

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

Class : XIth

Date :

Solutions

Subject : MATHS

DPP No. :1

Topic :-TRIGONOMETRIC FUNCTIONS

1 (c)

We have,

$$\sin x + \sin^2 x = 1 \Rightarrow \sin x = \cos^2 x$$

Now,

$$\begin{aligned} & \cos^{12} x + 3 \cos^{10} x + 3 \cos^8 x + \cos^6 x - 1 \\ &= \cos^6 x (\cos^6 x + 3 \cos^4 x + 3 \cos^2 x + 1) - 1 \\ &= \cos^6 x (\cos^2 x + 1)^3 - 1 \\ &= \sin^3 x (\sin x + 1)^3 - 1 \\ &= (\sin^2 x + \sin x)^3 - 1 \\ &= (\sin^2 x + \cos^2 x)^3 - 1 \quad [\because \sin x = \cos^2 x] \\ &= 1 - 1 = 0 \end{aligned}$$

2 (c)

Let $a = 3x + 4y$, $b = 4x + 3y$ and $c = 5x + 5y$.

Clearly, c is the largest side and thus the largest angle C is given by

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{-2xy}{2(12x^2 + 25xy + 12y^2)} < 0$$

$\Rightarrow C$ is an obtuse angle

3 (a)

Let $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$.

Then,

$$a - b = x + 2 > 0 \quad [\because x > 1]$$

$$a - c = x^2 - x > 0 \quad [\because x > 1]$$

So, a is the largest side

Hence, the largest angle is given by

$$\begin{aligned} \cos \theta &= \frac{b^2 + c^2 - a^2}{2bc} \\ \Rightarrow \cos \theta &= \frac{(x^2 - 1)^2 + (2x + 1)^2 - (x^2 + x + 1)^2}{2(x^2 - 1)(2x + 1)} \\ &= -\frac{1}{2} \end{aligned}$$

$$\Rightarrow \theta = 2\pi/3 = 120^\circ$$

4 (c)

We have,

$$\frac{1}{2} a p_1 = \Delta, \frac{1}{2} b p_2 = \Delta, \frac{1}{2} c p_3 = \Delta$$

$$\Rightarrow p_1 = \frac{2\Delta}{a}, p_2 = \frac{2\Delta}{b}, p_3 = \frac{2\Delta}{c}$$

$$\therefore \frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{a^2 + b^2 + c^2}{4\Delta^2}$$

$$\begin{aligned} \frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3} &= \frac{a}{2\Delta} + \frac{b}{2\Delta} - \frac{c}{2\Delta} = \frac{a+b-c}{2\Delta} \\ &= \frac{2(s-c)}{2\Delta} = \frac{s-c}{\Delta} \end{aligned}$$

5 (c)

We have,

$$\begin{aligned} \cos C &= \frac{63}{65} \Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = \frac{63}{65} \\ &\Rightarrow \frac{26^2 + 30^2 - c^2}{2 \times 26 \times 30} = \frac{63}{65} \end{aligned}$$

$$\Rightarrow 676 + 900 - c^2 = 1260 \Rightarrow c^2 = 64 \Rightarrow c = 8$$

Thus, we have

$$a = 26, b = 30 \text{ and } c = 8$$

$$\begin{aligned} \therefore 2s &= a + b + c \Rightarrow 2s = 26 + 30 + 8 = 64 \Rightarrow s \\ &= 32 \end{aligned}$$

$$\text{Also, } \Delta = \sqrt{s(s-a)(s-b)(s-c)} =$$

$$\sqrt{32 \times 6 \times 2 \times 24} = 96$$

$$\text{Hence, } r_2 = \frac{\Delta}{s-b} = \frac{96}{32-30} = 48$$

6 (c)

$$\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 90^\circ \dots \cos 100^\circ$$

$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots 0 \dots \cos 100^\circ = 0$$

7 (b)

We have,

$$\begin{aligned} &\sin \frac{\pi}{2} + \sin \frac{2\pi}{7} + \sin \frac{3\pi}{7} \\ &= \frac{1}{2 \sin \frac{\pi}{7}} \left\{ 2 \sin^2 \frac{\pi}{7} + 2 \sin \frac{\pi}{7} \sin \frac{2\pi}{7} + 2 \sin \frac{\pi}{7} \sin \frac{3\pi}{7} \right\} \\ &= \frac{1}{2 \sin \left(\frac{\pi}{7} \right)} \left\{ 1 - \cos \frac{2\pi}{7} + \cos \frac{\pi}{7} - \cos \frac{3\pi}{7} + \cos \frac{2\pi}{7} \right. \\ &\quad \left. - \cos \frac{4\pi}{7} \right\} \end{aligned}$$

$$= \frac{1}{2 \sin \frac{\pi}{7}} \left\{ 1 + \cos \frac{\pi}{7} \right\} = \frac{2 \cos^2 \frac{\pi}{14}}{4 \sin \frac{\pi}{14} \cos \frac{\pi}{14}} = \frac{1}{2} \cot \frac{\pi}{14}$$

8 (a)

$$\text{Let } f(x) = \sqrt{3} \cos x + \sin x$$

$$\Rightarrow f(x) = 2 \left(\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x \right) = 2 \sin \left(x + \frac{\pi}{3} \right)$$

$$\text{Since, } -1 \leq \sin \left(x + \frac{\pi}{3} \right) \leq 1$$

$$\text{Hence, } f(x) \text{ is maximum, if } x + \frac{\pi}{3} = \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{6} = 30^\circ$$

9 (b)

$$\sin^2 17.5^\circ + \sin^2 72.5^\circ$$

$$= \sin^2 17.5^\circ + \cos^2 17.5^\circ \left[\because \sin(90^\circ - \theta) = \cos \theta \right]$$

$$= 1 = \tan^2 45^\circ$$

10 (a)

We have,

$$a \sin A = b \sin B$$

$$\Rightarrow a \cdot ak = b \cdot bk \Rightarrow a = b \Rightarrow \Delta ABC \text{ is isosceles}$$

11 (b)

$$\text{We know that } \sin^2 \theta \geq 1$$

$$\Rightarrow \frac{4xy}{(x+y)^2} \geq 1$$

$$\Rightarrow 4xy \geq (x+y)^2$$

$$\Rightarrow (x-y)^2 \leq 0$$

$$\Rightarrow x - y = 0 \Rightarrow y = x$$

$$\text{And } x \neq 0, y \neq 0$$

12 (b)

$$\text{Given that, } \cos \theta = \frac{1}{2} \left(x + \frac{1}{x} \right)$$

$$\Rightarrow x + \frac{1}{x} = 2 \cos \theta \quad \dots(i)$$

$$\text{We know that, } x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x} \right)^2 - 2$$

$$= (2 \cos \theta)^2 - 2 = 4 \cos^2 \theta - 2$$

$$= 2 \cos 2\theta \quad [\text{from Eq.(i)}]$$

$$\therefore \frac{1}{2} \left(x^2 + \frac{1}{x^2} \right) = \frac{1}{2} \times 2 \cos 2\theta = \cos 2\theta$$

13 (d)

$$\begin{aligned} & \operatorname{sech}^{-1}(\sin \theta) \\ &= \cosh^{-1}(\operatorname{cosec} \theta) \\ &= \log \left[\operatorname{cosec} \theta + \sqrt{(\operatorname{cosec}^2 \theta - 1)} \right] \\ &= \log \left[\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \right] = \log \cot \frac{\theta}{2} \end{aligned}$$

14 (d)

Consider the curves $y = 2^{\cos x}$ and $y = |\sin x|$.
Clearly, both the curves are symmetrical about y-axis as $\cos x$ and $|\sin x|$ are even functions
Also, $y = 2^{\cos x}$ and $y = |\sin x|$ intersect at two points in $[0, 2\pi]$
Hence, there are four solutions of the given equation

15 (d)

We have,

$$\begin{aligned} \cos(\lambda \sin \theta) &= \sin(\lambda \cos \theta) \\ \Rightarrow \cos(\lambda \sin \theta) &= \cos \left(\frac{\pi}{2} - \lambda \cos \theta \right) \\ \Rightarrow \lambda \sin \theta &= \frac{\pi}{2} - \lambda \cos \theta \Rightarrow \cos \theta + \sin \theta = \frac{\pi}{2\lambda} \end{aligned}$$

This equation will have a solution if

$$\begin{aligned} \left| \frac{\pi}{2\lambda} \right| &\leq \sqrt{2} \quad \left[\because |a \cos \theta + b \sin \theta| \leq \sqrt{a^2 + b^2} \right] \\ \Rightarrow \frac{\pi}{2\lambda} &\leq \sqrt{2} \Rightarrow \lambda \geq \frac{\pi}{2\sqrt{2}} \quad [\because \lambda > 0] \end{aligned}$$

16 (c)

We have,

$$\begin{aligned} c_1 + c_2 &= 2b \cos A \text{ and } c_1 c_2 = b^2 - a^2 \\ \therefore c_1 - c_2 &= \sqrt{(c_1 + c_2)^2 - 4c_1 c_2} \\ \Rightarrow c_1 - c_2 &= \sqrt{4b^2 \cos^2 A - 4(b^2 - a^2)} \\ &= 2\sqrt{a^2 - b^2 \sin^2 A} \end{aligned}$$

17 (b)

We have,

$$\begin{aligned} \tan \alpha &= (1 + 2^{-x})^{-1} = \frac{2^x}{2^x + 1} \text{ and } \tan \beta \\ &= \frac{1}{2^{x+1} + 1} \\ \therefore \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \end{aligned}$$

$$\Rightarrow \tan(\alpha + \beta) = \frac{2^x(2^{x+1} + 1) + (2^x + 1)}{(2^x + 1)(2^{x+1} + 1) - 2^x}$$

$$\Rightarrow \tan(\alpha + \beta) = \frac{2(2^x)^2 + 2.2^x + 1}{2(2^x)^2 + 2.2^x + 1} = 1 \Rightarrow \alpha + \beta = \pi/4$$

18 (a)

Given, $f(x) = \sin x(1 + \cos x)$

It is minimum at $x = \frac{\pi}{3}$

$$\therefore f\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)\left(1 + \cos\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{3}}{2}\left(1 + \frac{1}{2}\right) = \frac{3\sqrt{3}}{4}$$

19 (c)

We have,

$$\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$$

$$= \frac{\cos\left\{\frac{\pi}{11} + \left(\frac{5-1}{2}\right)\frac{2\pi}{11}\right\} \sin\left(\frac{5\pi}{11}\right)}{\sin\left(\frac{\pi}{11}\right)}$$

$$= \frac{\cos\frac{5\pi}{11} \sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}} = \frac{1}{2} \frac{\sin\left(\frac{10\pi}{11}\right)}{\sin\frac{\pi}{11}} = \frac{1}{2}$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	C	C	C	A	C	C	C	B	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	B	D	D	D	C	B	A	C

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Solutions
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1 (c)

$$\begin{aligned}
 & \left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 + \cos \frac{5\pi}{8}\right) \left(1 + \cos \frac{7\pi}{8}\right) \\
 &= \left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 - \cos \frac{3\pi}{8}\right) \\
 &\quad \times \left(1 - \cos \frac{\pi}{8}\right) \\
 &= \left(1 - \cos^2 \frac{\pi}{8}\right) \left(1 - \cos^2 \frac{3\pi}{8}\right) \\
 &= \sin^2 \frac{\pi}{8} \cdot \sin^2 \frac{3\pi}{8} = \frac{1}{4} \left[2 \sin \frac{\pi}{8} \sin \frac{3\pi}{8}\right]^2 \\
 &= \frac{1}{4} \left[\cos \frac{\pi}{4} - \cos \frac{\pi}{2}\right]^2 = \frac{1}{4} \left[\frac{1}{\sqrt{2}} - 0\right]^2 = \frac{1}{8}
 \end{aligned}$$

2 (a)

We have,

$$\begin{aligned}
 2 \sin \frac{A}{2} &= \sqrt{1 + \sin A} + \sqrt{1 - \sin A} \\
 \Rightarrow 2 \sin \frac{A}{2} - \sqrt{(\cos A/2 + \sin A/2)^2} \\
 &\quad + \sqrt{(\cos A/2 - \sin A/2)^2} \\
 \Rightarrow 2 \sin A/2 &= |\cos A/2 + \sin A/2| \\
 &\quad + |\sin A/2 - \cos A/2| \\
 \Rightarrow \cos A/2 + \sin A/2 &\geq 0 \text{ and } \cos A/2 - \sin A/2 \leq 0
 \end{aligned}$$

$$\Rightarrow \pi/4 \leq A/2 \leq 3\pi/4 \text{ and } \pi/4 \leq A \leq 5\pi/4$$

$$\Rightarrow \pi/4 \leq A/2 \leq 3\pi/4$$

$$\Rightarrow 2n\pi + \pi/4 \leq A/2 \leq 2n\pi + 3\pi/4, n \in \mathbb{Z}$$

3 (a)

We have,

$$\begin{aligned}
 a \cos^2 \frac{C}{2} + c \cos^2 \frac{A}{2} &= \frac{3b}{2} \\
 \Rightarrow a \left\{ \frac{s(s-c)}{ab} \right\} + c \left\{ \frac{s(s-a)}{bc} \right\} &= \frac{3b}{2} \\
 \Rightarrow \frac{s}{b} (2s - a - c) &= \frac{3b}{2}
 \end{aligned}$$

$$\Rightarrow 2s = 3b \Rightarrow a + c = 2b \Rightarrow a, b, c \text{ are in A.P.}$$

4 (a)

We have,

$$\begin{aligned}
 \tan(\theta_1 + \theta_2 + \dots + \theta_n) &= \frac{S_1 - S_3 + S_5 - S_7 + \dots}{1 - S_2 + S_4 - S_6 + \dots} \\
 \therefore \tan 5\theta &= \frac{{}^5C_1 \tan \theta - {}^5C_3 \tan^3 \theta + {}^5C_5 \tan^5 \theta}{1 - {}^5C_2 \tan^2 \theta + {}^5C_4 \tan^4 \theta}
 \end{aligned}$$

5 (d)

It is given that a, b, c are in A.P.

