have,

$l = \lim_{x\to -2} \frac{\tan \pi\, x}{x+2} + \lim_{x\to\infty} \left(1 + \frac{1}{x^2}\right)^x$

$\Rightarrow l = \lim_{x\to -2} \frac{\tan(2\pi + \pi x)}{x+2} + \lim_{x\to\infty} \left(1 + \frac{1}{x^2}\right)^x$

$\Rightarrow l = \pi \lim_{x\to -2} \frac{\tan \pi\,(x+2)}{\pi\,(x+2)} + e^{\lim_{x\to\infty}\frac{x}{x^2}} = \pi + e^0 = \pi + 1$

20 **(d)**

RHL$= \lim_{h\to 0} f(1+h)$

$= \lim_{h\to 0} \frac{\sqrt{1 - \cos 2h}}{h}$

$= \lim_{h\to 0} \frac{\sqrt{2} \sin h}{h} = \sqrt{2}$

LHL$= \lim_{h\to 0} \ f(1-h) \lim_{h\to 0} \frac{\sqrt{1-\cos(-2h)}}{h}$

$= \lim_{h\to 0} \sqrt{2} \frac{\sin h}{-h} = -\sqrt{2}$

Here, LHL$\neq$RHL

So, limit does not exist.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	A	A	A	C	C	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	D	A	C	A	C	D	C	D

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :3

Topic :-LIMITS & DERIVATIVES

1 **(a)**

We have,

$\lim\limits_{x\to 2}\frac{2x^2-4f'(x)}{x-2}=\lim\limits_{x\to 0}\frac{4x-4f''(x)}{1}$ [Using L' Hospital's Rule]

$$\Rightarrow \lim_{x\to 2}\frac{2x^2-4f'(x)}{x-2}=8-4f''(2)=8-4=4$$

2 **(a)**

$$\lim_{x\to\pi/4}\frac{\int_2^{\sec^2 x} f(t)dt}{x^2-\pi^2/16}$$

$=\lim\limits_{x\to\pi/4}\frac{2\sec^2 x\tan x\, f(\sec^2 x)}{2x}$ [Using Leibniz and L' Hospital's rules]

$$=\frac{\sec^2\frac{\pi}{2}\, f\left(\sec^2\frac{\pi}{4}\right)\tan\frac{\pi}{4}}{\pi/4}=\frac{8}{\pi}f(2)$$

3 **(d)**

$$\lim_{x\to a}\frac{f(a)g(x)-f(x)g(a)}{x-a}\left[\frac{0}{0}\text{from}\right]$$

$=\lim\limits_{x\to a}\frac{f(a)g'(x)-f'(x)g(a)}{1-0}$ [by L' Hospital's rule]

$=f(a)g'(a)-f'(a)g(a)$

$=2(-1)-1(3)=-2-3=-5$

4 **(a)**

We have,

$$\lim_{x\to\infty}f(x)=\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$$

$$\Rightarrow \lim_{x\to\infty}f(x)=\lim_{x\to\infty}\left(1+\frac{4x+1}{x^2+x+2}\right)^x=e^{\lim\limits_{x\to\infty}\frac{x(4x+1)}{x^2+x+2}}=e^4$$

5 **(c)**

$$\lim_{x\to 0}\frac{\sin^{-1}x-x}{x^3\cos x}=\lim_{x\to 0}\frac{\frac{1}{\sqrt{1-x^2}}-1}{x^3(-\sin x)+3x^2\cos x}$$

[using L'Hospital's rule]

$$= \lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{\sqrt{1-x^2}.x^2(-x\sin x+3\cos x)} \times \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}$$

$$= \lim_{x\to 0} \frac{1}{\left[\begin{matrix}\sqrt{1-x^2}\left(1+\sqrt{1-x^2}\right)\\ (-x\sin x+3\cos x)\end{matrix}\right]}$$

$$= \frac{1}{1(1+1)(3)} = \frac{1}{6}$$

6 **(c)**

Here, $\lim_{x\to 0}(\sin x)^{1/x} + \lim_{x\to 0}\left(\frac{1}{x}\right)^{\sin x} = 0 + \lim_{x\to 0} e^{\log\left(\frac{1}{x}\right)^{\sin x}}$

$$\left[\begin{matrix}\lim_{x\to 0}(\sin x)^{\frac{1}{x}} \to 0\\ \text{as,}\quad 0<\sin x<1\end{matrix}\right]$$

$$= e^{\lim_{x\to 0}\frac{\log(1/x)}{\operatorname{cosec} x}} = e^{\lim_{x\to 0}\frac{x\left(-\frac{1}{x^2}\right)}{-\operatorname{cosec} x\cot x}}$$

[by L'Hospital's rule]

$$= e^{\lim_{x\to 0}\frac{\sin x}{x}\tan x} = e^0 = 1$$

7 **(b)**

$$\lim_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$$

$$= \lim_{x\to\infty}\left[1+\frac{-6}{3x+2}\right]^{\frac{x+1}{3}}$$

$$= \left[\lim_{x\to\infty}\left\{1+\frac{-6}{3x+2}\right\}^{\frac{3x+2}{-6}}\right]^{\frac{-6}{3x+2}\times\frac{x+1}{3}}$$

$$= [e]^{\lim_{x\to\infty}\frac{-6}{3x+2}\times\frac{x+1}{3}} \left[\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e\right]$$

$$= e^{\lim_{x\to\infty}\frac{-2x-2}{3x+2}} = e^{-2/3}$$

8 **(c)**

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

As $x\to 0 \Rightarrow \theta\to 0$

$$\therefore \lim_{\theta\to 0}\frac{1}{\tan\theta}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \lim_{\theta\to 0}\frac{1}{\tan\theta}\sin^{-1}(\sin 2\theta)$$

$$= \lim_{\theta\to 0}\frac{2\theta}{\tan\theta} = 2$$

9 **(c)**

$$\lim_{x\to 0}\frac{2\sin^2 3x}{x^2} = \lim_{x\to 0} 2\left(\frac{\sin 3x}{3x}\right)^2 \times \frac{9}{1} = 18$$

10 **(a)**

$$\lim_{x\to 0}\frac{a^x + a^{-x} - 2}{x^2} = \lim_{x\to 0}\frac{a^x \log a - a^{-x}\log a}{2x}$$

[by L' Hospital's rule]

$$= \lim_{x\to 0}\frac{a^x (\log a)^2 + a^{-x}(\log a)^2}{2}$$

$= (\log a)^2$ [by L' Hospital's rule]

11 **(b)**

LHL= $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = 1$

[$\because$ $(0-h)$ is rational]

RHL= $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = 1$

[$\because$ $(0+h)$ is rational]

Hence, LHL=RHL=1

12 **(c)**

$$\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x = \lim_{x\to\infty}\left[\frac{1-\frac{3}{x}}{1+\frac{2}{x}}\right]^x$$

$$= \frac{e^{-3}}{e^2} = e^{-5}$$

13 **(b)**

We have,

$$\lim_{x\to\infty}\frac{x}{2}\sin\left(\frac{\pi}{2x}\right)$$

$$= \lim_{x\to\infty}\frac{\sin\left(\frac{\pi}{2x}\right)}{\frac{\pi}{2x}}\cdot\frac{\pi}{4} = \frac{\pi}{4}\lim_{y\to 0}\frac{\sin y}{y} = \frac{\pi}{4}, \text{where } y = \frac{\pi}{2x}$$

14 **(b)**

$$\lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0}\frac{f'(x)}{1}$$

$$= \lim_{x\to 0}\frac{\tan^4 x}{1} = 0$$

15 **(c)**

RHL= $\lim_{x\to 1^+}\frac{1}{2}\{g(x)+(x)\}\sin x$

$$= \lim_{x\to 1^+}\frac{1}{2}\{1+x\}\sin x$$

$$= \frac{1}{2}.(1+1)\sin 1 = \sin 1$$

and LHL= $\lim_{x\to 1^-}\frac{\sin x}{x} = \sin 1$

Since, RHL=LHL=sin 1

$\therefore$ $\lim_{x\to 1} f(x) = \sin 1$

16 **(c)**

Given, $\lim_{x\to\infty}\left[\frac{x^3+1}{x^2+1} - (ax+b)\right] = 2$

$$\Rightarrow \quad \lim_{x\to\infty}\left[\frac{x(1-a)-b-\frac{a}{x}+\frac{(1-b)}{x^2}}{1+\frac{1}{x^2}}\right]=2$$

This limit will exist, if

$1-a=0$

and $b=-2$

$\Rightarrow \quad a=1$

and $b=-2$

17 **(b)**

$$\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=\lim_{x\to 2}\frac{\frac{x-2}{x^2-3x+2}-1}{x-2}$$

$$=\lim_{x\to 2}\frac{x-2-(x^2-3x+2)}{(x-2)(x^2-3x+2)}$$

$$=\lim_{x\to 2}\frac{-(x-2)^2}{(x-2)(x-2)(x-1)}$$

$$=-\lim_{x\to 2}\frac{1}{x-1}$$

$$=-1$$

18 **(c)**

$$\lim_{x\to 3}\frac{\int_3^{f(x)}2t^3dt}{x-3}=\lim_{x\to 3}\frac{2[f(x)]^3.f'(x)}{1}$$

$$=2[f(3)]^3.f'(3)=2\times 3^3\times\frac{1}{2}$$

$$=27$$

19 **(a)**

Given, $\lim_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)}=4$

Applying L' Hospital's rule,

$$\Rightarrow \quad \lim_{x\to a}\frac{f(a)g'(x)-g(a)f'(x)}{g'(x)-f'(x)}=4$$

$$\Rightarrow \quad \lim_{x\to a}\frac{kg'^{(x)}-kf'(x)}{g'(x)-f'(x)}=4$$

$$\Rightarrow \quad k=4$$

20 **(d)**

We have,

$\lim\limits_{x\to 0} \frac{\sin x}{\sqrt{x^2}} = \lim\limits_{x\to 0} \frac{\sin x}{|x|}$

Now, $\lim\limits_{x\to 0^-} \frac{\sin x}{|x|} = \lim\limits_{x\to 0} \frac{\sin x}{x|} = -1$

and, $\lim\limits_{x\to 0^+} \frac{\sin x}{|x|} = \lim\limits_{x\to 0} \frac{\sin x}{x} = 1$

Hence, $\lim\limits_{x\to 0} \frac{\sin x}{|x|}$ does not exist

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	D	A	C	C	B	C	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	B	C	C	B	C	A	D