$$\therefore 2b = a + c$$

Now,

$$\begin{aligned}
 \frac{\tan \frac{A}{2} + \tan \frac{C}{2}}{\cot \frac{B}{2}} &= \left(\tan \frac{A}{2} + \tan \frac{C}{2} \right) \tan \frac{B}{2} \\
 \Rightarrow \frac{\tan \frac{A}{2} + \tan \frac{C}{2}}{\cot \frac{B}{2}} &= \left\{ \frac{\Delta}{s(s-a)} + \frac{\Delta}{s(s-c)} \right\} \frac{\Delta}{s(s-b)} \\
 \Rightarrow \frac{\tan \frac{A}{2} + \tan \frac{C}{2}}{\cot \frac{B}{2}} &= \frac{\Delta^2}{s^2(s-b)} \left\{ \frac{1}{s-a} + \frac{1}{s-c} \right\} \\
 \Rightarrow \frac{\tan \frac{A}{2} + \tan \frac{C}{2}}{\cot \frac{B}{2}} &= \frac{\Delta^2 b}{s \Delta^2} \\
 \Rightarrow \frac{\tan \frac{A}{2} + \tan \frac{C}{2}}{\cot \frac{B}{2}} &= \frac{2b}{2s} = \frac{2b}{a+b+c} = \frac{2b}{3b} = \frac{2}{3} \quad [\because a+c=2b]
 \end{aligned}$$

6 (c)

We have,

$$\begin{aligned}
 2 \frac{\cos A}{a} + \frac{\cos B}{b} + 2 \frac{\cos C}{c} &= \frac{a}{bc} + \frac{b}{ac} \\
 \Rightarrow 2 \left(\frac{b^2 + c^2 - a^2}{2abc} \right) + \frac{c^2 + a^2 - b^2}{2abc} \\
 &\quad + 2 \left(\frac{a^2 + b^2 - c^2}{2abc} \right) = \frac{a^2 + b^2}{abc} \\
 \Rightarrow b^2 + c^2 &= a^2 \Rightarrow A = \frac{\pi}{2}
 \end{aligned}$$

7 (d)

$$2^{n-1} \tan(2^{n-1}\alpha) + 2^n \cot(2^n\alpha)$$

$$\begin{aligned}
&= 2^{n-1} \left[\frac{\sin 2^{n-1} \alpha}{\cos 2^{n-1} \alpha} + 2 \frac{\cos 2^n \alpha}{\sin 2^n \alpha} \right] \\
&= 2^{n-1} \left[\frac{\cos 2^n \alpha \cos 2^{n-1} \alpha + \sin 2^n \alpha}{\sin 2^{n-1} \alpha + \cos 2^n \alpha \cos 2^{n-1} \alpha} \right] \\
&= 2^{n-1} \left[\frac{\cos 2^{n-1} \alpha (1 + \cos 2^n \alpha)}{\sin 2^n \alpha \cos 2^{n-1} \alpha} \right] \\
&= 2^{n-1} \cot 2^{n-1} \alpha
\end{aligned}$$

Proceeding in similar way in last, we get

$$\begin{aligned}
&\tan \alpha + 2 \cot 2\alpha \\
&= \frac{\sin \alpha}{\cos \alpha} + 2 \frac{\cos 2\alpha}{\sin 2\alpha} \\
&= \frac{\cos 2\alpha \cos \alpha + \sin 2\alpha \sin \alpha + \cos 2\alpha \cos \alpha}{\sin 2\alpha \cos \alpha} \\
&= \frac{\cos \alpha (1 + \cos 2\alpha)}{2 \sin \alpha \cos^2 \alpha} \\
&= \frac{2 \cos^2 \alpha}{2 \sin \alpha \cos \alpha} \\
&= \frac{\cos \alpha}{\sin \alpha} = \cot \alpha
\end{aligned}$$

8 (c)

$$\cos^2 \left(\frac{\pi}{3} - x \right) - \cos^2 \left(\frac{\pi}{3} + x \right)$$

$$\begin{aligned}
&= \left[\cos \left(\frac{\pi}{3} - x \right) + \cos \left(\frac{\pi}{3} + x \right) \right] \left[\cos \left(\frac{\pi}{3} - x \right) - \cos \left(\frac{\pi}{3} + x \right) \right] \\
&= \left(2 \cos \frac{\pi}{3} \cos x \right) \left(2 \sin \frac{\pi}{3} \sin x \right)
\end{aligned}$$

$$= \sin \frac{2\pi}{3} \sin 2x = \frac{\sqrt{3}}{2} \sin 2x$$

Hence, maximum value of given expression is $\frac{\sqrt{3}}{2}$

9 (d)

We have,

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \Rightarrow \cos C = -1 \Rightarrow C = \pi$$

Which is impossible in a triangle

10 (c)

We have,

$$\frac{a}{\cos A} = \frac{b}{\cos B}$$

$$\Rightarrow 2R \sin A \cos B = 2R \sin B \cos A$$

$$\Rightarrow \sin(A - B) = 0 \Rightarrow A = B$$

$$\therefore 2 \sin A \cos B = \sin 2A = \sin(180^\circ - C) [$$

$$\therefore 2A + C = 180^\circ]$$

$$\Rightarrow 2 \sin A \cos B = \sin C$$

11 (d)

$$\text{Given, } 1 + \sin \theta + \sin^2 \theta + \dots \infty = 4 + 2\sqrt{3}$$

$$\Rightarrow \frac{1}{1 - \sin \theta} = 4 + 2\sqrt{3} \quad [\because 0 < \sin \theta < 1]$$

$$\Rightarrow 1 - \sin \theta = \frac{4 - 2\sqrt{3}}{16 - 12} = 1 - \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

12 (c)

We have,

$$\begin{aligned}
&a(b^2 + c^2) \cos A \\
&\quad + b(c^2 + a^2) \cos B + c(a^2 + b^2) \cos C \\
&= (ab^2 \cos A + ba^2 \cos B) \\
&\quad + (ac^2 \cos A + ca^2 \cos C) \\
&\quad + (bc^2 \cos B + cb^2 \cos C) \\
&= ab(b \cos A + a \cos B) + ca(c \cos A + a \cos C) \\
&\quad + bc(c \cos B + b \cos C) \\
&= abc + abc + abc = 3abc
\end{aligned}$$

14 (a)

$$\text{We have, } \tan(\pi \cos \theta) = \tan \left(\frac{\pi}{2} - \pi \sin \theta \right)$$

$$\therefore \sin \theta + \cos \theta = \frac{1}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \cos \left(\theta - \frac{\pi}{4} \right) = \frac{1}{2\sqrt{2}}$$

15 (a)

We have,

$$y = 5x^2 + 2x + 3$$

Clearly, it represents an upward opening parabola having its vertex at $(-1/5, 14/5)$

$$\therefore y \geq \frac{14}{5} > 2$$

$$\text{Now, } y = 2 \sin x \leq 2$$

Thus, the two curves do not intersect. Hence, there is no common point in the two curves

16 (d)

We have,

$$(a + b + c)(b + c - a) = \lambda bc$$

$$\Rightarrow 2s(2s - a) = \lambda bc$$

$$\Rightarrow \frac{s(s - a)}{bc} = \frac{\lambda}{4}$$

$$\Rightarrow \cos^2 \frac{A}{2} = \frac{\lambda}{4}$$

$$\Rightarrow 0 < \frac{\lambda}{4} < 1 \Rightarrow 0 < \lambda < 4 \quad \left[\because \cos^2 \frac{A}{2} \leq 1 \right]$$

17 (c)

The given expression can be written as

$$\begin{aligned}
 & (1 + \cot^2 A) \cot^2 A - (1 + \tan^2 A) \tan^2 A \\
 & \quad - (\cot^2 A - \tan^2 A) \{ (1 + \tan^2 A)(1 + \cot^2 A) - 1 \} \\
 & = \cot^2 A + \cot^4 A - \tan^2 A - \tan^4 A
 \end{aligned}$$

$$\begin{aligned}
 & -(\cot^2 A - \tan^2 A)(\cot^2 A + \tan^2 A + 1) \\
 & = \cot^2 A + \cot^4 A - \tan^2 A - \tan^4 A \\
 & \quad - (\cot^2 A - \tan^2 A) \\
 & \quad - (\cot^4 A - \tan^4 A) \\
 & = 0
 \end{aligned}$$

13 (b)

We have, $2b = a + c$

And,

$$\Delta = \frac{3}{5} \times \frac{\sqrt{3}}{4} \left(\frac{a+b+c}{3} \right)^2$$

$$\Rightarrow \Delta = \frac{3\sqrt{3}}{20} b^2$$

$$\Rightarrow s(s-a)(s-b)(s-c) = \frac{27}{400} b^4$$

$$\Rightarrow \left(\frac{a+b+c}{2} \right) \left(\frac{b+c-a}{2} \right) \left(\frac{c+a-b}{2} \right) \left(\frac{a+b-c}{2} \right) = \frac{27}{400} b^4$$

$$\Rightarrow \left(\frac{3b}{2} \right) \times \left(\frac{b+c-2b+c}{2} \right) \left(\frac{b}{2} \right) \left(\frac{2b-c+b-c}{2} \right) = \frac{27}{400} b^4$$

$$[\because 2b = a + c]$$

$$\Rightarrow \frac{3b}{2} \times \left(\frac{2c-b}{2} \right) \times \frac{b}{2} \times \left(\frac{3b-2c}{2} \right) = \frac{27}{400} b^4$$

$$\Rightarrow (2c-b)(3b-2c) = \frac{9b^2}{25}$$

$$\Rightarrow (6bc - 4c^2 - 3b^2 + 2bc) = \frac{9b^2}{25}$$

$$\Rightarrow 8bc - 4c^2 - 3b^2 = \frac{9b^2}{25}$$

$$\Rightarrow \frac{84}{25} b^2 - 8bc + 4c^2 = 0$$

$$\Rightarrow 21b^2 - 50bc + 25c^2 = 0$$

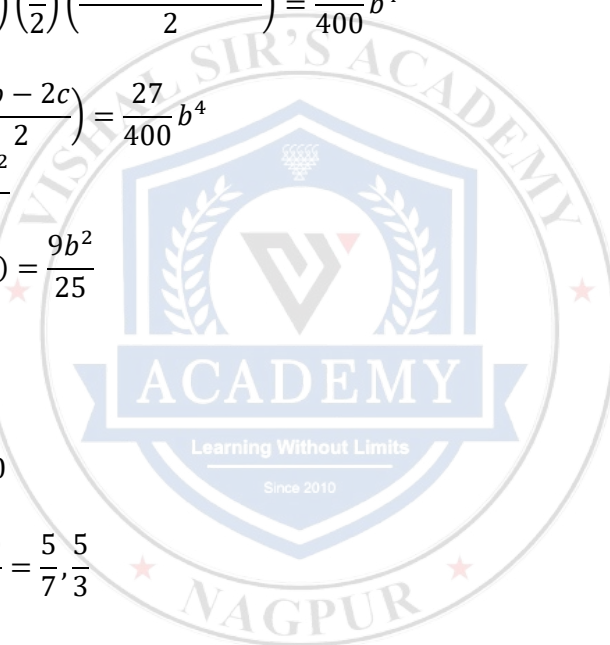
$$\Rightarrow (7b - 5c)(3b - 5c) = 0$$

$$\Rightarrow 7b = 5c \text{ or, } 3b = 5c \Rightarrow \frac{b}{c} = \frac{5}{7}, \frac{5}{3}$$

Now,

$$2b = a + c \Rightarrow \frac{2b}{c} = \frac{a}{c} + 1 \Rightarrow \frac{a}{c} = \frac{3}{7}, \frac{7}{3}$$

Hence, $a : b : c = 3 : 5 : 7$



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(a)

$$\begin{aligned}
& \sqrt{\frac{a+b}{a-b}} - \sqrt{\frac{a-b}{a+b}} \\
&= \sqrt{\frac{1+\frac{b}{a}}{1-\frac{b}{a}}} - \sqrt{\frac{1-\frac{b}{a}}{1+\frac{b}{a}}} \\
&= \sqrt{\frac{1+\tan \alpha}{1-\tan \alpha}} - \sqrt{\frac{1-\tan \alpha}{1+\tan \alpha}} \\
&= \frac{(1+\tan \alpha) - (1-\tan \alpha)}{\sqrt{1-\tan^2 \alpha}} \\
&= \frac{2 \tan \alpha}{\sqrt{1-\tan^2 \alpha}} = \frac{2 \sin \alpha}{\sqrt{\cos 2\alpha}}
\end{aligned}$$

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(b)

Since, $\sin \theta + \cos \theta = x$... (i)

$$\text{and } \sin^6 \theta + \cos^6 \theta = \frac{1}{4}[4 - 3(x^2 - 1)^2]$$

On equation Eq (i), we get

$$\sin 2\theta = x^2 - 1 \leq 1 \quad (\because \sin 2\theta \leq 1)$$

$$\Rightarrow x^2 \leq 2 \Rightarrow -\sqrt{2} \leq x \leq \sqrt{2}$$

$$\text{Now, } \sin^6 \theta + \cos^6 \theta = (\sin^2 \theta + \cos^2 \theta)^3 - 3 \sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta)$$

$$= 1 - 3 \sin^2 \theta \cos^2 \theta = 1 - \frac{3}{4} \sin^2 2\theta$$

$$= 1 - \frac{3}{4}(x^2 - 1)^2 = \frac{1}{4}[4 - 3(x^2 - 1)^2]$$

Thus, the given result will hold true only when $x^2 \leq 2$ and not for all real values of x

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(b)

We have,

$$\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$$

$$\Rightarrow \sin(B+C) \sin(B-C) = \sin(A+B) \sin(A-B)$$

$$\Rightarrow \sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$$

$$\Rightarrow b^2 - c^2 = a^2 - b^2 \Rightarrow a^2, b^2, c^2 \text{ are in A.P.}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	A	A	D	C	D	C	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	C	A	A	D	C	B	A	B	B



Topic :-TRIGONOMETRIC FUNCTIONS

1

(a)It is given that A, B, C are in A.P.

$$\therefore 2B = A + C \Rightarrow 3B = A + B + C \Rightarrow 3B = 180^\circ \Rightarrow B = 60^\circ$$

$$\Rightarrow \cos B = \frac{1}{2}$$

$$\Rightarrow \frac{c^2 + a^2 - b^2}{2ac} = \frac{1}{2}$$

$$\Rightarrow c^2 + a^2 - b^2 = ac$$

$$\Rightarrow (a - c)^2 = b^2 - ac$$

$$\Rightarrow |a - c| = \sqrt{b^2 - ac}$$

$$\Rightarrow |\sin A - \sin C| = \sqrt{\sin^2 B - \sin A \sin C}$$

$$\Rightarrow 2 \left| \sin \frac{A - C}{2} \right| \cos \frac{A + C}{2} = \sqrt{\frac{3}{4} - \sin A \sin C}$$

$$\Rightarrow 2 \left| \sin \frac{A - C}{2} \right| = \sqrt{3 - 4 \sin A \sin C}$$

$$\Rightarrow \frac{\sqrt{3 - 4 \sin A \sin C}}{|A - C|} = \frac{2 \left| \sin \frac{A - C}{2} \right|}{|A - C|}$$

$$\Rightarrow \lim_{A \rightarrow C} \frac{\sqrt{3 - 4 \sin A \sin C}}{|A - C|} = \lim_{A \rightarrow C} \left| \frac{\sin \left(\frac{A - C}{2} \right)}{\frac{A - C}{2}} \right| = 1$$

2

(c)

$$3 - \cos \theta + \cos \left(\theta + \frac{\pi}{3} \right)$$

$$= 3 - \cos \theta + \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$$

$$= 3 - \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta = 3 - \sin \left(\theta + \frac{\pi}{6} \right)$$

$$\text{Since, } -1 \leq \sin \theta \leq 1$$

3 Hence, the value of expression lies in $[2, 4]$

(c)

We have, $\cos A = m \cos B$

$$\Rightarrow \frac{\cos A}{\cos B} = \frac{m}{1}$$

$$\Rightarrow \frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m + 1}{m - 1}$$

$$\Rightarrow \frac{2 \cos \frac{A+B}{2} \cos \frac{B-A}{2}}{2 \sin \frac{A+B}{2} \sin \frac{B-A}{2}} = \frac{m + 1}{m - 1}$$

$$\Rightarrow \cot \frac{A+B}{2} = \left(\frac{m+1}{m-1} \right) \tan \frac{B-A}{2}$$

$$\text{But } \cot \frac{A+B}{2} = \lambda \tan \frac{B-A}{2}$$

$$\therefore \lambda = \frac{m+1}{m-1}$$

4

(c)

$$\begin{aligned} & \cos^4 \frac{\pi}{8} + \cos^4 \frac{7\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} \\ &= \cos^4 \frac{\pi}{8} + \cos^4 \frac{\pi}{8} + \cos^4 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) + \cos^4 \left(\frac{\pi}{2} + \frac{\pi}{8} \right) \\ &= 2 \left[\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} \right] \\ &= 2 \left[\left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \right)^2 - 2 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} \right] \\ &= 2 \left[1 - \frac{1}{2} \left(\sin \frac{\pi}{4} \right)^2 \right] \\ &= 2 \left[1 - \frac{1}{4} \right] = \frac{3}{2} \end{aligned}$$

5

(b)

$$\text{Given, } \sin \theta = \frac{12}{13} \text{ and } \cos \phi = -\frac{3}{5}$$

$$\therefore \cos \theta = \frac{5}{13} \text{ and } \sin \phi = -\frac{4}{5}$$

$$\therefore \sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$$

$$= \frac{12}{13} \times \left(-\frac{3}{5} \right) + \frac{5}{13} \times \left(-\frac{4}{5} \right)$$

$$= \frac{-36}{65} + \frac{(-20)}{65} = -\frac{56}{65}$$

6

(c)

We have,

$$\sec^2 \theta \operatorname{cosec}^2 \theta = \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} - \frac{4}{\sin^2 2\theta} \geq 4$$

and, $\sec^2 \theta \operatorname{cosec}^2 \theta = \frac{4}{\sin^2 2\theta} \geq 4$

Thus, the required equation is

$x^2 - \lambda x + \lambda = 0$, where $\lambda \geq 4$

7

(a)

$$\begin{aligned} & \frac{1}{m} \left[(m+n) + \frac{1}{(m+n)} \right] \\ &= \frac{1}{\sec \theta} \left[\sec \theta + \tan \theta + \frac{1}{\sec \theta + \tan \theta} \right] \\ &= \frac{[\sec^2 \theta + \tan^2 \theta + 2 \sec \theta \tan \theta + 1]}{\sec \theta (\sec \theta + \tan \theta)} \\ &= \frac{2 \sec^2 \theta + 2 \sec \theta \tan \theta}{\sec \theta (\sec \theta + \tan \theta)} \\ &= 2 \end{aligned}$$

8

(a)

$$\begin{aligned} \therefore \sin A \sin B &= \frac{1}{2} \times 2 \sin A \sin B \\ &= \frac{1}{2} [\cos(A-B) - \cos(A+B)] \\ &= \frac{1}{2} [\cos(A-B) - \cos 90^\circ] \quad (\because A+B+C=180^\circ \text{ and } \angle C=90^\circ, \text{ given}) \\ &= \frac{1}{2} \cos(A-B) \leq \frac{1}{2} \\ \therefore \text{Maximum value of } \sin A \sin B &= \frac{1}{2} \end{aligned}$$

9

(b)

In cyclic quadrilateral $ABCD$, we have

$A + C = \pi$ and $B + D = \pi$

$\therefore \cos A = -\cos C$ and $\cos B = -\cos D$

$\Rightarrow \cos A + \cos B + \cos C + \cos D = 0$

10

(d)

Let $A = \theta$, $B = 2\theta$ and $C = 3\theta$. Then,

$A + B + C = 180^\circ \Rightarrow 6\theta = 180^\circ \Rightarrow \theta = 30^\circ$

$\therefore A = 30^\circ$, $B = 60^\circ$ and $C = 90^\circ$

Now,

$a : b : c = \sin A : \sin B : \sin C$,

$\Rightarrow a : b : c = \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 \Rightarrow a : b : c = 1 : \sqrt{3} : 2$

11

(b)

We have,

$\sin(\pi \cos \theta) = \cos(\pi \sin \theta)$

$$\begin{aligned}\Rightarrow \sin(\pi \cos \theta) &= \sin\left(\frac{\pi}{2} + \pi \sin \theta\right) \\ \Rightarrow \pi \cos \theta &= \frac{\pi}{2} + \pi \sin \theta \\ \Rightarrow \pi \cos \theta - \pi \sin \theta &= \frac{\pi}{2} \\ \Rightarrow \cos \theta - \sin \theta &= \frac{1}{2} \\ \Rightarrow \frac{1}{\sqrt{2}} \cos \theta - \frac{1}{\sqrt{2}} \sin \theta &= \frac{1}{2\sqrt{2}} \Rightarrow \cos\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}\end{aligned}$$

12

(a)

The given equation is not meaningful for

$$|\cos x| = 1$$

So, let $|\cos x| \neq 1$

Now,

$$2^{1+|\cos x|+\cos^2 x+|\cos x|^3+\dots+\text{to } \infty} = 4$$

$$\Rightarrow \frac{1}{2^{1-|\cos x|}} = 2^2$$

$$\Rightarrow \frac{1}{1-|\cos x|} = 2$$

$$\Rightarrow 2 - 2|\cos x| = 1$$

$$\Rightarrow |\cos x| = \frac{1}{2}$$

$$\Rightarrow \cos x = \pm \frac{1}{2}$$

$$\Rightarrow \cos x = \cos \frac{\pi}{3}, \cos x = \cos \frac{2\pi}{3}$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, x = 2n\pi \pm \frac{2\pi}{3}, n \in \mathbb{Z}$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, x = (2n \pm 1)\pi \pm \frac{\pi}{3}$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

13

(b)

We have,

$$(\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 = 0$$

$$\Rightarrow (\cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta) - (\sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta) = 0$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta = -2(\cos \alpha \cos \beta - \sin \alpha \sin \beta)$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta = -2 \cos(\alpha + \beta)$$

14

(d)

The given expression can be written as $\frac{1+\cos y - \sin^2 y}{1+\cos y} + \frac{(1-\cos^2 y) - \sin^2 y}{\sin y(1-\cos y)}$

$$= \frac{\cos y (1 + \cos y)}{1 + \cos y} + 0 = \cos y$$

15

(a)

We have,

$$a = 2b$$

$$\Rightarrow 2R \sin A = 4R \sin B$$

$$\Rightarrow \sin A = 2 \sin B$$

$$\Rightarrow \sin 3B = 2 \sin B \quad [\because A = 3B]$$

$$\Rightarrow 3 \sin B - 4 \sin^3 B = 2 \sin B$$

$$\Rightarrow \sin B - 4 \sin^3 B = 0$$

$$\Rightarrow 1 - 4 \sin^2 B = 0 \Rightarrow \sin B = \frac{1}{2} \Rightarrow B = \frac{\pi}{6}$$

$$\therefore A = 3B = \frac{\pi}{2}$$

16

(b)

We know that

$$AD^2 = \frac{1}{4}(b^2 + c^2 + 2bc \cos A)$$

$$\therefore 4AD^2 = b^2 + c^2 + 2bc \cos \frac{\pi}{3} \Rightarrow 4AD^2 = b^2 + c^2 + bc$$

17

(a)

We know that,

$$\alpha - \beta = (\theta - \beta) - (\theta - \alpha)$$

$$\therefore \cos(\alpha - \beta) = \cos(\theta - \beta) \cos(\theta - \alpha) + \sin(\theta - \beta) \sin(\theta - \alpha)$$

$$= ab + \sqrt{1 - a^2} \sqrt{1 - b^2}$$

$$\text{and } \sin(\alpha - \beta) = \pm(a\sqrt{1 - b^2} - b\sqrt{1 - a^2})$$

$$\Rightarrow \sin^2(\alpha - \beta) = a^2 + b^2 - 2a^2b^2 - 2ab\sqrt{1 - a^2} \sqrt{1 - b^2}$$

$$\Rightarrow \sin^2(\alpha - \beta) = a^2 + b^2 - 2a^2b^2 - 2ab[\cos(\alpha - \beta) - ab]$$

$$\therefore \sin^2(\alpha - \beta) - a^2 + b^2 - 2ab \cos(\alpha - \beta)$$

$$\Rightarrow \sin^2(\alpha - \beta) + 2ab \cos(\alpha - \beta) = a^2 + b^2$$

18

(b)

$$\frac{1}{2} \tan \frac{x}{2} = \frac{1}{2} \cot \frac{x}{2} - \cot x \quad \left[\because \cot x = \frac{1 - \tan^2 \frac{x}{2}}{2 \tan \frac{x}{2}} \right]$$

$$\text{And } \frac{1}{2^2} \tan \frac{x}{2^2} = \frac{1}{2^2} \cot \left(\frac{x}{2^2} \right) - \frac{1}{2} \cot \left(\frac{x}{2} \right)$$

$$\text{Similarly, } \frac{1}{2^3} \tan \left(\frac{x}{2^3} \right) = \frac{1}{2^3} \cot \left(\frac{x}{2^3} \right) - \frac{1}{2^2} \cot \left(\frac{x}{2^2} \right)$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$\frac{1}{2^n} \tan \left(\frac{x}{2^n} \right) = \frac{1}{2^n} \cot \left(\frac{x}{2^n} \right) - \frac{1}{2^{n-1}} \cot \left(\frac{x}{2^{n-1}} \right)$$

On adding all the above results, we get

$$\frac{1}{2} \tan \frac{x}{2} + \frac{1}{2^2} \tan \left(\frac{x}{2^2} \right) + \dots + \frac{1}{2^n} \tan \left(\frac{x}{2^n} \right) = \frac{1}{2^n} \cot \left(\frac{x}{2^n} \right) - \cot x$$

19

(c)

It is given that

Area of $\triangle ABC$ = Area of $\triangle DEF$

$$\Rightarrow \frac{1}{2} AB \cdot AC \sin A = \frac{1}{2} CE \cdot EF \sin E$$

$$\Rightarrow \sin A = \sin E$$

$$\Rightarrow \sin 2E = \sin E$$

$$\Rightarrow 2E = \pi - E \Rightarrow E = \frac{\pi}{3} \Rightarrow A = 2E = \frac{2\pi}{3}$$

20

(c)

We have,

$$\begin{aligned} & \sin \frac{\pi}{18} \sin \frac{5\pi}{18} \sin \frac{7\pi}{18} \\ &= \cos \left(\frac{\pi}{2} - \frac{\pi}{18} \right) \cos \left(\frac{\pi}{2} - \frac{5\pi}{18} \right) \cos \left(\frac{\pi}{2} - \frac{7\pi}{18} \right) \\ &= \cos \frac{8\pi}{18} \cos \frac{4\pi}{18} \cos \frac{2\pi}{9} \\ &= \cos \frac{\pi}{9} \cos \frac{2\pi}{9} \cos \frac{4\pi}{9} = \frac{\sin(2^3 \cdot \pi/9)}{2^3 \sin \pi/9} = \frac{1}{2^3} = \frac{1}{8} \end{aligned}$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	C	C	B	C	A	A	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	B	D	A	B	A	B	C	C

Topic :-TRIGONOMETRIC FUNCTIONS

1 (c)

We have,

$$a = \sin^4 \theta + \cos^4 \theta \leq \sin^2 \theta + \cos^2 \theta \leq 1$$

Also,

$$a = \sin^4 \theta + \cos^4 \theta = (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta$$

$$\Rightarrow a = \sin^4 \theta + \cos^4 \theta = 1 - \frac{1}{2} \sin^2 2\theta$$

$$\Rightarrow \sin^2 2\theta = 2(1 - a) \Rightarrow 2(1 - a) \leq 1 \Rightarrow a \geq \frac{1}{2}$$

$$\text{Hence, } \frac{1}{2} \leq a \leq 1$$

2 (a)