SESSION : 2025-26

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :4

Topic :-LIMITS & DERIVATIVES

1 **(a)**

We have,

$$\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$$

$$=\left\{\frac{d}{dx}(x^2\sin x)\right\}_{\text{at } x=a}=2a\sin a+a^2\cos a$$

2 **(a)**

$$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$$

$$=\lim_{x\to 0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2x}$$

$$=\lim_{x\to 0}\frac{2f''(x)-12f''(2x)+16f''(4x)}{2}$$

$$=\frac{2f''(0)-12f''(0)+16f''(0)}{2}=\frac{6a}{2}=3a$$

3 **(b)**

$$\lim_{x\to 0}\frac{e^x+\log(1+x)-(1-x)^{-2}}{x^2}\left[\frac{0}{0}\text{form}\right]$$

$$=\lim_{x\to 0}\frac{e^x+(1+x)^{-1}-2(1-x)^{-3}}{2x}$$

[by L' Hospital's rule]

$$=\lim_{x\to 0}\frac{e^x-(1+x)^{-2}-6(1-x)^{-4}}{2}$$

[by L' Hospital's rule]

$$=\frac{e^0-1-6}{2}=-3$$

4 **(a)**

Given, $\lim_{x\to 0} kx\operatorname{cosec}(x)=\lim_{x\to 0} x\operatorname{cosec}(kx)$

$$\Rightarrow\quad k\lim_{x\to 0}\frac{x}{\sin x}=\lim_{x\to 0}\frac{x}{\sin kx}\times\frac{k}{k}$$

$$\Rightarrow\quad k=\frac{1}{k}$$

$$\Rightarrow\quad k=\pm 1$$

5 **(d)**

We have,

$$\lim_{x\to 0^+}\frac{|x|}{x}=\lim_{x\to 0^+}\frac{x}{x}=1$$

and, $\lim_{x\to 0^-}\frac{|x|}{x}=\lim_{x\to 0^+}\frac{|-x|}{-x}=\lim_{x\to 0^+}\frac{x}{-x}=-1$

Hence, $\lim_{x\to 0}\frac{|x|}{x}$ does not exist

6 **(b)**

We have,

$$\lim_{x\to 0}x^2\sin\frac{\pi}{x}$$

$= 0\times$ (A finite oscillating number) $=0$

7 **(c)**

$$\lim_{x\to\infty}\left(\frac{x^2+bx+4}{x^2+ax+5}\right)=\lim_{x\to\infty}\frac{\left(1+\frac{b}{x}+\frac{4}{x^2}\right)x^2}{\left(1+\frac{a}{x}+\frac{5}{x^2}\right)x^2}=1$$

8 **(b)**

Since, $f(x)$ is the integral function of $\frac{2\sin x-\sin 2x}{x^3}$, therefore by definition

$$f'(x)=\frac{2\sin x-\sin 2x}{x^3}$$

$$\Rightarrow\quad \lim_{x\to 0}f'(x)=\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}$$

$$=\lim_{x\to 0}=\frac{2\sin x}{x}.\frac{1-\cos x}{x^2}=1$$

9 **(a)**

We have,

$$\lim_{x\to\infty}\left(\frac{3x-4}{3x+4}\right)^{\left(\frac{x+1}{3}\right)}=\lim_{x\to\infty}\left(1+\frac{-6}{3x+2}\right)^{\frac{x+1}{2}}$$

$$=e^{\lim_{x\to\infty}\frac{-6}{3x+2}\times\frac{x+1}{3}}=e^{\lim_{x\to\infty}\frac{-2x-2}{3x+2}}=e^{-2/3}$$

10 **(c)**

$$\lim_{x\to 0}\frac{(1+x)^8-1}{(1+x)^2-1}$$

$$=\lim_{x\to 0}\frac{[(1+x)^4+1][(1+x)^2+1][(1+x)^2-1]}{(1+x)^2-1}$$

$=2\times 2=4$

Alternate

$$\lim_{x\to 0}\frac{(1+x)^8-1}{(1+x)^2-1}\left(\frac{0}{0}\text{form}\right)$$

$\lim_{x\to 0}\frac{8(1+x)^7}{2(1+x)}$ (by L' Hospital's rule)

$=4$

11 **(a)**

We have,

$$\lim_{x\to\pi/4}\frac{2\sqrt{2}-(\cos x+\sin x)^3}{1-\sin 2x}$$
$$=\lim_{x\to\pi/4}\frac{2^{3/2}-\{(\cos x+\sin x)^2\}^{3/2}}{2-(1+\sin 2x)}$$
$$=\lim_{x\to\pi/4}\frac{2^{3/2}-(1+\sin 2x)^{3/2}}{2-(1+\sin 2x)}$$
$$=\lim_{y\to 2}\frac{y^{3/2}-2^{3/2}}{y-2}\text{, where } y=1+\sin 2x$$
$$=\frac{3}{2}(2)^{3/2-1}=\frac{3}{2}\times\sqrt{2}=\frac{3}{\sqrt{2}}$$

12 **(b)**

We have,
$$\lim_{x\to-\infty}\left(3x+\sqrt{9x^2-x}\right)$$
$$=\lim_{y\to\infty}\left(-3y+\sqrt{9y^2+y}\right)\text{, where } y=-x$$
$$=\lim_{y\to\infty}\frac{-9y^2+9y^2+y}{(3y+\sqrt{9y^2}+y)}=\lim_{y\to\infty}\frac{y}{3y+\sqrt{9y^2}+y}=\frac{1}{3+3}=\frac{1}{6}$$

13 **(a)**
$$\lim_{x\to 0}\frac{e^{5x}-e^{4x}}{x}$$
$$=\lim_{x\to 0}\frac{\left[\begin{array}{c}\left(1+\frac{5x}{1!}+\frac{(5x)^2}{2!}+\cdots\right)-\\ \left(1+\frac{4x}{1!}+\frac{(4x)^2}{2!}+\ldots\right)\end{array}\right]}{x}$$
$$=\lim_{x\to 0}\frac{x\left[\begin{array}{c}\left(\frac{5}{1!}+\frac{25x}{2!}+\cdots\right)-\\ \left(\frac{4}{1!}+\frac{16}{2!}+\ldots\right)\end{array}\right]}{x}$$
$$=1$$

14 **(a)**

We have,
$$L=\lim_{x\to 0}\frac{a-\sqrt{a^2-x^2}-\frac{x^2}{4}}{x^4}$$
$$\Rightarrow L=\lim_{x\to 0}\frac{a-a\left(1-\frac{x^2}{a^2}\right)^{1/2}-\frac{x^2}{4}}{x^4}$$
$$\Rightarrow L=\lim_{x\to 0}\frac{a-a\left\{1-\frac{1}{2}\cdot\frac{x^2}{a^2}-\frac{1}{8}\cdot\frac{x^4}{a^4}+\frac{1}{16}\cdot\frac{x^6}{a^6}\ldots\right\}-\frac{x^2}{4}}{x^4}$$
$$\Rightarrow L=\lim_{x\to 0}\frac{\left(\frac{1}{2}\cdot\frac{x^2}{a}+\frac{1}{8}\cdot\frac{x^4}{a^3}-\frac{1}{16}\cdot\frac{x^6}{a^5}\ldots\right)-\frac{x^2}{4}}{x^4}$$
$$\Rightarrow L=\lim_{x\to 0}\frac{x^2}{2}\left(\frac{1}{a}-\frac{1}{2}\right)+\frac{1}{8a^3}-\frac{1}{16}\cdot\frac{x^2}{a^5}+\cdots$$

$\Rightarrow \frac{1}{a} - \frac{1}{2} = 0$ and in that case $L = \frac{1}{8a^3}$ $[\because L$ is finite$]$

$\Rightarrow a = 2$ and $L = \frac{1}{64}$

15 **(b)**

$\lim_{x \to 0} \log_e (\sin x)^x = \log_e \left[\lim_{x \to 0} (\sin x)^x\right]$

$= \log_e \left[\lim_{x \to 0} (1 + \sin x - 1)^{\frac{x(\sin x - 1)}{(\sin x - 1)}}\right]$

$= \log_e \left[e^{\lim_{x \to 0} x(\sin x - 1)}\right]$

$= \log_e 1$

16 **(a)**

We have,

$$\lim_{x \to 0^+} x^m (\log x)^n = \lim_{x \to 0^+} \frac{(\log x)^n}{x^{-m}}$$

$\Rightarrow \lim_{x \to 0^+} x^m (\log x)^n = \lim_{x \to 0^+} \frac{n(\log x)^{n-1}\frac{1}{x}}{-m\, x^{-m-1}}$ [By L' Hospital's Rule]

$\Rightarrow \lim_{x \to 0^+} x^m (\log x)^n = \lim_{x \to 0^+} \frac{n(\log x)^{n-1}}{-m\, x^{-m}}$

$\Rightarrow \lim_{x \to 0^+} x^m (\log x)^n = \lim_{x \to 0^+} \frac{n(n-1)(\log x)^{n-2}\frac{1}{x}}{(-m)^2 x^{-m-1}}$ [By L' Hospital's Rule]

$\Rightarrow \lim_{x \to 0^+} x^m (\log x)^n = \lim_{x \to 0^+} \frac{n(n-1)(\log x)^{n-2}}{m^2 x^{-m}}$

$=$ ………………

$= \lim_{x \to 0^+} \frac{n!}{(-m)^n x^{-m}} \left[\begin{array}{l}\text{Diff. numerator and} \\ \text{denominator } n \text{ times}\end{array}\right]$

$= 0$

17 **(a)**

$$\lim_{x \to \infty} \frac{\sqrt{1 + x^4} - (1 + x^2)}{x^2} = \lim_{x \to \infty} \frac{\sqrt{1 + \frac{1}{x^4}} - \left(1 + \frac{1}{x^2}\right)}{1}$$

$= \frac{1 - 1}{1} = 0$

18 **(c)**

Let $y = \lim_{x \to 0} (\operatorname{cosec} x)^{1/\log x}$

$\Rightarrow \quad \log y = \lim_{x \to 0} \frac{\log \operatorname{cosec} x}{\log x}$

$= \lim_{x \to 0} \frac{-\cot x}{1/x}$ [by L'Hospital's rule]

$= -\lim_{x \to 0} \frac{x}{\tan x} = -1$

$\Rightarrow \quad \log y = -1$

$\Rightarrow \quad y = \frac{1}{e}$

19 **(c)**

$$\lim_{x\to 1}\frac{2x-f(x)}{x-1}$$

$= \lim_{x\to 1}\frac{2-f\prime(x)}{1}$ (by L' Hospital's rule)

$$= 2 - f'(1)$$

$$= 2 - (1) = 1$$

20 **(d)**

$f(0) = \lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}$

$= \lim_{x\to 0}\frac{\sin(1-\cos x).\sin x}{4x^3}$

$= \frac{1}{4}.\lim_{x\to 0}\frac{\sin(1-\cos x)}{1-\cos x}.\frac{1-\cos x}{x^2}.\frac{\sin x}{x}$

$= \frac{1}{4}.1.\frac{1}{2}.1 = \frac{1}{8}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	B	A	D	B	C	B	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	A	A	B	A	A	C	C	D

SESSION : 2025-26

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :5

Topic :-LIMITS & DERIVATIVES

1 **(a)**

We have,

$$\lim_{x\to\infty} x^{3/2}\left(\sqrt{x^3+1}-\sqrt{x^3-1}\right)$$

$$=\lim_{x\to\infty}\frac{2x^{3/2}}{\sqrt{x^3+1}+\sqrt{x^3-1}}$$

$$=\lim_{x\to\infty}\frac{2x^{3/2}}{\sqrt{1+\frac{1}{x^3}}+\sqrt{1-\frac{1}{x^3}}}=\frac{2}{1+1}=1$$

2 **(a)**

Given, $\lim_{x\to a}\frac{a^x-x^a}{x^x-a^a}=-1$

$\Rightarrow \lim_{x\to a}\frac{a^x\log_e a-ax^{a-1}}{x^x(1+\log_e x)-0}=-1$ [by L' Hospital's rule]

$\Rightarrow \frac{a^a\log_e a-a^a}{a^a(1+\log_e a)}=-1$

$\Rightarrow 2\log_e a=0 \Rightarrow a=1$

3 **(c)**

We have,

$$\lim_{x\to\infty}\left\{\frac{x^3+1}{x^2+1}-(ax+b)\right\}=2$$

$$\Rightarrow \lim_{x\to\infty}\frac{x^3(1-a)-bx^2-ax+(1-b)}{x^2+1}=2$$

$\Rightarrow 1-a=0$ and $-b=2 \Rightarrow a=1, b=-2$

4 **(d)**

$$\lim_{x\to0}\frac{e^{x^2}-\cos x}{x^2}\left(\frac{0}{0}\text{from}\right)$$

$$=\lim_{x\to0}\frac{2xe^{x^2}-\sin x}{2x}\left(\frac{0}{0}\text{from}\right)$$

$$=\lim_{x\to0}\frac{2e^{x^2}+4x^2e^{x^2}+\cos x}{2}$$

$$=\frac{2+0+1}{2}=\frac{3}{2}$$

5 **(d)**

$$\lim_{n\to\infty}\frac{1}{1-n^3}\sum_{r=1}^{n} r^2=\lim_{n\to\infty}\frac{1}{n^3\left(\frac{1}{n^3}-1\right)}\frac{n(n+1)(2n+1)}{6}$$

$$=\lim_{n\to\infty}\frac{n^3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{n^3\left(\frac{1}{n^3}-1\right)6}=-\frac{1}{3}$$

6 **(a)**

$$\lim_{x\to\frac{\pi}{6}}\frac{\sin 2x}{\sin x}=\lim_{x\to\frac{\pi}{6}}\frac{2\sin x\cos x}{\sin x}$$

$$=2\lim_{x\to\frac{\pi}{6}}\cos x=\sqrt{3}$$

7 **(a)**

We have,

$$\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}=\lim_{x\to 0}\frac{1-\cos(2\sin^2 x/2)}{x^4}$$

$$=\lim_{x\to 0}\frac{2\sin^2(\sin^2 x/2)}{x^4}=2\lim_{x\to 0}\left\{\frac{\sin(\sin^2 x/2)}{x^2}\right\}^2$$

$$=2\lim_{x\to 0}\left\{\frac{\sin(\sin^2 x/2)}{\sin^2 x/2}\times\frac{\sin^2 x/2}{x^2/4}\times\frac{1}{4}\right\}^2=2\left(\frac{1}{2}\right)^2=\frac{1}{8}$$

8 **(d)**

Let $f(x)=ax^2+bx+c$

We have,

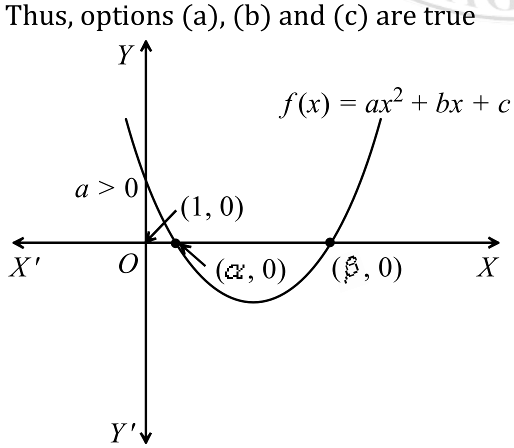
$$\lim_{x\to m}\frac{|ax^2+bx+c|}{ax^2+bx+c}=1$$

$\Rightarrow ax^2+bm+c>0$

$\Rightarrow f(m)>0$

$\Rightarrow$ Point $(m, f(m))$ must be on darkened part of the curve $y=f(x)$

Thus, options (a), (b) and (c) are true



9 **(b)**

$\lim_{x\to 1}\frac{x^m-1}{x^n-1}=\lim_{x\to 1}\frac{mx^{m-1}}{nx^{n-1}}$ [by L' Hospital's rule]

$= \frac{m}{n}$

10 **(d)**

$$\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2} = \lim_{x\to\alpha}\frac{2\sin^2\left(\frac{ax^2+bx+c}{2}\right)}{(x-\alpha)^2}$$

[Since α and β are the roots of $ax^2+bx+c=0$, so it can be written as $a(x-\alpha)(x-\beta) = 0$]

$$= \lim_{x\to\alpha}\frac{2\sin^2\left(\frac{a(x-\alpha)(x-\beta)}{2}\right)}{(x-\alpha)^2}$$

$$= \lim_{x\to\alpha}\frac{2\sin^2\left(\frac{a}{2}(x-\alpha)(x-\beta)\right)\left(\frac{a}{2}\right)^2(x-\beta)^2}{\left[\left(\frac{a}{2}\right)(x-\alpha)(x-\beta)\right]^2}$$

$$= \lim_{x\to\alpha} 2\left(\frac{a}{2}\right)^2(x-\beta)^2 = \frac{a^2}{2}(\alpha-\beta)^2$$

11 **(c)**

$\lim_{x\to 0}\left[\frac{2^x-1}{\sqrt{1+x}-1}\right] = \lim_{x\to 0}\frac{2^x\log_e 2}{\frac{1}{2\sqrt{1+x}}}$ [by L' Hospital's rule]

$= 2\log_e 2 = \log_e 4$

12 **(b)**

$$\lim_{x\to 0}\left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}}\right)$$

$$= \lim_{x\to 0}\left(\frac{x(\sqrt{1+x}+\sqrt{1-x})}{2x}\right)$$

$= 1$

13 **(c)**

Given, $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x} = k$

using L' Hospital's rule

$$\Rightarrow \quad \lim_{x\to 0}\frac{\left(\frac{1}{3+x}+\frac{1}{3-x}\right)}{1} = k$$

$$\Rightarrow \quad \frac{1}{3}+\frac{1}{3} = k \quad \Rightarrow \quad k = \frac{2}{3}$$

14 **(a)**

We have,

$$\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx} = e^{\lim_{x\to\infty}\frac{c+dx}{a+bx}} = e^{d/b}$$

15 **(c)**

$$\lim_{x\to 0}\left(\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}\right) = \lim_{x\to 0}\frac{\sec^2 x^2 . 2x}{\sin x + x\cos x}$$

$$= \lim_{x\to 0}\frac{2x.\sec^2 x^2}{x\left(\frac{\sin x}{x}+\cos x\right)}$$

$$= \frac{2 \times 1}{1 + 1} = 1 \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

16 **(c)**

It is fundamental co0ncept of indeterminate

ie, $\lim_{x \to \infty} \frac{\sin x}{x} = \frac{\sin \infty}{\infty}$

$$= 0 \times \text{finite term} = 0$$

17 **(a)**

Using expressions of $\cos x$ and $\log(1+x)$, the given limit is equal to

$$\lim_{x \to 0} \frac{x\left\{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \dots\right\} - \left\{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots\right\}}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{1}{2} - \frac{x}{2!} - \frac{x}{3} + \cdots\right) = \frac{1}{2}$$

18 **(c)**

Let $A = \lim_{n \to \infty} \frac{1}{n^k} \{(n+1)^k (n+2)^k \dots (n+n)^k \dots (n+n)^k\}^{1/n}$

Then,

$$A = \lim_{n \to \infty} \left\{\left(1 + \frac{1}{n}\right)^k \left(1 + \frac{2}{n}\right)^k \dots \left(1 + \frac{1}{n}\right)^k\right\}^{1/n}$$

$$\Rightarrow \log A = \lim_{n \to \infty} \frac{1}{n} \sum_{r=1^n} \log\left(1 + \frac{r}{n}\right)^k = k \lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{n} \log\left(1 + \frac{r}{n}\right)$$

$$\Rightarrow \log A = 2k(\log 2 - 1/2) = \log 4^k - k$$

$$\Rightarrow A = \left(\frac{4}{e}\right)^k$$

19 **(d)**

LHL$= \lim_{x \to 0} \frac{-\sin x}{x}$

$= -1$

RHL$= \lim_{x \to 0} \frac{\sin x}{x}$

$= 1$

$\Rightarrow$ LHL $\neq$ RHL

$\Rightarrow$ $\lim_{x \to 0} \frac{\sin |x|}{x}$ Does not exist.