Let $\sqrt{3} + 1 = r \cos \alpha$ and $\sqrt{3} - 1 = r \sin \alpha$, then

$$r = \sqrt{(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2}$$

$$= \sqrt{3 + 1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3}} = 2\sqrt{2}$$

$$\text{and } \tan \alpha = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{1-\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)} = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$

$$\Rightarrow \alpha = \frac{\pi}{12}$$

The given equation reduces to

$$2\sqrt{2} \cos(\theta - \alpha) = 2$$

$$\Rightarrow \cos\left(\theta - \frac{\pi}{12}\right) = \cos \frac{\pi}{4}$$

$$\Rightarrow \theta - \frac{\pi}{12} = 2n\pi \pm \frac{\pi}{4}$$

$$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$$

3 (d)

$$\sin(A + B) = \sin A \cos B + \sin B \cos A$$

$$= \frac{1}{\sqrt{10}} \cdot \sqrt{1 - \frac{1}{5}} + \frac{1}{\sqrt{5}} \sqrt{1 - \frac{1}{10}}$$

$$\left[\because \sin A = \frac{1}{\sqrt{10}}, \sin B = \frac{1}{\sqrt{5}} \right]$$

$$= \frac{1}{\sqrt{10}} \sqrt{\frac{4}{5}} + \frac{1}{\sqrt{5}} \sqrt{\frac{9}{10}} = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4}$$

$$\Rightarrow A + B = \frac{\pi}{4}$$

4 **(b)**

The given equation can be rewritten as

$$\tan \theta (\sin \theta + \sqrt{3}) = 0$$

$$\Rightarrow \tan \theta = 0, \text{ but } \sin \theta + \sqrt{3} \neq 0$$

$$\Rightarrow \tan \theta = 0 \Rightarrow \theta = n\pi, n \in I$$

5 **(a)**

We have, $y = \sin \theta - \cos \theta$ and $\sin \theta - \cos \theta$ lies between $-\sqrt{2}$ and $+\sqrt{2}$

$$\therefore -\sqrt{2} \leq y \leq \sqrt{2}$$

6 **(c)**

$$\text{Now, } \sin(\alpha - \beta) = \sin(\theta - \beta - (\theta - \alpha))$$

$$= \sin(\theta - \beta) = \cos(\theta - \alpha) - \cos(\theta - \beta) \sin(\theta - \alpha)$$

$$= ba - \sqrt{1 - b^2} \sqrt{1 - a^2}$$

$$\text{and } \cos(\alpha - \beta) = \cos(\theta - \beta - (\theta - \alpha))$$

$$= \cos(\theta - \beta) \cos(\theta - \alpha) + \sin(\theta - \beta) \sin(\theta - \alpha)$$

$$= a\sqrt{1 - b^2} + b\sqrt{1 - a^2}$$

$$\therefore \cos^2(\alpha - \beta) + 2ab \sin(\alpha - \beta)$$

$$= (a\sqrt{1 - b^2} + b\sqrt{1 - a^2})^2 + 2ab(ab - \sqrt{1 - a^2} \sqrt{1 - b^2})$$

$$= a^2 + b^2$$

7 **(d)**

We have,

$$8 \sec^2 \theta - 6 \sec \theta + 1 = 0$$

$$\Rightarrow (4 \sec \theta - 1)(2 \sec \theta - 1) = 0$$

$$\Rightarrow \sec \theta = \frac{1}{4}, \sec \theta = \frac{1}{2}$$

But, this is not possible as $|\sec \theta| \geq 1$

8 **(c)**

We have,

$$x^3 - 13x^2 + 54x - 72 = 0$$

$$\Rightarrow (x - 3)(x^2 - 10x + 24) = 0$$

$$\Rightarrow (x-3)(x-4)(x-6) = 0 \Rightarrow x = 3, 4, 6$$

Let $a = 3, b = 4$ and $c = 6$

$$\therefore \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc} = \frac{61}{144}$$

9 **(b)**

$$\begin{aligned}\cos^4 \theta - \sin^4 \theta &= (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta) \\ &= \cos 2\theta = 2 \cos^2 \theta - 1\end{aligned}$$

10 **(b)**

$$\begin{aligned}\cos 15^\circ \cos 7\frac{1}{2}^\circ \sin 7\frac{1}{2}^\circ \\ &= \frac{1}{2} \cos 15^\circ \sin 15^\circ = \frac{1}{4} \sin 30^\circ \\ &= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}\end{aligned}$$

11 **(b)**

$$\begin{aligned}\sqrt{\frac{1-\sin \theta}{1+\sin \theta}} + \sqrt{\frac{1+\sin \theta}{1-\sin \theta}} &= \frac{1-\sin \theta + 1+\sin \theta}{\sqrt{1-\sin^2 \theta}} \\ &= \frac{2}{\sqrt{\cos^2 \theta}} = \frac{2}{|\cos \theta|} \\ &= -\frac{2}{\cos \theta} = -2 \sec \theta \left(\because \frac{\pi}{2} < \theta < \pi \right)\end{aligned}$$

12 **(c)**

$$\begin{aligned}\cos^2 A(3-4\cos^2 A)^2 + \sin^2 A(3-4\sin^2 A)^2 \\ &= (3\cos A - 4\cos^3 A)^2 + (3\sin A - 4\sin^3 A)^2 \\ &= (-\cos 3A)^2 + (\sin 3A)^2 = 1\end{aligned}$$

13 **(a)**

It is given that A, B, C are in A.P.

$$\therefore 2B = A + C$$

$$\Rightarrow 3B = A + B + C \Rightarrow 3B = 180^\circ \Rightarrow B = 60^\circ$$

Also,

$$b : c = \sqrt{3} : \sqrt{2}$$

$$\Rightarrow \frac{\sin B}{\sin C} = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{3}}{2 \sin C} = \frac{\sqrt{3}}{\sqrt{2}} \Rightarrow \sin C = \frac{1}{\sqrt{2}} \Rightarrow C = 45^\circ$$

$$\therefore A = 180^\circ - (60^\circ + 45^\circ) = 75^\circ$$

14 **(b)**

$$\text{We have, } \tan x = \frac{b}{a}$$

$$\begin{aligned}
& \therefore \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} \\
&= \sqrt{\frac{1+\frac{b}{a}}{1-\frac{b}{a}}} + \sqrt{\frac{1-\frac{b}{a}}{1+\frac{b}{a}}} \\
&= \sqrt{\frac{1+\tan x}{1-\tan x}} + \sqrt{\frac{1-\tan x}{1+\tan x}} \\
&= \sqrt{\frac{\cos x + \sin x}{\cos x - \sin x}} + \sqrt{\frac{\cos x - \sin x}{\cos x + \sin x}} \\
&= \frac{\cos x + \sin x + \cos x - \sin x}{\sqrt{\cos^2 x - \sin^2 x}} = \frac{2 \cos x}{\sqrt{\cos 2x}}
\end{aligned}$$

15 (c)

We have,

$$\sin A + \cos A = m \text{ and } \sin^3 A + \cos^3 A = n$$

$$\text{Now, } \sin A + \cos A = m$$

$$\Rightarrow (\sin A + \cos A)^3 = m^3$$

$$\Rightarrow \sin^3 A + \cos^3 A + 3 \sin A \cos A (\sin A + \cos A) = m^3$$

$$\Rightarrow n + 3 \sin A \cos A m = m^3 \dots (i)$$

Again,

$$\sin A + \cos A = m$$

$$\Rightarrow \sin^2 A + \cos^2 A + 2 \sin A \cos A = m^2$$

$$\Rightarrow \sin A \cos A = \frac{m^2 - 1}{2} \dots (ii)$$

From (i) and (ii), we have

$$n + 3m \frac{(m^2 - 1)}{2} = m^3$$

$$\Rightarrow 2n + 3m^3 - 3m = 2m^3 \Rightarrow m^3 - 3m + 2n = 0$$

16 (b)

$$\therefore \sec x - 1 = (\sqrt{2} - 1) \tan x$$

$$\Rightarrow 1 - \cos x = (\sqrt{2} - 1) \sin x$$

$$\Rightarrow \sin \frac{x}{2} \left\{ \sin \frac{x}{2} - (\sqrt{2} - 1) \cos \frac{x}{2} \right\} = 0$$

$$\Rightarrow \sin \frac{x}{2} = 0 \text{ or } \tan \frac{x}{2} = \sqrt{2} - 1 = \tan \frac{\pi}{8}$$

$$\Rightarrow \frac{x}{2} = n\pi \text{ or } \frac{x}{2} = n\pi + \frac{\pi}{8}$$

$$\therefore x = 2n\pi, 2n\pi + \frac{\pi}{4}$$

17 (a)

$$\alpha - \beta = (\theta - \beta) - (\theta - \alpha)$$

$$\therefore \cos(\alpha - \beta) = \cos(\theta - \beta) \cos(\theta - \alpha) + \sin(\theta - \beta) \sin(\theta - \alpha)$$

$$\text{And } \sin(\alpha - \beta) = \sin(\theta - \beta) \cos(\theta - \alpha) - \sin(\theta - \alpha) \cos(\theta - \beta)$$

$$\Rightarrow \cos(\alpha - \beta) = b \cdot a + \sqrt{1 - a^2} \sqrt{1 - b^2}$$

$$\text{And } \sin(\alpha - \beta) = (a\sqrt{1 - b^2}) - (b\sqrt{1 - a^2})$$

$$\text{Now, } \sin^2(\alpha - \beta) = (a\sqrt{1 - b^2})^2 + (b\sqrt{1 - a^2})^2 - 2ab\sqrt{1 - a^2}\sqrt{1 - b^2}$$

$$\Rightarrow \sin^2(\alpha - \beta) = a^2(1 - b^2) + b^2(1 - a^2) - 2ab\{\cos(\alpha - \beta) - ab\}$$

$$\Rightarrow \sin^2(\alpha - \beta) + 2ab \cos(\alpha - \beta) = a^2 - a^2b^2 + b^2 - b^2a^2 + 2a^2b^2$$

$$\Rightarrow \sin^2(\alpha - \beta) + 2ab \cos(\alpha - \beta) = a^2 + b^2$$

18 (a)

$$\text{We have, } S = \sin \theta + \sin 2\theta + \sin 3\theta + \dots + \sin n\theta$$

We know that,

$$\sin \theta + \sin(\theta + \beta) + \sin(\theta + 2\beta) + \dots n \text{ terms}$$

$$= \frac{\sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \sin \left[\frac{\theta + \theta + (n-1)\beta}{2} \right]$$

$$\text{Put, } \beta = \theta$$

$$\therefore S = \frac{\sin \frac{n\theta}{2} \cdot \sin \frac{(n+1)\theta}{2}}{\sin \frac{\theta}{2}}$$

19 (c)

$$\text{Given, } \frac{\tan 3\theta - 1}{\tan 3\theta + 1} = \sqrt{3}$$

$$\Rightarrow \tan 3\theta - 1 - \sqrt{3} \tan 3\theta - \sqrt{3} = 0$$

$$\Rightarrow \tan 3\theta = \frac{1 + \sqrt{3}}{1 - \sqrt{3}} = \tan(45^\circ + 60^\circ)$$

$$\Rightarrow \tan 3\theta = \tan \frac{7\pi}{12}$$

$$\Rightarrow 3\theta = n\pi + \frac{7\pi}{12}$$

$$\Rightarrow \theta = \frac{n\pi}{3} + \frac{7\pi}{12}$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	D	B	A	C	D	C	B	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	A	B	C	B	A	A	C	C

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

Class : XIth

Date :

Solutions

Subject : MATHS

DPP No. :5

Topic :-TRIGONOMETRIC FUNCTIONS

1 (a)

We have,

$$2 \sin \theta = r^4 - 2r^2 + 3$$

$$\Rightarrow 2 \sin \theta = (r^2 - 1)^2 + 2$$

Clearly, LHS ≤ 2 and RHS ≥ 2

So, the equation is meaningful if each side is equal to 2

Clearly, RHS = 2 for $r^2 = 1$

For $r^2 = 1$, we have

$$2 \sin \theta = 2$$

$$\Rightarrow \sin \theta = 1 \Rightarrow \theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2} \quad [\because 0 \leq \theta \leq 5\pi]$$

Also, $r^2 = 1 \Rightarrow r = \pm 1$

Hence, the total number of pairs of the form (r, θ) is $2 \times 3 = 6$

2 (a)

We have,

$$\frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{a^2 + b^2 + c^2}{4\Delta^2}$$

Also,

$$\cot A + \cot B + \cot C = \frac{2R}{abc} (b^2 + c^2 - a^2 + c^2 + a^2 - b^2 + a^2 + b^2 - c^2)$$

$$\Rightarrow \cot A + \cot B + \cot C = \frac{R(a^2 + b^2 + c^2)}{abc} = \frac{a^2 + b^2 + c^2}{4\Delta}$$

$$\text{Hence, } \frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{\cot A + \cot B + \cot C}{\Delta}$$

3 (c)

We have,

$$\sin 2\theta + 2 = 4 \sin \theta + \cos \theta$$

$$\Rightarrow 2 \sin \theta \cos \theta - \cos \theta + 2 - 4 \sin \theta = 0$$

$$\Rightarrow \cos \theta (2 \sin \theta - 1) - 2(2 \sin \theta - 1) = 0$$

$$\Rightarrow (2 \sin \theta - 1)(\cos \theta - 2) = 0$$

$$\Rightarrow 2 \sin \theta - 1 = 0 \quad [\because \cos \theta - 2 \neq 0]$$

$$\Rightarrow \sin \theta = \frac{1}{2}$$

$$\Rightarrow \theta = 2\pi + \frac{\pi}{6}, 2\pi + \frac{5\pi}{6}, 4\pi + \frac{\pi}{6}, 4\pi + \frac{5\pi}{6}$$

Hence, the equation has 4 solutions

ALITER The curves $y = \sin x$ and $y = \frac{1}{2}$ intersect at 4 points in $[\pi, 5\pi]$. So, the equation has 4 solutions

4 **(c)**

For a triangle inscribed in a circle, we have

$$\frac{a}{2 \sin A} = \frac{b}{2 \sin B} = \frac{c}{2 \sin C} = R$$

$$\therefore \sin^2 A + \sin^2 B + \sin^2 C$$

$$= \frac{a^2}{4R^2} + \frac{b^2}{4R^2} + \frac{c^2}{4R^2} (a^2 + b^2 + c^2)$$

It is given that

$$\frac{a^2 + b^2 + c^2}{2} = 2(2R)^2 \Rightarrow a^2 + b^2 + c^2 = 16R^2$$

$$\therefore \sin^2 A + \sin^2 B + \sin^2 C = \frac{1}{4R^2} (16R^2) = 4$$

5 **(d)**

We have,

$$\begin{aligned} & \tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ \\ &= (\tan 9^\circ + \tan 81^\circ) - (\tan 27^\circ + \tan 63^\circ) \\ &= (\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ) \\ &= \frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\sin 27^\circ \cos 27^\circ} \\ &= \frac{1}{\sin 18^\circ} - \frac{1}{\sin 54^\circ} \\ &= 2 \frac{\sin 54^\circ - \sin 18^\circ}{\sin 54^\circ \sin 18^\circ} = 2 \frac{\cos 36^\circ - \sin 18^\circ}{\sin 18^\circ \cos 36^\circ} = 4 \end{aligned}$$

6 **(c)**

Given, $\sin 4A + \sin 2A = \cos 4A + \cos 2A$

$$\Rightarrow 2 \sin 3A \cos A = 2 \cos 3A \cos A$$

$$\therefore \tan 3A = 1 \text{ and } \cos A = 0$$

$$\Rightarrow A = \frac{\pi}{12} \text{ and } A = \frac{\pi}{2} \notin \left(0, \frac{\pi}{4}\right)$$

$$\therefore \tan 4A = \tan \frac{\pi}{3} = \sqrt{3}$$

7 **(c)**

We have,

$$\sin A + \sin B = \frac{a+b}{c}$$

$$\Rightarrow \sin A + \sin B = \frac{\sin A + \sin B}{\sin C} \Rightarrow \sin C = 1$$

8 **(a)**

We have,

$$\sin(\alpha + \beta) = 1, \sin(\alpha - \beta) = \frac{1}{2}$$

$$\Rightarrow \alpha + \beta = \frac{\pi}{2} \text{ and } \alpha - \beta = \frac{\pi}{6}$$

$$\Rightarrow \alpha = \frac{\pi}{3}, \beta = \frac{\pi}{6}$$

$$\therefore \tan(\alpha + 2\beta) \tan(2\alpha + \beta)$$

$$= \tan\left(\frac{2\pi}{3}\right) \tan\frac{5\pi}{6} = \left(-\cot\frac{\pi}{6}\right) \left(-\cot\frac{\pi}{3}\right) = 1$$

9

(c)

Let $a_0 = \cos \theta$. Then,

$$a_1 = \sqrt{\frac{1}{2}(1 + a_0)} = \sqrt{\frac{1}{2}(1 + \cos \theta)} = \cos \frac{\theta}{2}$$

$$a_2 = \sqrt{\frac{1}{2}(1 + a_1)} = \sqrt{\frac{1}{2}\left(1 + \cos \frac{\theta}{2}\right)} = \cos\left(\frac{\theta}{2^2}\right)$$

and so on

$$\text{Now, } \frac{1 - a_0^2}{a_1 a_2 a_3 \dots \text{to } \infty}$$

$$= \frac{\sin \theta}{\cos \frac{\theta}{2} \cos \frac{\theta}{2^2} \cos \frac{\theta}{2^3} \dots \text{to } \infty}$$

$$= \lim_{n \rightarrow \infty} \frac{\sin \theta}{\cos \frac{\theta}{2} \cos \frac{\theta}{2^2} \cos \frac{\theta}{2^3} \dots \text{to } \infty}$$

$$= \lim_{n \rightarrow \infty} \frac{\{2^n \sin(\theta/2^n)\} \sin \theta}{\sin(2^n \times \theta/2^n)} = \lim_{n \rightarrow \infty} \frac{\sin(\theta/2^n) \cdot \theta}{(\theta/2^n)} = \theta = a_0$$

10

(b)

Let the angles of triangle ABC be $A = \theta, B = 2\theta$ and $C = 7\theta$. Then,

$$A + B + C = 180^\circ \Rightarrow 10\theta = 180^\circ \Rightarrow \theta = 18^\circ$$

$$\therefore A = 18^\circ, B = 36^\circ \text{ and } C = 126^\circ$$

Clearly, c is the greatest side and a is the smallest side.

Now,

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{c}{a} = \frac{\sin C}{\sin A} = \frac{\sin 126^\circ}{\sin 18^\circ} = \frac{\cos 36^\circ}{\sin 18^\circ} = \frac{\sqrt{5} + 1}{\sqrt{5} - 1}$$

11

(b)

We have,

$$A = \frac{2\pi}{3} \text{ and } \Delta = \frac{9\sqrt{3}}{2} \text{ cm}^2$$

$$\therefore \Delta = \frac{1}{2} bc \sin A \Rightarrow \frac{9\sqrt{3}}{2} = \frac{1}{2} bc \sin \frac{2\pi}{3} \Rightarrow bc = 18$$

Also,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \cos \frac{2\pi}{3} = \frac{(b-c)^2 + 2bc - a^2}{2bc}$$

$$\Rightarrow -\frac{1}{2} = \frac{27 + 36 - a^2}{36} \Rightarrow a^2 = 81 \Rightarrow a = 9 \text{ cm}$$

12 (b)

We have,

$$s = 8k \text{ and } \Delta = \sqrt{8k \times 3k \times 2k \times 3k} = 12k^2$$

$$\therefore r = \frac{\Delta}{s} \Rightarrow \frac{12k^2}{8k} = 6 \Rightarrow k = 4$$

13 (d)

We have,

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x} 2^{\cos x}} \quad [\because AM \geq GM]$$

$$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq \sqrt{2^{\sin x + \cos x}}$$

$$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2\sqrt{2^{-\sqrt{2}}} \quad [\because -\sqrt{2} \leq \sin x + \cos x \leq \sqrt{2}]$$

$$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{1-\frac{1}{\sqrt{2}}}$$

14 (a)

$$\text{Let } A = \frac{1}{3 \sin \theta - 4 \cos \theta + 7}$$

Now, A will be minimum when $3 \sin \theta - 4 \cos \theta + 7$ is maximum

$\therefore$ Maximum value of

$$3 \sin \theta - 4 \cos \theta + 7 = \sqrt{3^2 + 4^2} + 7 = 12$$

$$\therefore \text{Minimum value of } \frac{1}{3 \sin \theta - 4 \cos \theta + 7} \text{ is } \frac{1}{12}$$

15 (b)

We have,

$$\operatorname{cosec} \theta = \frac{p+q}{p-q}$$

Now,

$$\cos\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \frac{1 - \tan \frac{\theta}{2}}{1 + \tan \frac{\theta}{2}} = \frac{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}$$

$$\Rightarrow \cos\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \sqrt{\left(\frac{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}\right)^2} = \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}}$$

$$\Rightarrow \cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \sqrt{\frac{\operatorname{cosec} \theta - 1}{\operatorname{cosec} \theta + 1}} = \sqrt{\frac{\frac{p+q}{p-q} - 1}{\frac{p+q}{p-q} + 1}} = \sqrt{\frac{q}{p}}$$

16 (a)

$$\begin{aligned}
\sin t + \cos t &= \frac{1}{5} \\
\Rightarrow \frac{2 \tan \frac{t}{2}}{1 + \tan^2 \frac{t}{2}} + \frac{1 - \tan^2 \frac{t}{2}}{1 + \tan^2 \frac{t}{2}} &= \frac{1}{5} \\
\Rightarrow 10 \tan \frac{t}{2} + 5 - 5 \tan^2 \frac{t}{2} &= 1 + \tan^2 \frac{t}{2} \\
\Rightarrow 6 \tan^2 \frac{t}{2} - 10 \tan \frac{t}{2} - 4 &= 0 \\
\Rightarrow \left(6 \tan \frac{t}{2} + 2\right) \left(\tan \frac{t}{2} - 2\right) &= 0 \\
\Rightarrow \tan \frac{t}{2} = \frac{-1}{3}, 2 \text{ for } 0 < t < \pi \\
\tan \frac{t}{2} &= 2
\end{aligned}$$

17 (a)

We have,

$$\begin{aligned}
\sin \alpha \cos^3 \alpha &> \sin^3 \alpha \cos \alpha \\
\Rightarrow \sin \alpha \cos \alpha (\cos^2 \alpha - \sin^2 \alpha) &> 0 \\
\Rightarrow \cos \alpha (1 - \tan^2 \alpha) &> 0 \quad [\because \sin \alpha > 0 \text{ for } 0 < \alpha < \pi] \\
\Rightarrow \cos \alpha > 0 \text{ and } 1 - \tan^2 \alpha &> 0 \\
\Rightarrow \cos \alpha < 0 \text{ and } 1 - \tan^2 \alpha &< 0 \\
\Rightarrow \alpha \in (0, \pi/4) \text{ or } \alpha \in (3\pi/4, \pi)
\end{aligned}$$

18 (a)

We have, $\alpha + \beta + \gamma = \pi$

$$\begin{aligned}
\text{Now, } \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma & \\
= \sin^2 \alpha + \sin^2 (\beta - \gamma) \sin(\beta + \gamma) & \\
= \sin^2 \alpha + \sin (\pi - \alpha) \sin(\beta + \gamma) & \quad (\because \alpha + \beta - \gamma = \pi) \\
= \sin^2 \alpha + \sin \alpha \sin(\beta + \gamma) & \\
= \sin \alpha [\sin \alpha + \sin(\beta + \gamma)] & \\
= \sin \alpha [\sin(\pi - (\beta - \gamma)) + \sin(\beta + \gamma)] & \\
= \sin \alpha [\sin(\beta - \gamma) + \sin(\beta + \gamma)] & \\
= \sin \alpha [2 \sin \beta \cos \gamma] & \\
= 2 \sin \alpha \sin \beta \cos \gamma &
\end{aligned}$$

19 (c)

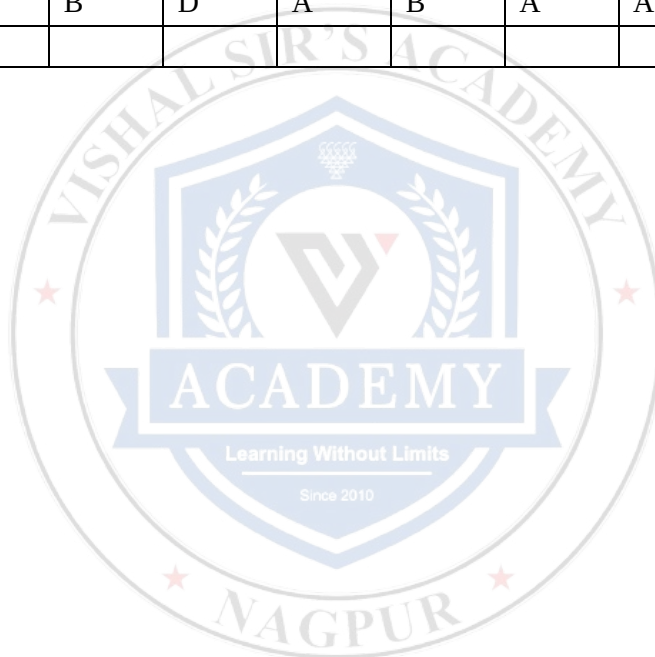
$$\begin{aligned}
\sec x \cos 5x &= -1 \\
\Rightarrow \cos 5x &= -\cos x \\
\Rightarrow 5x &= 2n\pi \pm (\pi - x)
\end{aligned}$$

$$\Rightarrow x = \frac{(2n+1)\pi}{6} \text{ or } \frac{(2n-1)\pi}{6}$$

The possible values of x which lies in the interval $(0, 2\pi)$ are

$$\frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{6}, \frac{7\pi}{4}, \frac{9\pi}{6} \text{ and } \frac{11\pi}{6}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	A	C	C	D	C	C	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	B	D	A	B	A	A	A	C



Topic :-TRIGONOMETRIC FUNCTIONS

1 (a)

$$\cos(\alpha + \beta) = -\frac{12}{13}$$

Here, $0 < (\alpha + \beta) < \pi$

$$\therefore \sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$$

$$= \sqrt{1 - \frac{144}{169}}$$

$$= \frac{5}{13}$$

$$\text{Now, } \sin \beta = \sin[(\alpha + \beta) - \alpha]$$

$$= \sin(\alpha + \beta) \cos \alpha - \cos(\alpha + \beta) \sin \alpha$$

$$= \frac{5}{13} \cdot \frac{3}{5} - \left(-\frac{12}{13}\right) \cdot \frac{4}{5}$$

$$= \frac{15}{65} + \frac{48}{65}$$

$$= \frac{63}{65}$$

2 (c)

We have,

$$\sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14} \sin \frac{7\pi}{14}$$

$$= \sin \left(\frac{\pi}{2} - \frac{3\pi}{7}\right) \sin \left(\frac{\pi}{2} - \frac{2\pi}{7}\right) \sin \left(\frac{\pi}{2} - \frac{\pi}{7}\right) \sin \frac{\pi}{2}$$

$$= \cos \frac{3\pi}{7} \cos \frac{2\pi}{7} \cos \frac{\pi}{7}$$

$$= -\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7}$$

$$= -\frac{\sin(2^3\pi/7)}{2^3 \sin \pi/7} = -\frac{\sin(8\pi/7)}{8 \sin \pi/7} = \frac{1}{8}$$

3 (b)

We have,

$$\sin \theta \cos \alpha + \cos \theta \sin \alpha = 2k \sin \theta \cos \theta$$

$$\Rightarrow \cos \alpha \frac{2t}{1+t^2} + \sin \alpha \frac{1-t^2}{1+t^2} = 2k \frac{2t}{1+t^2} \times \frac{1-t^2}{1+t^2}$$

where $t = \tan \frac{\theta}{2}$

$$\Rightarrow \sin \alpha t^4 - (2 \cos \alpha + 4k)t^3 + t(4k - 2 \cos \alpha) - \sin \alpha = 0$$

$$\Rightarrow S_1 = 2 \cos \alpha + 4k, S_2 = 0$$

$$S_3 = 2 \cos \alpha - 4k, S_4 = -1$$

where S_r denotes the sum of the product of roots taken r at a time

Now,

$$\tan \left(\frac{\theta_1}{2} + \frac{\theta_2}{2} + \frac{\theta_3}{2} + \frac{\theta_4}{2} \right) = \frac{S_1 - S_3}{1 - S_2 + S_4} = \infty = \tan \left(\frac{\pi}{2} \right)$$

$$\Rightarrow \frac{\theta_1}{2} + \frac{\theta_2}{2} + \frac{\theta_3}{2} + \frac{\theta_4}{2} = n\pi + \frac{\pi}{2}, n \in \mathbb{Z}$$

$$\Rightarrow \theta_1 + \theta_2 + \theta_3 + \theta_4 = (2n + 1)\pi, n \in \mathbb{Z}$$