20 **(a)**

We have,

$$\lim_{x \to \infty} \left\{\frac{x^2 \sin\left(\frac{1}{x}\right) - x}{1 - |x|}\right\}$$

$$= \lim_{x \to \infty} \frac{x^2 \sin(x^{-1}) - x}{1 - x}$$

$$= \lim_{x \to \infty} \frac{\left(\frac{\sin(x^{-1})}{x^{-1}}\right) - 1}{x^{-1} - 1} = \frac{1 - 1}{0 - 1} = 0$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	C	D	D	A	A	D	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	C	A	C	C	A	C	D	A

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :6

Topic :-LIMITS & DERIVATIVES

1 **(b)**

We have,

$l_1 = \lim\limits_{x \to -2} (x + |x|) = -2 + 2 = 0$

$l_2 = \lim\limits_{x \to -2} (2x + |x|) = -4 + 2 = -2$

and $l_3 = \lim\limits_{x \to \pi/2} \frac{\cos x}{x - \pi/2} = \lim\limits_{x \to \pi/2} \frac{\sin(\pi/2)}{-(\pi/2 - x)} = -1$

$\therefore l_2 < l_3 < l_1$

2 **(b)**

We have,

$$\lim_{x \to \infty} \left(\frac{x+2}{x+1}\right)^{x+3} = \lim_{x \to \infty} \left(1 + \frac{1}{x+1}\right)^{x+3} = e^{\lim\limits_{x \to \infty} \frac{x+3}{x+1}} = e$$

3 **(b)**

We have,

$$\lim_{x \to 0} \frac{1}{x^{12}} \left\{1 - \cos\frac{x^2}{2} - \cos\frac{x^4}{4} + \cos\frac{x^2}{2}\cos\frac{x^4}{4}\right\}$$

$$= \lim_{x \to 0} \frac{\left(1 - \cos\frac{x^2}{2}\right)\left(1 - \cos\frac{x^4}{4}\right)}{x^8}$$

$$= \frac{1}{64} \lim_{x \to 0} \frac{1 - \cos\frac{x^2}{2}}{\left(\frac{x^2}{2}\right)^2} \times \frac{1 - \cos\frac{x^4}{4}}{\left(\frac{x^4}{4}\right)^2}$$

$$= \frac{1}{64} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{256} \qquad \left[\because \lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta^2} = \frac{1}{2}\right]$$

4 **(c)**

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$$

$$= \lim_{x \to \infty} \sqrt{\frac{x - \frac{\sin x}{x}}{1 + \frac{\cos^2 x}{x}}}$$

$$= \sqrt{\frac{1 - 0}{1 + 0}}$$

$\left[\because \frac{\sin x}{x} \to 0, \frac{\cos^2 x}{x} \to 0 \text{ as } x \to \infty\right]$
$= 1$

5 **(c)**

We have,

$\lim\limits_{x\to1}(2-x)^{\tan\frac{\pi x}{2}} = \lim\limits_{x\to1}\{1+(1-x)\}^{\tan\frac{\pi x}{2}} = e^{\lim\limits_{x\to1}(1-x)\tan\frac{\pi x}{2}}$

$= e^{\lim\limits_{h\to0}-h\tan\left(\frac{\pi}{2}+\frac{\pi h}{2}\right)} = e^{\lim\limits_{h\to0}h\cot\frac{\pi h}{2}} = e^{\lim\limits_{h\to0}\frac{h}{\tan\left(\frac{\pi h}{2}\right)}} = e^{2/\pi}$

6 **(a)**

We know that, if $r < 1$, then

$\lim\limits_{n\to\infty} r^n = 0$

And if $r > 1$, then

$\lim\limits_{n\to\infty} r^n = \infty$

Here, $\lim\limits_{n\to\infty} r^n = 0$

$\therefore \quad r < 1 \quad ie, \quad r = \frac{4}{5}$

7 **(a)**

$\lim\limits_{x\to0}\frac{5^x-5^{-x}}{2x} = \lim\limits_{x\to0}\frac{5^x\log5-5^{-x}\log5}{2}$

[by L' Hospital's rule]

$= \frac{\log5+\log5}{2}$

$= \log 5$

8 **(d)**

$\lim\limits_{x\to0}\frac{\sqrt{1-\cos2x}}{\sqrt{2}x} = \lim\limits_{x\to0}\frac{\sqrt{2\sin^2 x}}{\sqrt{2}x}$

$= \lim\limits_{x\to0}\frac{\sin|x|}{x} = f(x)$ [say]

Now, $f(0+0) = \lim\limits_{h\to0}\frac{|\sin(0+h)|}{0+h} = 1$

$f(0-0) = \lim\limits_{h\to0}\frac{|\sin(0-h)|}{-h} = -1$

$\because \quad f(0+0) \neq f(0-0)$

$\therefore$ The limit of function does not exist.

9 **(c)**

We have,

$\lim\limits_{x\to\infty} a^x \sin\left(\frac{b}{a^x}\right) = \lim\limits_{x\to\infty}\frac{\sin\left(\frac{b}{a^x}\right)}{\left(\frac{b}{a^x}\right)}.b = 1\cdot b = b$

10 **(d)**

$$\lim_{x\to\infty}\left(\frac{x^3}{3x^2-4} - \frac{x^2}{3x+2}\right)$$
$$= \lim_{x\to\infty}\frac{x^3(3x+2)-x^2(3x^2-4)}{(3x^2-4)(3x+2)}$$

$$= \lim_{x\to\infty} \frac{2x^3 + 4x^2}{9x^3 + 6x^2 - 12x - 8}$$
$$= \lim_{x\to\infty} \frac{2 + 4/x}{9 + 6/x - 12/x^2 - 8/x^3} = \frac{2}{9}$$

11 **(a)**

We have,

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = \lim_{x\to 0} \left\{\frac{(e^{x^2} - 1)}{x^2} + \frac{(1 - \cos x)}{x^2}\right\}$$
$$= \lim_{x\to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x\to 0} \frac{1 - \cos x}{x^2} = 1 + \frac{1}{2} = \frac{3}{2}$$

12 **(d)**

Let $x = \frac{1}{y}$. Then,

$$\lim_{x\to\infty} \left\{\frac{a_1^{1/x} + a_2^{1/x} + \cdots + a_n^{1/x}}{n}\right\}^{nx}$$
$$= \lim_{y\to 0} \left\{\frac{a_1^y + a_2^y + \cdots + a_n^y}{n}\right\}^{n/y}$$
$$= \lim_{y\to 0} \left\{\frac{1 + a_1^y + a_2^y + \cdots + a_n^y - n}{n}\right\}^{n/y}$$
$$= e^{\lim_{y\to 0}\left\{\frac{a_1^y - 1}{y} + \frac{a_2^y - 1}{y} + \cdots + a_n^y - 1\right\}^{n/y}}$$
$$= e^{\log a_1 + \log a_2 + \cdots + \log a_n} = e^{\log(a_1 a_2 \ldots\ldots a_n)} = a_1 a_2 a_3 \ldots a_n$$

13 **(b)**

$$\lim_{x\to 0} \frac{\sin^2 x + \cos x - 1}{x^2}$$
$$= \lim_{x\to 0} \frac{\cos x + \cos^2 x}{x^2}$$
$$= \lim_{x\to 0} \cos x \,.\, \frac{1 - \cos x}{x^2}$$
$$= 1.\frac{1}{2} = \frac{1}{2}$$

14 **(b)**

We have,

$$\lim_{x\to 0} \frac{\sin 4x}{1 - \sqrt{1 - x}} = \lim_{x\to 0} \frac{\sin 4x}{x}(1 + \sqrt{1 - x})$$
$$= \lim_{x\to 0} 4\left(\frac{\sin 4x}{4x}\right)(1 + \sqrt{1 - x}) = 8$$

15 **(a)**

$$\lim_{x\to 1} \frac{x^8 - 2x + 1}{x^4 - 2x + 1} = \lim_{x\to 1} \frac{8x^7 - 2}{4x^3 - 2}$$
$= \frac{8-2}{4-2} = 3$ [using L' Hospital's rule]

166 **(a)**

$$\lim_{x\to\infty}\sqrt{a^2x^2+ax+1}-\sqrt{a^2x^2+1}$$
$$=\lim_{x\to\infty}\frac{a^2x^2+ax+1-a^2x^2-1}{\sqrt{a^2x^2+ax+1}+\sqrt{a^2x^2+1}}$$
$$=\lim_{x\to\infty}\frac{ax}{x\left[\sqrt{a^2+\frac{a}{x}+\frac{1}{x^2}}+\sqrt{a^2+\frac{1}{x^2}}\right]}$$
$$=\frac{a}{\sqrt{a^2}+\sqrt{a^2}}=\frac{1}{2}$$

17 **(d)**

LHL= $\lim_{x\to0^-}(-1)^{[x]}$

$=\lim_{h\to0}(-1)^{[0-h]}=(-1)^{-1}=-1$

RHL= $\lim_{x\to0^+}(-1)^{[x]}=\lim_{h\to0}(-1)^{[0+h]}$

$=(-1)^0=1$

∵ LHL≠RHL

∴ Limit does not exist.

18 **(a)**

$$\lim_{x\to0}\frac{(1-e^x)\sin x}{(x+x^2)x}$$
$$=\lim_{x\to0}\frac{\left(-x-\frac{x^2}{2!}-\frac{x^3}{3!}-\dots\right)}{x(1+x)}\times\lim_{x\to0}\frac{\sin x}{x}$$
$$=-1\times1=-1$$

19 **(c)**

$$\lim_{x\to\infty}\left(\frac{x+3}{x+1}\right)^{x+2}=\lim_{x\to\infty}\left[\left(1+\frac{2}{x+1}\right)^{\left(\frac{x+2}{2}\right)}\right]^{\frac{2}{x+2}\times(x+2)}$$
$$=\lim_{x\to\infty}\left(1+\frac{2}{x+1}\right)^{\left(\frac{x+2}{2}\right)\left[\frac{2(x+2)}{x+1}\right]}$$
$$=e^{\lim_{x\to\infty}\frac{(2+4/x)}{(1+1/x)}}$$
$$=e^2$$

20 **(b)**

Since, α is a repeated root.

∴ $ax^2+bx+c=a(x-\alpha)^2$

Now, $\lim_{x\to\alpha}\frac{\sin(ax^2+bx+c)}{(x-\alpha)^2}$

$$=\lim_{x\to\alpha}\frac{\sin a(x-\alpha)^2}{a(x-\alpha)^2}\times a$$
$$=\lim_{x\to\alpha}a(1)=a$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	C	C	A	A	D	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	B	B	A	A	D	A	C	B

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :7

Topic :-LIMITS & DERIVATIVES

1 **(b)**

We have,

$\lim\limits_{x \to 0} \frac{e^x - 1 - x}{x^2} = \lim\limits_{x \to 0} \frac{e^x - 1}{2x} = \frac{1}{2}$ [Using L' Hospital's Rule]

2 **(c)**

$$\lim_{x \to \infty} \frac{(2x+1)^{40}(4x-1)^5}{(2x+3)^{45}} = \lim_{x \to \infty} \frac{x^{40}\left(2+\frac{1}{x}\right)^{40} x^5\left(4-\frac{1}{x}\right)^5}{x^{45}\left(2+\frac{3}{x}\right)^{45}}$$

$$= \frac{(2+0)^{40}(4-0)^5}{(2+0)^{45}} = \frac{2^{50}}{2^{45}} = 32$$

3 **(c)**

$\therefore \lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} ([x-3] + |x-4|)$

$= \lim\limits_{h \to 0} ([3-h-3] + |3-h-4|)$

$= \lim\limits_{h \to 0} ([-h] + 1 + h)$

$= -1 + 1 + 0 = 0$

4 **(d)**

We have,

$\lim\limits_{x \to 2^-} \{x + (x - [x]^2\} = \lim\limits_{x \to 2^-} x + \lim\limits_{x+2^-} (x - [x])^2$

5 **(d)**

We have,

$$\lim_{x \to \infty} \left(\frac{x^2+2x+3}{2x^2+x+5}\right)^{\frac{3x-2}{3x+2}} = \lim_{x \to \infty} \left(\frac{1+\frac{2}{x}+\frac{3}{x^2}}{2+\frac{1}{x}+\frac{5}{x^2}}\right)^{\frac{1-2/3x}{1+2/3x}} = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$

6 **(c)**

$$\lim_{x \to 0} \left[\frac{e^{\sin x} - e^x}{\sin x - x}\right] = \lim_{x \to 0} \left[\frac{e^x(e^{\sin x - x} - 1)}{\sin x - x}\right]$$

$$= \lim_{x \to 0} e^x \lim_{x \to 0} \left[\frac{e^{\sin x - x} - 1}{\sin x - x}\right]$$

$= e^0 \times 1 = 1$

7 **(b)**

We have,

$\lim_{x\to 1}(\log_2 2x)^{\log_x 5}$

$= \lim_{x\to 1}(\log_2 2 + \log_2 x)^{\log_x 5}$

$= \lim_{x\to 1}(1+\log_2 x)^{1/\log_5 x} = e^{\lim_{x\to 1}\log_2 x.\frac{1}{\log_5 x}}$

$= e^{\lim_{x\to 1}\log_2 5} = e^{\log_2 5}$

8 **(a)**

$f(x)$ is a positive increasing function

$$\Rightarrow \quad 0 < f(x) < f(2x) < f(3x)$$

$$\Rightarrow \quad 0 < 1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$$

$$\Rightarrow \quad \lim_{x\to\infty} 1 < \lim_{x\to\infty}\frac{f(2x)}{f(x)} < \lim_{x\to\infty}\frac{f(3x)}{f(x)}$$

By Sandwich theorem

$$\lim_{x\to\infty}\frac{f(2x)}{f(x)} = 1$$

9 **(c)**

LHL= $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x^2 - 3$

$= 9 - 3 = 6$

And RHL= $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 2x + 5$

$= 2\times 3 + 5 = 11$

$\therefore$ 6 and 11 are the roots of equation

$\therefore$ Required equation is

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$\Rightarrow \quad x^2 - (11+6)x + (11\times 6) = 0$

$\Rightarrow \quad x^2 - 17x + 66 = 0$

10 **(c)**

We have,

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), x \neq 0 \\ 0, \quad x = 0 \end{cases}$$

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(0-h) = \lim_{x\to 0} -h\sin\left(-\frac{1}{h}\right)$

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0} h\sin\left(\frac{1}{h}\right) = 0$

Similarly, we have $\lim_{x\to 0^+} f(x) = 0$

Hence, $\lim_{x\to 0} f(x) = 0$

11 **(d)**

We have, $f(x)\frac{|x+\pi|}{\sin x}$

$\therefore \lim_{x\to -\pi^-} f(x) = \lim_{h\to 0} f(-\pi - h) = \lim_{h\to 0}\frac{|-\pi-h+\pi|}{\sin(-\pi-h)}$

$\Rightarrow \lim_{x\to -\pi^-} f(x) = -\lim_{h\to 0}\frac{h}{\sin(\pi+h)} = \lim_{h\to 0}\frac{h}{\sin h} = 1$

and,

$$\lim_{x\to -\pi^+} f(x) = \lim_{h\to 0} f(-\pi + h) = \lim_{h\to 0} \frac{|-\pi + h + \pi|}{\sin(-\pi + h)}$$

$$\Rightarrow \lim_{x\to -\pi^+} f(x) = -\lim_{h\to 0} \frac{h}{\sin h} = -1$$

Hence, $\lim_{x\to -\pi} f(x)$ does not exist

12 **(d)**

We have,

$$\lim_{x\to 3} \frac{f(x) - f(3)}{x - 3} = f'(3)$$

Now, $f'(x) = \frac{x}{(18-x^2)^{3/2}} \Rightarrow f'(3) = \frac{3}{(9)^{3/2}} = \frac{1}{9}$

13 **(c)**

We have,

$$\lim_{x\to\infty} \frac{5^{x+1} - 7^{x+1}}{5^x - 7^x} = \lim_{x\to\infty} \frac{5.\left(\frac{5}{7}\right)^x - 7}{\left(\frac{5}{7}\right)^x - 1} = \frac{5\times 0 - 7}{0 - 1} = 7$$

14 **(d)**

We have,

$A_i = \frac{x-a_i}{|x-a_i|}, i = 1, 2, \dots, n$ and $a_1 < a_2 < \cdots < a_{n-1} < a_n$

Let x be in the left neighbourhood of a_m. Then,

$x - a_i < 0$ for $i = m, m+1, \dots, n$

and,

$x - a_i > 0$ for $i = 1, 2, \dots, m-1$

$$A_i = \begin{cases} = \dfrac{(x - a_i)}{-(x - a_i)} = -1 \text{ for } i = m, m+1, \dots, n \\ \quad = \dfrac{x - a_i}{x - a_i} = 1 \text{ for } i = 1, 2, \dots, m-1 \end{cases}$$

Similarly, if x is in the right neighbourhood of a_m. Then,

$x - a_i < 0$ for $i = m+1, \dots, n$ and $x - a_i > 0$ for $i = 1, 2, \dots, m$

$$\therefore A_i = \begin{cases} A_i = \dfrac{x - a_i}{-(x - a_i)} = -1 \text{ for } i = \ m+1, \dots, n \\ \quad A_i = \dfrac{x - a_i}{x - a_i} = 1 \text{ for } i = 1, 2, \dots, m \end{cases}$$

Thus, we have

$$\lim_{x\to a_m^-} (A_1\, A_2 \dots A_n) = (-1)^{n-m+1}$$

and,

$$\lim_{x\to a_m^+} (A_1\, A_2 \dots A_n) = (-1)^{n-m}$$

Hence, $\lim_{x\to a_m} (A_1\, A_2 \dots A_n)$ does not exist

15 **(a)**

We have,

$$\lim_{x\to-1}\frac{(1+x)(1-x^2)(1+x^3)(1-x^4)\dots(1+x^{4n-1})(1-x^{4n})}{[(1+x)(1-x^2)(1+x^3)(1-x^4)\dots(1-x^{2n})]^2}$$
$$=\lim_{x\to-1}\frac{(1+x^{2n+1})(1-x^{2n+2})\dots(1+x^{4n-1})(1-x^{4n})}{(1+x)(1-x^2)(1+x^3)(1-x^4)\dots(1-x^{2n})}$$
$$=\lim_{x\to-1}\left\{\frac{1+x^{2n+1}}{1+x}\times\frac{1-x^{2n+2}}{1-x^2}\times\frac{1+x^{2n+3}}{1+x^3}\times\dots\times\frac{1-x^{4n}}{1-x^{2n}}\right\}$$
$$=\lim_{x\to-1}\left\{\frac{x^{2n+1}+1}{x+1}\times\frac{x^{2n+2}-1}{x^2-1}\times\frac{x^{2n+3}+1}{x^3+1}\times\dots\times\frac{x^{4n}-1}{x^{2n}-1}\right\}$$
$$=\lim_{x\to-1}\left\{\frac{x^{2n+1}-(-1)^{2n+1}}{x-(-1)}\times\frac{x^{2n+2}-(-1)^{2n+2}}{x^2-(-1)^2}\right\}$$
$$\times\left\{\frac{x^{2n+3}-(-1)^{2n+3}}{x^3-(-1)^3}\times\dots\times\frac{x^{4n}-(-1)^{4n}}{x^{2n}-(-1)^{2n}}\right\}$$
$$=\frac{2n+1}{1}\times\frac{2n+2}{2}\times\frac{2n+3}{3}\times\dots\times\frac{4n}{2n}$$
$$=\frac{4n!}{\{(2n)!\}^2}={}^{4n}C_{2n}$$

16 **(a)**

We have,

$$\lim_{x\to\infty}\frac{\log x}{x^n}=\lim_{x\to\infty}\frac{1}{n\,x^n}=0\quad\text{[By L' Hospital's Rule]}$$

17 **(b)**

$$\lim_{x\to9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3}=\lim_{x\to9}\frac{f(x)-9}{x-9}\times\frac{\sqrt{x}+3}{\sqrt{f(x)}+3}$$
$$=f'(9)\times\left(\frac{\sqrt{9}+3}{\sqrt{f(9)}+3}\right)$$
$$=f'(9)\times\frac{3+3}{3+3}=3$$

18 **(a)**

We have,

$$\lim_{n\to\infty}x^{2n}=\begin{cases}0,\text{if }|x|<1\\1,\text{if }|x|=1\\\infty,\text{if }|x|>1\end{cases}$$
$$\therefore f(x)=\lim_{n\to\infty}\frac{1-\frac{1}{x^{2n}}}{1+\frac{1}{x^{2n}}}=\lim_{n\to\infty}\begin{cases}-1,|x|<1\\0,|x|=1\\1,|x|>1\end{cases}$$

19 **(b)**

$$\lim_{x\to1}\frac{e^{-x}-e^{-1}}{x-1}=\frac{1}{e}\lim_{x\to1}\frac{e^{-(x-1)}-1}{x-1}=-\frac{1}{e}$$

20 **(c)**

$$\lim_{n\to\infty}\frac{x^n}{x^n+1}=\lim_{n\to\infty}\frac{x^n}{(1+1/x^n)x^n}$$
$$=\lim_{n\to\infty}\frac{1}{1+\frac{1}{x^n}}=1$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	C	D	D	C	B	A	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	D	C	D	A	A	B	A	B	C

Topic :-LIMITS & DERIVATIVES

1 **(b)**

We have,

$x^2+4x+5=(x+2)^2+1\geq 1$ for all x

$\therefore a=1$

$$b=\lim_{\theta\to 0}\frac{1-\cos^2\theta}{\theta^2}=1$$

$$\therefore \sum_{r=0}^{n}{}^nC_r\ a^r\ b^{n-r}=(a+b)^n=2^n$$

2 **(c)**

We have,

$$\lim_{n\to\infty}\left\{1+\left(\frac{x}{y}\right)^n\right\}^{1/n}=\lim_{n\to\infty}y\,(1+0)^{1/n}=y\times 1^0=y$$