4 **(b)**

Let r be the radius of the circle. Then,

$$\frac{3\pi}{4} = \frac{15\pi}{r} \Rightarrow r = 20 \text{ cm}$$

5 **(a)**

We know that

$$\cot \alpha - \tan \alpha = 2 \cot 2\alpha$$

$$\therefore \cot \theta - \tan \theta - 2 \tan 2\theta - 4 \tan 4\theta - 8 \cot 8\theta$$

$$= 2 \cot 2\theta - 2 \tan 2\theta - 4 \tan 4\theta - 8 \cot 8\theta$$

$$= 2(2 \cot 4\theta) - 4 \tan 4\theta - 8 \cot 8\theta$$

$$= 4 \cot 4\theta - 4 \tan 4\theta - 8 \cot 8\theta$$

$$= 4(\cot 4\theta - \tan 4\theta) - 8 \cot 8\theta$$

$$= 4 \times 2 \cot 8\theta - 8 \cot 8\theta = 0$$

6 **(d)**

We have,

$$b = \sqrt{3}, c = 1 \text{ and } A = 30^\circ$$

$$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow \frac{\sqrt{3}}{2} = \frac{4 - a^2}{2\sqrt{3}} \Rightarrow a = 1$$

$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow 2 = \frac{\sqrt{3}}{\sin B} = \frac{1}{\sin C}$$

$$\Rightarrow \sin B = \frac{\sqrt{3}}{2} \text{ and } \sin C = \frac{1}{2}$$

$$\Rightarrow B = 120^\circ \text{ and } C = 30^\circ \quad [\because b > c \therefore B > C]$$

7 **(c)**

Here, $a = 3, b = 4$

$$\therefore \text{maximum value} = \sqrt{3^2 + 4^2} = 5$$

8 **(a)**

Let ABC be the triangle such that $a = 2, b = \sqrt{6}$ and $c = \sqrt{3} - 1$

Clearly, $b > a > c$

So, B is the greatest angle and C is the smallest angle

Now,

$$\cos B = \frac{c^2 + a^2 - b^2}{2ac}$$

$$\Rightarrow \cos B = \frac{(\sqrt{3}-1)^2 + 4 - 6}{4(\sqrt{3}-1)^2} = -\frac{1}{2} \Rightarrow B = 120^\circ$$

And,

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \cos C = \frac{4 + 6 - (\sqrt{3}-1)^2}{4\sqrt{6}} = \frac{\sqrt{3}+1}{2\sqrt{2}} \Rightarrow C = 15^\circ$$

9

(b)

We have,

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$\Rightarrow \frac{1}{r} = \frac{1}{16} + \frac{1}{48} + \frac{1}{24} \Rightarrow r = 8$$

10

(c)

Given, $3 \sin^2 x - 7 \sin x + 2 = 0$

$$(3 \sin x - 1)(\sin x - 2) = 0$$

$$\Rightarrow \sin x = \frac{1}{3} \text{ or } 2$$

$$\Rightarrow \sin x = \frac{1}{3} \quad [\because \sin x \neq 2]$$

Let $\sin^{-1} \frac{1}{3} = \alpha$, $0 < \alpha < \frac{\pi}{2}$, then $\alpha, \pi - \alpha, 2\pi + \alpha, 3\pi - \alpha, 4\pi + \alpha, 5\pi - \alpha$ are the solution in $[0, 5\pi]$

Hence, required number of solutions are 6

11

(a)

We have, $\cos^2 \theta = \cos 2\theta$

$$\Rightarrow \cos^2 \theta = 2\cos^2 \theta - 1$$

$$\Rightarrow \cos^2 \theta = 1 \Rightarrow \theta = n\pi$$

12

(a)

The given equation is

$$3^{\sin 2x + 2 \cos^2 x} + 3^{1 - \sin 2x + 2 \sin^2 x} = 28$$

$$\Rightarrow 3^{\sin 2x + 2 \cos^2 x} + 3^{3 - (\sin 2x + 2 \cos^2 x)} = 28$$

$$\Rightarrow y + \frac{27}{y} = 28, \text{ where } y = 3^{\sin 2x + 2 \cos^2 x}$$

$$\Rightarrow y^2 - 28y + 27 = 0 \Rightarrow y = 27 \text{ or } y = 1$$

If $y = 27$, then

$$3^{\sin 2x + 2 \cos^2 x} = 3^3$$

$$\Rightarrow \sin 2x + 2 \cos^2 x = 3$$

$$\Rightarrow \sin 2x + 2 \cos 2x = 2$$

$$\Rightarrow \sin 2x = 2 \cos 2x = 1$$

$$\Rightarrow \sin x = 0 \text{ or, } \tan x = \frac{1}{2}$$

$$\text{If } y = 1$$

$$\Rightarrow 3^{\sin 2x + 2 \cos^2 x} = 1$$

$$\Rightarrow \sin 2x + 2 \cos^2 x = 0$$

$$\Rightarrow 2 \cos x (\sin x + \cos x) = 0$$

$$\Rightarrow \cos x = 0 \text{ or, } \tan x = -1$$

13 (c)

$$\text{Let } f(x) = 27^{\cos 2x} 81^{\sin 2x} = 3^{3 \cos 2x + 4 \sin 2x}$$

$$= 3^{5\left(\frac{3}{5} \cos 2x + \frac{4}{5} \sin 2x\right)}$$

$$\text{Let } \frac{3}{5} = \sin \phi$$

$$\Rightarrow \frac{4}{5} = \cos \phi$$

$$\text{Then, } f(x) = 3^{5(\sin \phi \cos 2x + \cos \phi \sin 2x)}$$

$$= 3^{5(\sin(\phi + 2x))}$$

For minimum value of given function,

$\sin(\phi + 2x)$ will be minimum

$$\text{ie, } \sin(\phi + 2x) = -1$$

$$\therefore f(x) = 3^{5(-1)} = \frac{1}{243}$$

14 (c)

We have,

$$\sec 2x - \tan 2x = \frac{1 - \sin 2x}{\cos 2x}$$

$$\Rightarrow \sec 2x - \tan 2x = \frac{1 - \cos\left(\frac{\pi}{2} - 2x\right)}{\sin\left(\frac{\pi}{2} - 2x\right)}$$

$$\Rightarrow \sec 2x - \tan 2x = \frac{2 \sin^2(\pi/4 - x)}{2 \sin(\pi/4 - x) \cos(\pi/4 - x)} = \tan\left(\frac{\pi}{4} - x\right)$$

15 (a)

$$\because \sin^5 x - \cos^5 x = \frac{\sin x - \cos x}{\sin x \cos x}$$

$$\Rightarrow \sin x \cos x \left[\frac{\sin^5 x - \cos^5 x}{\sin x - \cos x} \right] = 1$$

$$\Rightarrow \frac{1}{2} \sin 2x [\sin^4 x + \sin^3 x \cos x + \sin^2 x \cos^2 x + \sin x \cos^3 x + \cos^4 x] = 1$$

$$\Rightarrow \sin 2x [(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x + \sin x \cos x (\sin^2 x + \cos^2 x) + \sin^2 x + \cos^2 x] = 2$$

$$\Rightarrow \sin 2x [1 - \sin^2 x \cos^2 x + \sin x \cos x] = 2$$

$$\Rightarrow \sin^3 2x - 2 \sin^2 2x - 4 \sin 2x + 8 = 0$$

$$\Rightarrow (\sin 2x - 2)^2 (\sin 2x + 2) = 0$$

$$\Rightarrow \sin 2x = \pm 2, \text{ which is not possible for any } x$$

16 (b)

$$\cos(\alpha + \beta) = \frac{4}{5} \Rightarrow \alpha + \beta \in \text{1st quadrant and}$$

$$\sin(\alpha - \beta) = \frac{5}{13}$$

$$\Rightarrow \alpha - \beta \in \text{1st quadrant}$$

$$\Rightarrow 2\alpha = (\alpha + \beta) + (\alpha - \beta)$$

$$\therefore \tan 2\alpha = \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \tan(\alpha - \beta)}$$

$$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{56}{33}$$

17 (d)

We have,

$$\begin{aligned} & \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} \\ &= \frac{1}{2 \sin \frac{\pi}{7}} \left\{ 2 \sin \frac{\pi}{7} \cos \frac{2\pi}{7} + 2 \sin \frac{\pi}{7} \cos \frac{4\pi}{7} + 2 \sin \frac{\pi}{7} \cos \frac{6\pi}{7} \right\} \\ &= \frac{1}{2 \sin \frac{\pi}{7}} \left\{ \sin \frac{3\pi}{7} - \sin \frac{\pi}{7} + \sin \frac{5\pi}{7} - \sin \frac{3\pi}{7} + \sin \pi - \sin \frac{5\pi}{7} \right\} \\ &= -\frac{1}{2} \end{aligned}$$

18 (a)

We have,

$$\begin{aligned} & 2a \cos^2 \left(\frac{C}{2} \right) + 2c \cos^2 \left(\frac{A}{2} \right) = 3b \\ & \Rightarrow a(1 + \cos C) + c(1 + \cos A) = 3b \\ & \Rightarrow a + c + (a \cos C + c \cos A) = 3b \\ & \Rightarrow a + c + b = 3b \Rightarrow a + c = 2b \Rightarrow a, b, c \text{ are in A.P.} \end{aligned}$$

19 (d)

$$\text{Given that, } \frac{1 - \cos 2\theta}{1 + \cos 2\theta} = 3$$

$$\Rightarrow \frac{2 \sin^2 \theta}{2 \cos^2 \theta} = 3$$

$$\Rightarrow \tan^2 \theta = (\sqrt{3})^2$$

$$\Rightarrow \tan^2 \theta = \tan^2 \frac{\pi}{3}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$$

20 (b)

We have,

$$a = 5 \text{ cm}, b = 4 \text{ cm and } \cos(A - B) = \frac{31}{32}$$

$$\therefore \tan \frac{A - B}{2} = \frac{a - b}{a + b} \cot \frac{C}{2}$$

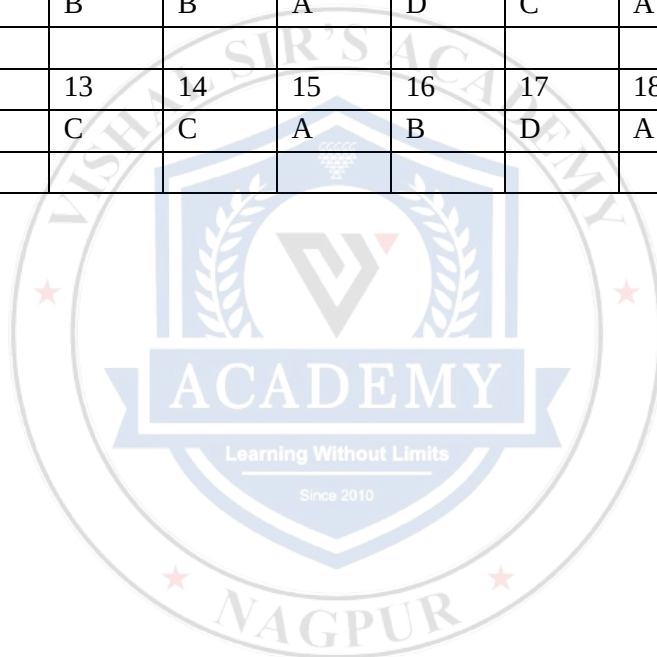
$$\Rightarrow \sqrt{\frac{1 - \cos(A - B)}{1 + \cos(A - B)}} = \frac{a - b}{a + b} \sqrt{\frac{1 + \cos C}{1 - \cos C}}$$

$$\Rightarrow \frac{1 - \frac{31}{32}}{1 + \frac{31}{32}} = \left(\frac{5 - 4}{5 + 4}\right)^2 \left(\frac{1 + \cos C}{1 - \cos C}\right)$$

$$\Rightarrow \frac{81}{63} = \frac{1 + \cos C}{1 - \cos C} \Rightarrow \cos C = \frac{1}{8}$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	B	A	D	C	A	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	C	C	A	B	D	A	D	B



Topic :-TRIGONOMETRIC FUNCTIONS

1 (a)

 Given, $\sin 2x + \cos 4x = 2$

$$\Rightarrow \sin 2x + 1 - 2 \sin^2 2x = 2$$

$$\Rightarrow 2 \sin^2 2x - \sin 2x + 1 = 0$$

 Now, Discriminant, $D = (-1)^2 - 4 \cdot 2 \cdot 1 = -7 < 0$

Hence, it is an imaginary equation, so the real root does not exist.

2 (d)

We have,

$$\sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$$

$$\Rightarrow \sin \theta_1 = \sin \theta_2 = \sin \theta_3 = 1 \quad [\because -1 \leq \sin x \leq 1]$$

$$\Rightarrow \theta_1 = \theta_2 = \theta_3 = \frac{\pi}{2} \Rightarrow \cos \theta_1 + \cos \theta_2 + \cos \theta_3 = 0$$

3 (b)

We have,

$$k \sin x + (1 - 2 \sin^2 x) = 2k - 7$$

$$\Rightarrow 2 \sin^2 x - k \sin x + 2(k - 4) = 0$$

$$\Rightarrow \sin x = \frac{k \pm \sqrt{k^2 - 16k + 64}}{4} = \frac{k \pm (k - 8)}{4} = \frac{1}{2}(k - 4), 2$$

$$\Rightarrow \sin x = \frac{1}{2}(k - 4) \quad [\because \sin x \neq 2]$$

$$\text{Now, } -1 \leq \sin x \leq 1 \Rightarrow -1 \leq \frac{k-4}{2} \leq 1 \Rightarrow 2 \leq k \leq 6$$

4 (b)

$$\text{Given that, } \cos 2B = \frac{\cos(A+C)}{\cos(A-C)}$$

$$= \frac{\cos A \cos C - \sin A \sin C}{\cos A \cos C + \sin A \sin C}$$

$$\Rightarrow \frac{1 - \tan^2 B}{1 + \tan^2 B} = \frac{1 - \tan A \tan C}{1 + \tan A \tan C}$$

$$\Rightarrow 1 + \tan^2 B - \tan A \tan C - \tan A \tan C \tan^2 B$$

$$= 1 - \tan^2 B + \tan A \tan C - \tan A \tan C \tan^2 B$$

$$\Rightarrow 2 \tan^2 B = 2 \tan A \tan C$$

$$\Rightarrow \tan^2 B = \tan A \tan C$$

Hence, $\tan A$, $\tan B$ and $\tan C$ will be in GP

5 **(c)**

We have,

$$\begin{aligned} & \left(\frac{\cos A + \cos B}{\sin A - \sin B} \right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B} \right)^n \\ &= \left(\cot \frac{A-B}{2} \right)^n + \left(-\cot \frac{A-B}{2} \right)^n \\ &= \{1 + (-1)^n\} \cot^n \left(\frac{A-B}{2} \right) \\ &= 0 \times \cot^n \left(\frac{A-B}{2} \right) = 0 \quad [\because n \text{ is odd}] \end{aligned}$$