3 **(b)**

$$\lim_{x\to 1}(\log ex)^{1/\log x}=\lim_{x\to 1}[\log e+\log x]^{1/\log x}$$

$$=\lim_{x\to 1}[1+\log x]^{1/\log x}$$

$$=e^{\lim_{x\to 1}\frac{\log x}{\log x}}=e$$

4 **(a)**

We have,

$$\lim_{x\to\infty}\left\{ax-\frac{x^2+1}{x+1}\right\}=b\Rightarrow\lim_{x\to\infty}\frac{(a-1)x^2+ax-1}{x+1}=b$$

Since b is a finite number. Therefore, degree of numerator must be less than or equal to that of the denominator

$\therefore a-1=0\Rightarrow a=1$

Now,

$$\lim_{x\to\infty}\frac{(a-1)x^2+ax-1}{x+1}=b\Rightarrow\lim_{x\to\infty}\frac{ax-1}{x+1}=b\Rightarrow a=b$$

Hence, $a=b=1$

5 **(b)**

Given limit

$$=\lim_{x\to 0}\frac{\int_0^x\frac{t\log(1+t)}{t^4+4}dt}{x^3}$$

Using L' Hospital's rule,

$$=\lim_{x\to 0}\frac{\frac{x\log(1+x)}{x^4+4}}{3x^2}$$

$$=\lim_{x\to 0}\frac{\log(1+x)}{3x}.\frac{1}{x^4+4}$$

$$=\frac{1}{3}.\frac{1}{4}=\frac{1}{12}$$

6 **(d)**

We know, $\lim\limits_{n\to\infty} x^{2n} = \begin{cases} \infty, & \text{if } x^2 > 1 \\ 1, & \text{if } x^2 = 1 \\ 0, & \text{if } x^2 < 1 \end{cases}$

Given, $f(x) = \lim\limits_{n\to\infty} \frac{\log(2+x) - x^{2n}\sin x}{1+x^{2n}}$

For $x^2 = 1$, $f(x) = \lim\limits_{n\to\infty} \frac{\log 3 - \sin 1}{2}$

$= \frac{1}{2}(\log 3 - \sin 1)$

For $x^2 < 1$,

$f(x) = \log(2+x)$

Foe $x^2 > 1$,

$$f(x) = \lim_{n\to\infty} \frac{\left(\frac{1}{x^{2n}}\right)\log(2+x) - \sin x}{\left(1+\frac{1}{x^{2n}}\right)}$$

$= -\sin x$

$$\therefore \quad f(x) = \begin{cases} \log(2+x), & x^2 < 1 \\ \frac{1}{2}(\log 3 - \sin 1), & x = 1 \\ -\sin x, & x^2 > 1 \end{cases}$$

$\therefore \lim\limits_{x\to 1^-} f(x) = \lim\limits_{h\to 0} \log(2+1-h)$

$=\log 3$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{h\to 0}[-\sin(1+h)]$

$= -\sin 1$

It is clear that both limits exist and $\lim\limits_{x\to 1^-} f(x) \neq \lim\limits_{x\to 1^+} f(x)$

7 **(d)**

We have,

$$\lim_{x\to\infty} \frac{\int_0^{2x} x\, e^{x^2}\, dx}{e^{4x^2}} = \lim_{x\to\infty} \frac{\int_0^{2x} e^{x^2} d\,(x^2)}{2e^{4x^2}} = \lim_{x\to\infty} \frac{\left[e^{x^2}\right]^{2x}}{2e^{4x^2}}$$

$$\Rightarrow \lim_{x\to\infty} \frac{\int_0^{2x} x\, e^{x^2}\, dx}{e^{4x^2}} = \lim_{x\to\infty} \frac{e^{4x^2} - 1}{2\, e^{4x^2}} = \lim_{x\to\infty} \left(\frac{1}{2} - \frac{1}{e^{4x^2}}\right) = \frac{1}{2}$$

8. **(c)**

We have,

$$\lim_{x\to\pi/2} \tan^2 x \left(\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2}\right)$$

$$= \lim_{x\to\pi/2} \tan^2 x \frac{(2\sin^2 x + 3\sin x + 4 - \sin^2 x - 6\sin x - 2)}{\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2}}$$

$$= \lim_{x\to\pi/2} \frac{\tan^2 x\,(\sin^2 x - 3\sin x + 2)}{\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2}}$$

$$= \lim_{x\to\pi/2} \frac{\sin^2 x\,(\sin x - 1)(\sin x - 2)}{(1-\sin^2 x)(\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2})}$$

$$= \lim_{x\to\pi/2} \frac{-\sin^2 x\,(\sin x - 2)}{(1+\sin x)(\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2})}$$

$$= \frac{1}{2(\sqrt{9}+\sqrt{9})} = \frac{1}{12}$$

9 **(b)**

$$\lim_{x\to\infty}\left(1+\frac{1}{x+1}\right)^{x+3}$$

$$= e^{\lim_{x\to\infty}\frac{x+3}{x+1}} = e$$

10 **(a)**

$$f(x) = \cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \frac{\pi}{2} - 3\tan^{-1}x$$

and $g(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$

$$\therefore \quad \lim_{x\to a}\frac{f(x)-f(a)}{g(x)-g(a)}$$

$$= \lim_{x\to a}\frac{\frac{\pi}{2} - 3\tan^{-1}x - \frac{\pi}{2} + 3\tan^{-1}a}{2\tan^{-1}x - 2\tan^{-1}a}$$

$$= -\frac{3}{2}\lim_{x\to a}\frac{\tan^{-1}x - \tan^{-1}a}{\tan^{-1}x - \tan^{-1}a} = -\frac{3}{2}$$

11 **(b)**

We have,

$$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)$$

$$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{h\to 0}\frac{k((2-h)^2-4)}{2-(2-h)} = k\lim_{h\to 0}\frac{h(h-4)}{h} = -4k$$

$$\lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h) = \lim_{h\to 0}\frac{k((2+h)^2-4)}{2-(2+h)}$$

$$\Rightarrow \lim_{x\to 2^+} f(x) = k\lim_{h\to 0}\frac{h(h+4)}{-h} = -4k$$

$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ for all $k \in R$

12 **(c)**

We have,

$$\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x\cos x}$$

$$= \lim_{x\to 0}\frac{(1-\cos x)(1+\cos x+\cos^2 x)}{x\sin x\cos x}$$

$$= \lim_{x\to 0}\frac{2\sin^2_{x/2}}{x.2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)} \times \frac{(1-\cos x+\cos^2 x)}{\cos x}$$

$$= \lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)}{2(x/2)} \times \frac{1+\cos x+\cos^2 x}{\cos\left(\frac{x}{2}\right)\cos x} = \frac{1}{2}\times 3 = \frac{3}{2}$$

13 **(b)**

We have,

$$\lim_{x\to\infty}\sqrt{\frac{x+\sin x}{x-\cos x}} = \lim_{x\to\infty}\sqrt{\frac{1+\frac{\sin x}{x}}{1-\frac{\cos x}{x}}} = \sqrt{\frac{1+0}{1-0}} = 1$$

14 **(a)**

Since, $f'(a)$ exists.

$$\therefore \quad f'(a)=\lim_{h\to 0}\frac{f(x)-\,f(a)}{x-a}$$

Now, $\lim\limits_{x\to a}\frac{xf(a)-af(x)}{x-a}$

$$=\lim_{x\to a}\frac{xf(a)-af(a)+af(a)-af(x)}{x-a}$$

$$=\lim_{x\to a}\frac{f(a)-(x-a)}{(x-a)}-a\lim_{x\to a}\frac{f(x)-\,f(a)}{(x-a)}$$

$$=f(a)-a\,f'(a)$$

15 **(d)**

We have,

$$\lim_{h\to 0}\frac{\sin(a+3h)-3\sin(a+2h)+3\sin(a+h)-\sin a}{h^3}$$

$$=\lim_{h\to 0}\frac{\{\sin(a+3h)-\sin a\}-3\{\sin(a+2h)-\sin(a+h)\}}{h^3}$$

$$=\lim_{h\to 0}\frac{2\sin\frac{3h}{2}\cos\left(a+\frac{3h}{2}\right)-6\cos\left(a+\frac{3h}{2}\right)\sin\frac{h}{2}}{h^3}$$

$$=\lim_{h\to 0}\frac{2\cos\left(a+\frac{3h}{2}\right)\left(\sin\frac{3h}{2}-3\sin\frac{h}{2}\right)}{h^3}$$

$$=-8\lim_{h\to 0}\cos\left(a+\frac{3h}{2}\right)\frac{\sin^3\frac{h}{2}}{h^3}$$

$$=-\lim_{h\to 0}\cos\left(a+\frac{3h}{2}\right)\left\{\frac{\sin\frac{h}{2}}{h/2}\right\}^3=-\cos a$$

16 **(b)**

We have,

$$\lim_{x\to 0}\left\{\frac{1^x+2^x+\cdots+n^x}{n}\right\}^{1/x}$$

$$=\lim_{x\to 0}\left\{1+\frac{1^x-1}{n}+\frac{2^x-1}{n}+\cdots+\frac{n^x-1}{n}\right\}^{1/x}$$

$$=e^{\lim\limits_{x\to 0}\frac{1}{n}\left\{\frac{1^x-1}{n}+\frac{2^x-1}{n}+\cdots+\frac{n^x-1}{n}\right\}}$$

$$=e^{\frac{1}{n}[\log 1+\log 2+\cdots+\log n]}$$

$$=e^{\frac{1}{n}(\log n!)}=e^{\log(n!)^{\frac{1}{n}}}=(n!)^{\frac{1}{n}}$$

17 **(b)**

We have,

$$\lim_{x\to 0}\frac{\int_0^{x^2}\cos t^2\,dt}{x\sin x}$$

$= \lim_{x\to\infty} \frac{2x\cos x^4}{x\cos x+\sin x}$ [Using L' Hospital's Rule]

$$= \lim_{x\to\infty} \frac{2\cos x^4 - 8x^4 \sin x^4}{2\cos x - x\sin x} = \frac{2-0}{2-0} = 1$$

18 **(c)**

We have,

$$f(x)+g(x)+h(x) = \frac{x^2-4x+17-4x-2}{x^2+x-12} = \frac{(x-3)(x-5)}{(x-3)(x+4)}$$

$$\therefore \lim_{x\to 3}[f(x)+g(x)+h(x)] = \lim_{x\to 3}\frac{(x-3)(x-5)}{(x-3)(x+4)} = -\frac{2}{7}$$