6 **(a)**

We have,

$$\begin{aligned} a \tan \theta + b \sec \theta &= c \\ \Rightarrow b \sec \theta &= c - a \tan \theta \\ \Rightarrow b^2 \sec^2 \theta &= c^2 + a^2 \tan^2 \theta - 2ac \tan \theta \\ \Rightarrow b^2(1 + \tan^2 \theta) &= c^2 + a^2 \tan^2 \theta - 2ac \tan \theta \\ \Rightarrow \tan^2 \theta(b^2 - a^2) + 2ac \tan \theta + b^2 - c^2 &= 0 \end{aligned}$$

Since $\tan \alpha$ and $\tan \beta$ are roots of this equation

$$\therefore \tan \alpha + \tan \beta = \frac{-2ac}{b^2 - a^2} \text{ and } \tan \alpha \tan \beta = \frac{b^2 - c^2}{a^2 - c^2}$$

Now,

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{\frac{-2ac}{b^2 - a^2}}{1 - \frac{b^2 - c^2}{a^2 - c^2}} = \frac{2ac}{a^2 - c^2}$$

7 **(b)**

$$\begin{aligned} \tan \theta + \frac{\tan \theta + \sqrt{3}}{1 - \sqrt{3} \tan \theta} + \frac{\tan \theta - \sqrt{3}}{1 + \sqrt{3} \tan \theta} &= 3 \\ \Rightarrow \tan \theta + \frac{8 \tan \theta}{1 - 3 \tan^2 \theta} &= 3 \\ \Rightarrow \frac{9 \tan \theta - 3 \tan^3 \theta}{1 - 3 \tan^2 \theta} &= 3 \\ \Rightarrow 3 \tan 3\theta = 3 &\Rightarrow \tan 3\theta = 1 \end{aligned}$$

8 **(a)**

$$\text{Since, } y = 1 + 4 \sin^2 x \cos^2 x$$

$$\Rightarrow y = 1 + \sin^2 2x$$

We know that, $0 \leq \sin^2 2x \leq 1$

$$\Rightarrow 1 \leq 1 + \sin^2 2x \leq 2$$

$$\Rightarrow 1 \leq y \leq 2$$

9

(a)

$$\begin{aligned} & \sin^2 \alpha + \sin^2 \beta - \sin^2 \gamma \\ &= \sin^2 \alpha + \sin(\beta - \gamma) \sin(\beta + \gamma) \\ &= \sin^2 \alpha \sin(\pi - \alpha) \sin(\beta + \gamma) \quad [\because \alpha + \beta - \gamma = \pi] \\ &= \sin \alpha [\sin \alpha + \sin(\beta + \gamma)] \\ &= \sin \alpha [\sin\{\pi - (\beta - \gamma)\} + \sin(\beta + \gamma)] \\ &= \sin \alpha [\sin(\beta - \gamma) + \sin(\beta + \gamma)] \\ &= \sin \alpha [2 \sin \beta \cos \gamma] \\ &= 2 \sin \alpha \sin \beta \cos \gamma \end{aligned}$$

10

(a)

We have,

$$\begin{aligned} & \cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} \\ &= \frac{s(s-a)}{\Delta} + \frac{s(s-b)}{\Delta} + \frac{s(s-c)}{\Delta} = \frac{s}{\Delta} (3s - 2s) = \frac{s^2}{\Delta} \end{aligned}$$

And,

$$\begin{aligned} & \cot A + \cot B + \cot C \\ &= \frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C} \\ &= \frac{b^2 + c^2 - a^2}{2bc \sin A} + \frac{c^2 + a^2 - b^2}{2ac \sin B} + \frac{a^2 + b^2 - c^2}{2ab \sin C} \\ &= \frac{b^2 + c^2 - a^2}{4\Delta} + \frac{c^2 + a^2 - b^2}{4\Delta} + \frac{a^2 + b^2 - c^2}{4\Delta} \\ &= \frac{a^2 + b^2 + c^2}{4\Delta} \end{aligned}$$

$$\begin{aligned} \therefore \frac{\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}}{\cot A + \cot B + \cot C} &= \frac{\frac{s^2}{\Delta}}{\frac{a^2 + b^2 + c^2}{4\Delta}} \\ &= \frac{(2s)^2}{a^2 + b^2 + c^2} = \frac{(a + b + c)^2}{a^2 + b^2 + c^2} \end{aligned}$$

11

(a)

$$\begin{aligned} & \because (\tan \alpha - \cot \alpha)^2 \geq 0 \\ & \Rightarrow \tan^2 \alpha + \cot^2 \alpha - 2 \geq 0 \\ & \Rightarrow \tan^2 \alpha + \cot^2 \alpha \geq 2 \end{aligned}$$

12

(a)

We have,

$$\begin{aligned} & \tan m \theta = \tan n \theta \\ & \Rightarrow m \theta = t \pi + n \theta, \text{ where } t \in \mathbb{Z} \\ & \Rightarrow \theta = \frac{r \pi}{m - n}, r \in \mathbb{Z} \end{aligned}$$

Clearly, these values of θ form an A.P. with common difference $\frac{\pi}{m-n}$

13 (a)

We have,

$$\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$$

$$\Rightarrow \frac{\sin(B+C)}{\sin(A+B)} = \frac{\sin(A-B)}{\sin(B-C)}$$

$$\Rightarrow \sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$$

$$\Rightarrow b^2 - c^2 = a^2 - b^2$$

$$\Rightarrow a^2, b^2, c^2 \text{ are in A.P.}$$

14 (a)

$$\text{Let } \sec \theta - \tan \theta = \lambda \quad \dots(i)$$

Then,

$$(\sec \theta + \tan \theta) = \frac{1}{\sec \theta - \tan \theta}$$

$$\Rightarrow \sec \theta + \tan \theta = \frac{1}{\lambda} \quad \dots(ii)$$

$$\therefore 2 \tan \theta = \frac{1}{\lambda} + \lambda \quad [\text{On subtracting (i) from (ii)}]$$

$$\Rightarrow 2x - \frac{1}{2x} = \frac{1}{\lambda} - \lambda$$

$$\Rightarrow \lambda = \frac{1}{2x}, -2x \Rightarrow \sec \theta - \tan \theta = \frac{1}{2x}, -2x$$

15 (d)

We observe that the LHS of the given equation is not defined for $x = n\pi, n \in \mathbb{Z}$

Now,

$$\cot x - \operatorname{cosec} x = 2 \sin x$$

$$\Rightarrow \cot x - 1 = 2 \sin^2 x$$

$$\Rightarrow 2 \cos^2 x + \cos x - 3 = 0$$

$$\Rightarrow (2 \cos x + 3)(\cos x - 1) = 0$$

$$\Rightarrow \cos x = 1 \quad [\because 2 \cos x + 3 \neq 0]$$

$$\Rightarrow x = 0, 2\pi$$

But, $x \neq n\pi, n \in \mathbb{Z}$

Hence, the given equation has no solution

16 (d)

$$\text{Given, } \frac{\sin x}{\sin y} = \frac{1}{2}, \frac{\cos x}{\cos y} = \frac{3}{2}$$

$$\Rightarrow \frac{\tan x}{\tan y} = \frac{1}{3}$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{4 \tan x}{1 - 3 \tan^2 x}$$

$$\text{Also, } \sin y = 2 \sin x, \cos y = \frac{2}{3} \cos x$$

$$\Rightarrow \sin^2 y + \cos^2 y = 4 \sin^2 x + \frac{4 \cos^2 x}{9} = 1$$

$$\Rightarrow 36 \tan^2 x + 4 = 9 \sec^2 x = 9(1 + \tan^2 x)$$

$$\Rightarrow 27 \tan^2 x = 5$$

$$\Rightarrow \tan x = \frac{\sqrt{5}}{3\sqrt{3}}$$

$$\Rightarrow \tan(x + y) = \frac{\frac{4\sqrt{5}}{3\sqrt{3}}}{1 - \frac{15}{27}} = \sqrt{15}$$

17 (d)

$$\begin{aligned} \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} &= \frac{1 + \tan 9^\circ}{1 - \tan 9^\circ} \\ &= \tan(45^\circ + 9^\circ) \\ &= \tan 54^\circ \end{aligned}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	B	B	C	A	B	A	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	A	A	D	C	D	A	A	D

Class : XIth
Subject : MATHS
Date :
DPP No. :8

Solutions

Topic :- TRIGONOMETRIC FUNCTIONS

1 (a)

Let ABC be the triangle such that $a = 2\sqrt{2}$ cm, $b = 2\sqrt{3}$ cm and $\angle A = \frac{\pi}{4}$

$$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{12 + c^2 - 8}{4\sqrt{3}c}$$

$$\Rightarrow 4 + c^2 = 2\sqrt{6}c$$

$$\Rightarrow c^2 - 2\sqrt{6}c + 4 = 0$$

$$\Rightarrow c = \frac{2\sqrt{6} \pm \sqrt{24 - 16}}{2}$$

$$\Rightarrow c = \sqrt{6} \pm \sqrt{2} \Rightarrow c = \sqrt{6} + \sqrt{2} \quad [\because c \text{ is the largest side}]$$

2 (b)

We have,

$$r \sin \theta = 3 \text{ and } r = 4(1 + \sin \theta)$$

$$\Rightarrow r = 4 + \frac{12}{r}$$

$$\Rightarrow r^2 - 4r - 12 = 0$$

$$\Rightarrow (r - 6)(r + 2) = 0$$

$$\Rightarrow r = 6 \quad [\because r > 0]$$

$$\therefore r \sin \theta = 3 \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Hence, the total number of ordered pairs of the form (r, θ) is $1 \times 2 = 2$

3 (d)

We have,

$$\begin{aligned} & \sin 65^\circ + \sin 43^\circ - \sin 29^\circ - \sin 7^\circ \\ &= (\sin 65^\circ + \sin 43^\circ) - (\sin 29^\circ + \sin 7^\circ) \\ &= 2 \sin 54^\circ \cos 11^\circ - 2 \sin 18^\circ \cos 11^\circ \\ &= 2 \cos 11^\circ (\cos 36^\circ - \sin 18^\circ) \\ &= 2 \cos 11^\circ \left(\frac{\sqrt{5} + 1}{4} - \frac{\sqrt{5} - 1}{4} \right) = \cos 11^\circ \end{aligned}$$

4 (c)

We have,

$$\sin B = \frac{1}{5} \sin(2A + B)$$

$$\begin{aligned} & \Rightarrow \frac{\sin(2A + B)}{\sin B} = \frac{5}{1} \\ & \Rightarrow \frac{\sin(2A + B) + \sin B}{\sin(2A + B) - \sin B} = \frac{5 + 1}{5 - 1} \\ & \Rightarrow \frac{2 \sin(A + B) \cos A}{2 \sin A \cos(A + B)} = \frac{3}{2} \\ & \Rightarrow \frac{\tan(A + B)}{\tan A} = \frac{3}{2} \end{aligned}$$

5 (b)

Since, $A + B + C = \pi$

$$\Rightarrow a = \pi - (B + C)$$

We have, $\cos A = \cos B \cos C$

$$\Rightarrow \cos[\pi - (B + C)] = \cos B \cos C$$

$$\Rightarrow -\cos(B + C) = \cos B \cos C$$

$$\Rightarrow -[\cos B \cos C - \sin B \sin C] = \cos B \cos C$$

$$\Rightarrow \sin B \sin C = 2 \cos B \cos C$$

$$\Rightarrow \tan B \tan C = 2$$

6 (a)

We have,

$$\sin x + \operatorname{cosec} x = 2 \Rightarrow (\sin x - 1)^2 = 0 \Rightarrow \sin x = 1$$

$$\therefore \sin^n x + \operatorname{cosec}^n x = 1 + 1 = 2$$

7 (a)

We have,

$$\begin{aligned} & \frac{a^2 - b^2}{a^2 + b^2} = \frac{\sin(A - B)}{\sin(A + B)} \\ & \Rightarrow \frac{\sin^2 A - \sin^2 B}{\sin^2 A + \sin^2 B} = \frac{\sin^2 A - \sin^2 B}{\sin^2(A + B)} \\ & \Rightarrow (\sin^2 A - \sin^2 B)(\sin^2 A + \sin^2 B - \sin^2 C) = 0 \\ & \Rightarrow \sin(A + B) \sin(A - B)(\sin^2 A + \sin^2 B - \sin^2 C) = 0 \end{aligned}$$

$$\Rightarrow \sin(A - B) = 0 \text{ or, } \sin^2 A + \sin^2 B = \sin^2 C$$

$$\Rightarrow A = B \text{ or, } a^2 + b^2 = c^2$$

$\Rightarrow \triangle ABC$ is either right angled or isosceles

8 (c)

We have,

$$\cos A = \cos B \cos C$$

$$\Rightarrow \cos\{\pi - (B + C)\} = \cos B \cos C$$

$$\Rightarrow -\cos(B + C) = \cos B \cos C$$

$$\Rightarrow 2 \cos B \cos C = \sin B \sin C$$

$$\Rightarrow \cot B \cot C = \frac{1}{2}$$

9 (c)

We have,

(a)

We have,

$$\sin 7\theta + 6 \sin 5\theta + 17 \sin 3\theta + 12 \sin \theta$$

$$\begin{aligned} &= \frac{\sin 6\theta + 5 \sin 4\theta + 12 \sin 2\theta}{(\sin 7\theta + \sin 5\theta) + 5(\sin 5\theta + \sin 3\theta) + 12(\sin 3\theta + \sin \theta)} \\ &= \frac{\sin 6\theta + 5 \sin 4\theta + 12 \sin 2\theta}{2 \sin 6\theta \cos \theta + 10 \sin 4\theta \cos \theta + 24 \sin 2\theta \cos \theta} \\ &= \frac{\sin 6\theta + 5 \sin 4\theta + 12 \sin 2\theta}{2 \cos \theta (\sin 6\theta + 5 \sin 4\theta + 12 \sin 2\theta)} \\ &= \frac{\sin 6\theta + 5 \sin 4\theta + 12 \sin 2\theta}{2 \cos \theta} \\ &= 2 \cos \theta \end{aligned}$$

$$2s = a + b + c = 13 + 14 + 15$$

$$\Rightarrow s = 21$$

$$\Rightarrow s - a = 8, s - b = 7 \text{ and } s - c = 6$$

Now,

$$\frac{1}{r_1} : \frac{1}{r_2} : \frac{1}{r_3} = \frac{s-a}{\Delta} : \frac{s-b}{\Delta} : \frac{s-c}{\Delta}$$

$$\Rightarrow \frac{1}{r_1} : \frac{1}{r_2} : \frac{1}{r_3} = s-a : s-b : s-c = 8 : 7 : 6$$