19 **(a)**

$$\lim_{x\to 0}\left[\frac{8\sin x + x\cos x}{3\tan x + x^2}\right] = \lim_{x\to 0}\left[\frac{\frac{8\sin x}{x}+\cos x}{\frac{3\tan x}{x}+x}\right]$$

$$= \frac{\lim_{x\to 0}\left[\frac{8\sin x}{x}+\cos x\right]}{\lim_{x\to 0}\left[\frac{3\tan x}{x}+x\right]}$$

$$= \frac{9}{3} = 3$$

20 **(d)**

We have,

$$\lim_{x\to 0}\frac{\log_e(3+x)-\log_e(3-x)}{x} = k$$

$$\Rightarrow \lim_{x\to 0}\frac{\log_e\left(1+\frac{x}{3}\right)-\log_e\left(1-\frac{x}{3}\right)}{x} = k$$

$$\Rightarrow \lim_{x\to 0}\frac{1}{3}\times\frac{\log_e\left(1+\frac{x}{3}\right)}{\frac{x}{3}} + \frac{1}{3} + \lim_{x\to 0}\frac{\log\left(1-\frac{x}{3}\right)}{\left(-\frac{x}{3}\right)} = k$$

$$\Rightarrow \frac{1}{3}+\frac{1}{3} = k \Rightarrow k = \frac{2}{3}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	B	A	B	D	D	C	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	A	D	B	B	C	A	D

SESSION : 2025-26

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :9

Topic :-LIMITS & DERIVATIVES

1 **(b)**

We have,

$$\lim_{x\to\infty}\left(\frac{x^2+6}{x^2-6}\right)^x=\lim_{x\to\infty}\left(1+\frac{12}{x^2-6}\right)^x=e^{\lim_{x\to\infty}\frac{12x}{x^2-6}}=e^0=1$$

2 **(a)**

$$\lim_{x\to\infty}\left[1+\frac{4x+1}{x^2+x+2}\right]^x=\lim_{x\to\infty}e^{\lim_{x\to\infty}\frac{4x^2+x}{x^2+x+2}}=e^4$$

3 **(a)**

We have,

$\lim_{x\to1}\frac{G(x)-G(1)}{x-1}=G'(1)$ [By def. of derivative]

Now,

$$G(x)=-\sqrt{25-x^2}\Rightarrow G'(x)=\frac{x}{\sqrt{25-x^2}}\Rightarrow G'(1)=\frac{1}{\sqrt{24}}$$

4 **(a)**

$$\lim_{x\to1}\cos^{-1}\left(\frac{1-\sqrt{x}}{1-x}\right)$$

$$=\lim_{x\to1}\cos^{-1}\left(\frac{1-\sqrt{x}}{(1-\sqrt{x})(1+\sqrt{x})}\right)$$

$$=\lim_{x\to1}\cos^{-1}\left(\frac{1}{1+\sqrt{x}}\right)$$

$$=\cos^{-1}\left(\frac{1}{2}\right)$$

$$=\frac{\pi}{3}$$

5 **(a)**

We have,

$$\lim_{x\to1}\frac{\sin(e^{x-1}-1)}{\log x}=\lim_{h\to0}\frac{\sin(e^h-1)}{\log(1+h)}$$

$$=\lim_{h\to0}\frac{\sin(e^h-1)}{(e^h-1)}\times\frac{(e^{-h}-1)}{h}\times\frac{h}{\log(1+h)}=1$$

6 **(b)**

$$\lim_{x\to\frac{\pi}{6}} \frac{3\sin x-\sqrt{3}\cos x}{6x-\pi}=\lim_{x\to\frac{\pi}{6}} \frac{3\cos x+\sqrt{3}\cos x}{6}$$

$$=\frac{3\cos\frac{\pi}{6}+\sqrt{3}\sin\frac{\pi}{6}}{6}$$

$$=\frac{3.\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}{6}$$

$$=\frac{1}{\sqrt{3}}$$

7 **(c)**

We have,

$$\lim_{x\to 2}\int_2^{f(x)} \frac{4\,t^3}{x-2}dt=\lim_{x\to 2}\frac{[t^4]_2{}^{f(x)}}{x-2}=\lim_{x\to 2}\frac{\{f(x)\}^4-16}{x-2}$$

$$=\lim_{x\to 2}\frac{4\{f(x)\}^3\,f\prime(x)}{1} \quad \text{[Applying L' Hospital's Rule]}$$

$$=4\big(f(2)\big)^3 f'(2)=32f'(2)$$

8 **(a)**

$$\lim_{x\to\frac{\pi}{4}}\frac{\int_2^{\sec^2 x} f(t)dt}{x^2-\frac{\pi^2}{16}} \qquad \left[\frac{0}{0}\text{ from}\right]$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{f(\sec^2 x)2\sec x\sec x\tan x}{2x}$$

$$\therefore \quad L=\frac{2f(2)}{\pi/4}=\frac{8f(2)}{\pi}$$

9 **(c)**

We have,

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\sqrt{\frac{x-\sin x}{x+\cos^2 x}}=\lim_{x\to\infty}\frac{\sqrt{1-\frac{\sin x}{x}}}{1+\frac{\cos^2 x}{x}}$$

$$\Rightarrow \lim_{x\to\infty} f(x)=\sqrt{\frac{1-0}{1+0}}=1\left[\because \frac{\sin x}{x}\to 0,\frac{\cos^2 x}{x}\to 0 \text{ as } x\to\infty\right]$$

10 **(a)**

We have,

$$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2} \qquad \left[\frac{0}{0}\text{ form}\right]$$

$$=\lim_{x\to 0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2} \quad \text{[Using L' Hospital's Rule]}$$

$$=\lim_{x\to 0}\frac{2f''(x)12f''(2x)+16f''(4x)}{2} \quad \text{[Using L' Hospital's Rule]}$$

$$= \frac{2\, f''(0) - 12f''\,(0) + 16f''\,(0)}{2}$$

$$= 3f''(0) = 3 \times 2 = 6$$

11 **(b)**

$$\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$$

$$= \lim_{h \to 0} \frac{\cot\left(\frac{\pi}{2} + h\right) - \cos\left(\frac{\pi}{2} + h\right)}{(-2h)^3}$$

$$= \lim_{h \to 0} \frac{-\tan h + \sin h}{(-2h)^3}$$

$$= \lim_{h \to 0} \frac{\sin h (1 - \cos h)}{\cos h \times 8h^3}$$

$$= \frac{1}{8} \lim_{h \to 0} \frac{\sin h}{h} \times \frac{2\sin^2 h/2}{4(h/2)^2} \times \frac{1}{\cos h} = \frac{1}{16}$$

12 **(c)**

We have,

$$\lim_{x \to \infty} \left(\frac{x+3}{x-1}\right)^{x+3} = \lim_{x \to \infty} \left(1 + \frac{4}{x-1}\right)^{x+3} = e^{\lim_{x \to \infty} \frac{4(x+3)}{(x+1)}} = e^4$$

13 **(d)**

$$\lim_{x \to 7} \frac{2 - \sqrt{x-3}}{x^2 - 49} = \lim_{x \to 7} \frac{-\frac{1}{2\sqrt{x-3}}}{2x}$$

$$= -\frac{1}{4.7.2} = -\frac{1}{56}$$

14 **(b)**

$$\lim_{x \to \infty} \frac{\sqrt{x^2+1} - \sqrt[3]{x^3+1}}{\sqrt[4]{x^4+1} - \sqrt[5]{x^4+1}}$$

$$= \lim_{x \to \infty} \frac{\sqrt{1 + \frac{1}{x^2}} - \sqrt[3]{1 + \frac{1}{x^3}}}{\sqrt[4]{1 + \frac{1}{x^4}} - \sqrt[5]{\frac{1}{x} + \frac{1}{x^5}}} = \frac{1-1}{1-0} = 0$$

15 **(a)**

$$\lim_{x \to 2} \frac{\int_6^{f(x)} 4\, t^3\, dt}{x - 2} \lim_{x \to 2} \frac{4\{f(x)\}^3}{1} . f'(x)$$

$$= 4\{f(2)\}^3 . f'(2)$$

$$= 4 \times (6)^3 . \frac{1}{48}$$

$$= 18$$

16 **(b)**

Since, $g(x)g(y) = g(x) + g(y) + g(xy) - 2$ ...(i)

Now, at $x = 0$, $y = 2$, we get

$g(0)g(2) = g(0) + g(2) + g(0) - 2$

$\Rightarrow \quad 5g(0) = 5 + 2g(0) - 2 \qquad [\because \ g(2) = 5]$

$\Rightarrow \quad g(0) = 1$

$g(x)$ is given in a polynomial and by the relation given $g(x)$ cannot be linear.

Let $g(x) = x^2 + k$

$\Rightarrow \quad g(x) = x^2 + 1 \qquad [\because \; g(0) = 1]$

$\therefore \quad g(x)$ is satisfied in Eq. (i)

$\therefore \quad \lim_{x \to 3} g(x) = g(3) = 3^2 + 1 = 10$

17 **(b)**

We have,

$$\lim_{x \to \pi/4} \frac{1 - \cot^3 x}{2 - \cot x - \cot^3 x}$$

$= \lim_{y \to 1} \frac{1 - y^3}{2 - y - y^3}$, where $y = \cot x$

$$= \lim_{y \to 1} \frac{y^3 - 1}{y^3 + y - 2}$$

$$= \lim_{y \to 1} \frac{(y-1)(y^2 + y + 1)}{(y-1)(y^2 + y + 2)} = \lim_{y \to 1} \frac{y^2 + y + 1}{y^2 + y + 2} = \frac{3}{4}$$

18 **(c)**

We have,

$$\lim_{x \to 1^-} \{1 - x + [x-1] + [1-x]\}$$

$$= \lim_{h \to 0} \{1 - (1-h) + [1 - h - 1] + [1 - (1-h)]\}$$

$$= \lim_{h \to 0} \{h + [-h] + [h]\} = \lim_{h \to 0} (h - 1 + 0) = -1$$

and,

$$\lim_{x \to 1^+} \{1 - x + [x-1] + [1-x]\}$$

$$= \lim_{h \to 0} \{1 - (1+h) + [1 + h - 1] + [1 - (1+h)]\}$$

$$= \lim_{h \to 0} \{-h + [h] + [-h]\} = \lim_{h \to 0} (-h + 0 - 1) = -1$$

Hence, $\lim_{x \to 1} f(x) = -1$

19 **(a)**

Given, $\lim_{x \to 1} \frac{ax^2 + bx + c}{(x-1)^2} = 2$

This limit will exist, if

$ax^2 + bx + c = 2(x-1)^2$

$\Rightarrow \quad ax^2 + bx + c = 2x^2 - 4x + 2$

$\Rightarrow \quad a = 2, \qquad b = -4, \qquad c = 2$

20 **(a)**

$$\lim_{x \to 1} \frac{2x^2 + x - 3}{3x^3 - 3x^2 + 2x - 2} = \lim_{x \to 1} \frac{(2x+3)(x-1)}{(3x^2+2)(x-1)}$$

$$= \lim_{x \to 1} \frac{2x + 3}{3x^2 + 2} = 1$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.										
Q.	11	12	13	14	15	16	17	18	19	20
A.										

SESSION : 2025-26

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :10

Topic :-LIMITS & DERIVATIVES

1 **(a)**

$$\left(\lim_{x\to\infty}\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\times\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\right)$$

$$=\lim_{y\to 0}\frac{(\sqrt{1+\sqrt{y}})/\sqrt{y}}{\sqrt{\frac{1}{y}+\frac{\sqrt{1+\sqrt{y}}}{\sqrt{y}}}+\sqrt{\frac{1}{y}}}\qquad\left[\text{put }x=\frac{1}{y}\right]$$

$$=\lim_{y\to 0}\frac{\sqrt{1+\sqrt{y}}}{\sqrt{1+\sqrt{y(1+\sqrt{y})}+1}}=\frac{1}{2}$$

2 **(d)**

We have,

$$\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}=\lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{x+4}=e^{\lim_{x\to\infty}\frac{5(x+4)}{x+1}}=e^5$$

3 **(c)**

We have,

$$\lim_{x\to a}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$$

$$=2\lim_{x\to\alpha}\frac{\sin^2\left\{\frac{(ax^2+bx+c)}{2}\right\}}{(x-\alpha)^2}$$

$$=2\lim_{x\to\alpha}\frac{\sin^2\left\{\frac{a(x-\alpha)(x-\beta)}{2}\right\}}{(x-\alpha)^2}\qquad\begin{bmatrix}\because \alpha,\beta\text{ are roots of}\\ ax^2+bx+c=0\\ \therefore ax^2+bx+c\\ =a(x-\alpha)(x-\beta)\end{bmatrix}$$

$$= 2\lim_{x\to\alpha}\left[\frac{\sin\left\{\frac{a(x-\alpha)(x-\beta)}{2}\right\}}{\frac{a(x-\alpha)(x-\beta)}{2}}\right]^2 \times \frac{a^2}{4}(x-\beta)^2$$

$$= 2(1)^2 \times \frac{a^2}{4}(\alpha-\beta)^2 = \frac{a^2}{2}(\alpha-\beta)^2$$

4 **(a)**

$$\lim_{x\to 0}(1-ax)^{1/x} = \lim_{x\to 0}\left[(1-ax)^{-\frac{1}{ax}}\right]^{-a} = e^{-a}$$

5 **(c)**

$$\lim_{n\to\infty}\frac{1-(10)^n}{1+(10)^{n+1}}$$

$$= \lim_{n\to\infty}\frac{10^n}{10^{n+1}}\left(\frac{\frac{1}{10^n}-1}{\frac{1}{10^{n+1}}+1}\right)$$

$\Rightarrow \quad -\frac{\alpha}{10} = \frac{1}{10}\left(\frac{0-1}{0+1}\right) = -\frac{1}{10}$ [given]

$\Rightarrow \quad \alpha = 1$

6 **(d)**

We have,

$$\lim_{n\to\infty}\frac{\sqrt{n^2+1}+\sqrt{n}}{\sqrt[4]{n^3+n}-\sqrt[4]{n}}$$

$$= \lim_{n\to\infty}\frac{n\left\{\sqrt{1+\frac{1}{n^2}}+\frac{1}{\sqrt{n}}\right\}}{n^{3/4}\left\{\sqrt[4]{1+\frac{1}{n^2}}-\frac{1}{\sqrt{n}}\right\}}$$

$$= \lim_{n\to\infty} n^{1/4}\frac{\left\{\sqrt{1+\frac{1}{n^2}}+\frac{1}{\sqrt{n}}\right\}}{\left\{\sqrt[4]{1+\frac{1}{n^2}}-\frac{1}{\sqrt{n}}\right\}} = \infty \times 2 = \infty$$

7 **(d)**

LHL= $\lim_{x\to 2^-}\frac{5}{\sqrt{2}-\sqrt{x}}$

$$= \lim_{h\to 0}\frac{5}{(\sqrt{2}-\sqrt{2-h})}\times\frac{(\sqrt{2}+\sqrt{2-h})}{(\sqrt{2}+\sqrt{2-h})}$$

$$= \lim_{h\to 0}\frac{5(\sqrt{2}+\sqrt{2-h})}{2-2+h} = \infty$$

RHL= $\lim_{x\to 2^+}\frac{5}{\sqrt{2}-\sqrt{x}}$

$$= \lim_{h\to 0}\frac{5}{(\sqrt{2}-\sqrt{2+h})}\times\frac{(\sqrt{2}+\sqrt{2+h})}{(\sqrt{2}+\sqrt{2+h})}$$

$$= \lim_{h\to 0}\frac{5(\sqrt{2}+\sqrt{2+h})}{2-2-h} = -\infty$$

$\therefore$ LHL$\neq$RHL

Hence, limit does not exist.

8 **(a)**

$$\lim_{n\to\infty}\frac{2^{-n}(n^2+5n+6)}{(n+4)(n+5)}=\lim_{n\to\infty}\frac{n^2(1+\frac{5}{n}+\frac{6}{n^2})}{2^n.n^2\left(1+\frac{4}{n}\right)(1+\frac{5}{n})}$$

$= 0$

9 **(b)**

$$\lim_{x\to 0} f(x)=\lim_{x\to 0} x.\sin\frac{1}{x}$$

$= 0 \times$ finite term $= 0$

10 **(b)**

$$\lim_{x\to a}\frac{\log(x-a)}{\log(e^x-e^a)}=\lim_{x\to a}\frac{\frac{1}{x-a}}{\frac{e^x}{e^x-e^a}}$$

[by L' Hospital's rule]

$$=\lim_{x\to a}\frac{e^x-e^a}{e^x(x-a)}$$

$$=\lim_{x\to a}\frac{e^x}{e^x(x-a)+e^x}$$

[by L' Hospital's rule]

$$=\frac{e^a}{0+e^a}=1$$

11 **(c)**

We have,

$$\lim_{x\to\infty}\left(1-\frac{2}{x+1}\right)^x=e^{\lim_{x\to\infty}\frac{-2x}{x+1}}=e^{-2}$$

12 **(d)**

$$\lim_{x\to 0}\left[\frac{3^x+3^{-x}-2}{x^2}\right]$$

$$=\lim_{x\to 0}\frac{\begin{bmatrix}1+x\log 3+\frac{x^2}{2!}(\log 3)^2+\cdots+\\ 1-x\log 3+\frac{x^2}{2!}(\log 3)^2-\ldots-2\end{bmatrix}}{x^2}=(\log 3)^2$$

13 **(d)**

We have,

$$\lim_{x\to 0}\left(\frac{e^x+e^{-x}-2}{x^2}\right)^{\frac{1}{x^2}}$$

$$=\lim_{x\to 0}\left\{\frac{2\left(\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots\right)}{x^2}\right\}^{\frac{1}{x^2}}$$

$$=\lim_{x\to 0}\left(1+2\frac{x^2}{4!}+2\frac{x^4}{6!}+\cdots\right)^{\frac{1}{x^2}}=e^{\lim_{x\to 0}\left(\frac{x^2}{4!}+\frac{2x^4}{6!}+\cdots\right)\times\frac{1}{x^2}}=e^{1/12}$$

14 **(b)**

Let $y = \lim_{x\to\infty}\left(\frac{\pi}{2} - \tan^{-1}x\right)^{1/x}$

$\Rightarrow \quad \log y = \lim_{x\to\infty}\frac{1}{x}\log\left(\frac{\pi}{2} - \tan^{-1}x\right) \qquad \left[\frac{\infty}{\infty}\text{ form}\right]$

$= \lim_{x\to\infty}\frac{\left(-\frac{1}{1+x^2}\right)}{\left(\frac{\pi}{2}-\tan^{-1}x\right)}$ [using L'Hospital's rule]

$= \lim_{x\to\infty}\frac{\frac{2x}{(1+x^2)^2}}{-\left(\frac{1}{1+x^2}\right)} = \lim_{x\to\infty}\frac{-2x}{1+x^2}$

[using L'Hospital's rule]

$\Rightarrow \quad \log y = 0 \quad \Rightarrow \quad y = 1$

15 **(d)**

We have,

$$\lim_{x\to1}\left\{\frac{x^3+2x^2+x+1}{x^2+2x+3}\right\}^{\frac{1-\cos(x-1)}{(x-1)^2}}$$

$$= \lim_{x\to1}\left\{\frac{x^3+2x^2+x+1}{x^2+2x+3}\right\}^{\frac{2\sin^2(x-1)/2}{(x-1)^2}} = \left(\frac{5}{6}\right)^{\frac{2}{4}\left\{\frac{\sin\left(\frac{x-1}{2}\right)}{\frac{x-1}{2}}\right\}^2} = \sqrt{\frac{5}{6}}$$

16 **(a)**

Here, $\lim_{x\to-3}x^2+2x-3=0$

$\therefore \lim_{x\to-3}3x^2+ax+a-7$ must be zero, in order to limit exist.

$\Rightarrow \quad 3(-3)^2 + a(-3) + a - 7 = 0$

$\Rightarrow \quad 27 - 2a - 7 = 0$

$\Rightarrow \quad 2a = 20$

$\Rightarrow \quad a = 10$

17 **(b)**

$$\lim_{x\to\infty}\left(1-\frac{4}{x-1}\right)^{3x-1} = \lim_{x\to\infty}\left[\left(1-\frac{4}{x-1}\right)^{\frac{-(x-1)}{4}}\right]^{-4\left(\frac{3x-1}{x-1}\right)}$$

$$= e^{-4\lim_{x\to\infty}(3-1/x)(1-1/x)} = e^{-12}$$

18 **(b)**

We have,

$$\lim_{n\to\infty}\left\{\frac{1}{1-n^2}+\frac{2}{1-n^2}+\frac{3}{1-n^2}+\cdots+\frac{n}{1-n^2}\right\}$$

$$= \lim_{n\to\infty}\frac{1+2+3+\cdots+n}{1-n^2} = \lim_{n\to\infty}\frac{n}{2(1-n)} = -\frac{1}{2}$$

20 **(c)**

$$\lim_{n\to\infty}\frac{3.2^{n+1}-4.5^{n+1}}{5.2^n+7.5^n}$$

$$= \lim_{n\to\infty}\frac{5^n\left(6.\left(\frac{2}{5}\right)^n-20\right)}{5^n\left(5.\left(\frac{2}{5}\right)^n+7\right)} = -\frac{20}{7}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	C	A	C	D	D	A	B	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	D	B	D	A	B	B	C	C