10



11(a)

Let the angles be $A = x - d, B = x, C = x + d$. Then,

$$x - d + x + x + d = 180^\circ \Rightarrow x = 60^\circ$$

Therefore, two larger angles are $B = 60^\circ$ and C

Hence, $b = 9$ and $c = 10$

Now,

$$\begin{aligned}\cos B &= \frac{c^2 + a^2 - b^2}{2ac} \\ \Rightarrow \frac{1}{2} &= \frac{100 + a^2 - 81}{20a} \Rightarrow a^2 - 10a + 19 = 0 \Rightarrow a = 5 \pm \sqrt{6}\end{aligned}$$

853 (b)

Since, $\cos 2x, \frac{1}{2}, \sin 2x$ are in AP

$$\begin{aligned}\Rightarrow \cos 2x + \sin 2x &= 1 \\ \Rightarrow \sin 2x &= 1 - \cos 2x = 2 \sin^2 x \\ \Rightarrow 2 \sin x (\cos x - \sin x) &= 0 \\ \Rightarrow \sin x &= 0 \text{ or } \cos x - \sin x = 0 \\ \Rightarrow x &= n\pi \text{ or } \tan x = 1 \\ \Rightarrow x &= n\pi \text{ or } x = n\pi + \frac{\pi}{4}\end{aligned}$$

Thus, required values of x are $n\pi$ and $n\pi + \frac{\pi}{4}$

12 (b)

$$\begin{aligned}\cos \frac{\pi}{18} + \cos \frac{2\pi}{18} + \dots + \cos \frac{16\pi}{18} + \cos \frac{17\pi}{18} + \cos \pi \\ = \cos \frac{\pi}{18} + \cos \frac{2\pi}{18} + \dots - \cos \frac{2\pi}{18} - \cos \frac{\pi}{18} + \cos \pi \\ = \cos \pi = -1\end{aligned}$$

13 (b)

Given that, $\sin \theta + \cos \theta = 1$

$$\begin{aligned}\Rightarrow \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta &= \frac{1}{\sqrt{2}} \\ \Rightarrow \sin \left(\theta + \frac{\pi}{4} \right) &= \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4} \\ \Rightarrow \theta + \frac{\pi}{4} &= n\pi + (-1)^n \frac{\pi}{4} \\ \Rightarrow \theta &= n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}\end{aligned}$$

14 (d)

We have,

$$0 < x < \pi \Rightarrow \sin x > 0$$

Now,

$$\begin{aligned}1 + \sin x + \sin^2 x + \dots \infty &= 4 + 2\sqrt{3} \\ \Rightarrow \frac{1}{1 - \sin x} &= 4 + 2\sqrt{3}\end{aligned}$$

$$\Rightarrow \sin x = 1 - \frac{1}{4 + 2\sqrt{3}}$$

$$\Rightarrow \sin x = \frac{3 + 2\sqrt{3}}{4 + 2\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

15 (b)

$$\frac{\sin x + \sin y}{\cos x + \cos y} = \frac{a}{b}$$

$$\Rightarrow \frac{2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)}{2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)} = \frac{a}{b}$$

$$\Rightarrow \tan\left(\frac{x+y}{2}\right) = \frac{a}{b}$$

16 (b)

The given equation can be written as

$$\cos(\pi \tan \theta) = \cos\left(\frac{\pi}{2} - \pi \cot \theta\right)$$

$$\Rightarrow \pi \tan \theta = 2n\pi \pm \left(\frac{\pi}{2} - \pi \cot \theta\right), n \in \mathbb{Z}$$

$$\Rightarrow \tan \theta = 2n \pm \left(\frac{1}{2} - \cot \theta\right), n \in \mathbb{Z}$$

$$\Rightarrow \tan \theta - \cot \theta = 2n - \frac{1}{2}, n \in \mathbb{Z} \text{ [Taking negative sign]}$$

$$\Rightarrow \frac{\tan^2 \theta - 1}{\tan \theta} = 2n - \frac{1}{2}$$

$$\Rightarrow \frac{\tan^2 \theta - 1}{2 \tan \theta} = n - \frac{1}{4}$$

$$\Rightarrow \frac{1 - \tan^2 \theta}{2 \tan \theta} = -n + \frac{1}{4}$$

$$\Rightarrow \cot 2\theta = m + \frac{1}{4}, \text{ where } m = -n \in \mathbb{Z}$$

17 (c)

From Questions 47, we have

$$\Delta = \frac{1}{2}ap_1, \Delta = \frac{1}{2}bp_2, \Delta = \frac{1}{2}cp_3$$

Now,

p_1, p_2, p_3 are in A.P.

$$\Rightarrow \frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c} \text{ are in A.P.}$$

$$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in A.P.} \Rightarrow a, b, c \text{ are in H.P.}$$

18 (d)

$$\cos \frac{\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5} = \frac{\sin 2^4 \frac{\pi}{5}}{2^4 \sin \frac{\pi}{5}} \left(\because \cos x \cos 2x \cos 4x \dots \cos 2^nx = \frac{\sin 2^nx}{2^n \sin x} \right)$$

$$= \frac{\sin \frac{16\pi}{5}}{16 \sin \frac{\pi}{5}} = \frac{\sin \left(3\pi + \frac{\pi}{5} \right)}{16 \sin \frac{\pi}{5}}$$

$$= \frac{-\sin \frac{\pi}{5}}{16 \sin \frac{\pi}{5}} = -\frac{1}{16}$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	B	D	C	B	A	A	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	B	B	B	D	B	B	C	D



Topic :-TRIGONOMETRIC FUNCTIONS

1 (c)

$$\operatorname{cosec} 15^\circ + \sec 15^\circ = \frac{2(\sin 15^\circ + \cos 15^\circ)}{2 \sin 15^\circ \cos 15^\circ}$$

$$= 2 \left[\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \right] / \sin 30^\circ$$

$$= \frac{4\sqrt{3}}{\sqrt{2}} = 2\sqrt{6}$$

2 (d)

We have, $\sin A = \frac{4}{5}$ and $\cos B = -\frac{12}{13}$

Now, $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$$= \sqrt{1 - \frac{16}{25}} \left(-\frac{12}{13} \right) - \frac{4}{5} \sqrt{1 - \frac{144}{169}}$$

$$= -\frac{3}{5} \times \frac{12}{13} - \frac{4}{5} \left(-\frac{5}{13} \right)$$

$$= -\frac{36}{65} + \frac{20}{65} = -\frac{16}{65}$$

3 (a)

Given that, $\frac{1}{\tan \theta} + \tan \theta = m$

$$\Rightarrow 1 + \tan^2 \theta = m \tan \theta$$

$$\Rightarrow \sec^2 \theta = m \tan \theta \quad \dots(i)$$

and $\sec \theta - \cos \theta = n$

$$\Rightarrow \sec^2 \theta - 1 = n \sec \theta$$

$$\Rightarrow \tan^2 \theta = n \sec \theta$$

$$\Rightarrow \tan^4 \theta = n^2 \sec^2 \theta = n^2 \cdot m \tan \theta \quad [\text{from Eq.(i)}]$$

$$\Rightarrow \tan^3 \theta = n^2 m \quad (\because \tan \theta \neq 0)$$

$$\Rightarrow \tan \theta = (n^2 m)^{1/3} \quad \dots(ii)$$

From Eq. (i), we get

$$\sec^2 \theta = m (n^2 m)^{1/3}$$

As we know that, $\sec^2 \theta - \tan^2 \theta = 1$

$$\Rightarrow m(mn^2)^{1/3} - (n^2 m)^{2/3} = 1$$

$$\Rightarrow m(mn^2)^{1/3} - n(nm^2)^{1/3} = 1$$

4 (c)

We have,

$$(\sin A + \sin B + \sin C)(\sin A + \sin B - \sin C) = 3 \sin A \sin B$$

$$\Rightarrow (\sin A + \sin B)^2 - \sin^2 C = 3 \sin A \sin B$$

$$\Rightarrow \sin^2 A + \sin^2 B - \sin^2 C = \sin A \sin B$$

$$\Rightarrow \sin^2 A + \sin(B + C) \sin(B - C) = \sin A \sin B$$

$$\Rightarrow \sin^2 A + \sin A \sin(B - C) = \sin A \sin B$$

$$\Rightarrow \sin A [\sin(B + C) + \sin(B - C)] = \sin A \sin B$$

$$\Rightarrow 2 \sin A \sin B \cos C = \sin A \sin B$$

$$\Rightarrow \cos C = 1/2 \quad [\because \sin A \sin B \neq 0]$$

$$\Rightarrow C = 60^\circ$$

5 (b)

$$\text{Given, } \cos 2x + 7 = a(2 - \sin x)$$

$$\Rightarrow 1 - 2 \sin^2 x + 7 = 2a - a \sin x$$

$$\Rightarrow 2 \sin^2 x - a \sin x + (2a - 8) = 0$$

$$\therefore \sin x = \frac{a \pm \sqrt{(-a)^2 - 8(2a - 8)}}{2 \times 2}$$

$$= \frac{a \pm (a - 8)}{4}$$

For (+) sign,

$$\sin x = \frac{a - 4}{2}$$

For (-) sign,

$$\sin x = 2 \text{ which is not possible}$$

We know $-1 \leq \sin x \leq 1$

$$\therefore -1 \leq \frac{a - 4}{2} \leq 1 \Rightarrow 2 \leq a \leq 6$$

6 (c)

$$\cos^2 B + \cos^2 C = \cos^2 B + \cos^2 \left(\frac{\pi}{2} - B \right)$$

$$= \cos^2 B + \sin^2 B = 1$$

7 (d)

We have,

$$b^2 \sin 2C + c^2 \sin 2B$$

$$= b^2 \cdot (2 \sin C \cos C) + c^2 \cdot (2 \sin B \cos B)$$

$$\begin{aligned}
&= 2(b \sin C)(b \cos C) + 2(c \sin B)(c \cos B) \\
&= 2(c \sin B)(b \cos c) + 2(c \sin B)(c \cos B) \\
&\left[\because \frac{b}{\sin B} = \frac{c}{\sin C} \right] \\
&= 2c \sin B(b \cos C + c \cos B) = 2ac \sin B = 4\Delta
\end{aligned}$$

8 (a)

Since the angles of $\triangle ABC$ are in A.P.

$$\therefore 2B = A + C \Rightarrow 3B = A + B + C \Rightarrow 3B = 180^\circ \Rightarrow B = 60^\circ$$

$$\text{Now, } \frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\Rightarrow \sin A = \frac{a}{b} \sin B = \frac{24}{22} \sin 60^\circ = \frac{6\sqrt{3}}{11}$$

$$\Rightarrow \cos A = \frac{\sqrt{13}}{11}$$

We have,

$$\sin C = \sin\{180^\circ - (A + B)\}$$

$$\Rightarrow \sin C = \sin(A + B)$$

$$\Rightarrow \sin C = \sin A \cos B + \cos A \sin B$$

$$\Rightarrow \sin C = \left(\frac{6\sqrt{3}}{11}\right)\left(\frac{1}{2}\right) + \frac{\sqrt{13}}{11}\left(\frac{\sqrt{3}}{2}\right) = \frac{6\sqrt{3} + \sqrt{39}}{22}$$

$$\therefore c = \frac{b \sin C}{\sin B} \Rightarrow c = 12 + 2\sqrt{13}$$

9 (a)

$$\text{We have, } \angle BFC = \frac{\pi}{2} = \angle BEC$$

So, the circle with BC as diameter will pass through E and F . Clearly, the circle with BC as diameter is the circumcircle of $\triangle BEF$ such that $\angle FBE = 90^\circ - A$

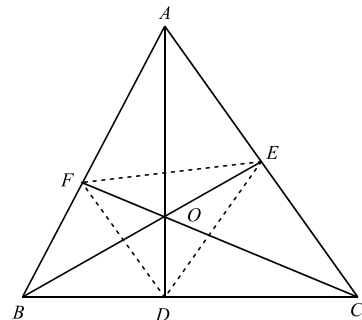
$$\therefore FE = 2\left(\frac{a}{2}\right) \sin \angle FBE \quad [\text{Using : } a = 2R \sin A]$$

$$\Rightarrow FE = a \cos A$$

Let R_1 be the radius of the circumcircle of $\triangle DEF$. Then,

$$R_1 = \frac{FE}{2 \sin \angle FDE} = \frac{a \cos A}{2 \sin(180^\circ - 2A)}$$

$$\Rightarrow R_1 = \frac{a \cos A}{4 \sin A \cos A} = \frac{a}{4 \sin A} = \frac{R}{2}$$



10 (b)

Clearly, the equation $x^2 + \sqrt{2}x + 1 = 0$ has imaginary roots. So, the two equations have both common roots

$$\begin{aligned}\therefore \frac{a}{1} &= \frac{b}{\sqrt{2}} = \frac{c}{1} \\ \Rightarrow \frac{\sin A}{1} &= \frac{\sin B}{\sqrt{2}} = \frac{\sin C}{1} \\ \Rightarrow \frac{\sin A}{1/\sqrt{2}} &= \frac{\sin B}{1} = \frac{\sin C}{1/\sqrt{2}} \\ \Rightarrow A &= \frac{\pi}{4}, B = \frac{\pi}{2}, C = \frac{\pi}{4}\end{aligned}$$

11 (a)

We have,

$$s = \frac{3a}{2} \text{ and } \Delta = \frac{\sqrt{3}}{4}a^2 \therefore r = \frac{\Delta}{s} = \frac{a}{2\sqrt{3}}$$

Let the length of each side of the square inscribed in the incircle be x . Then,

$$\begin{aligned}x^2 + x^2 &= (\text{Diameter})^2 \\ \Rightarrow 2x^2 &= \frac{a^2}{3} \Rightarrow x^2 = \frac{a^2}{6} \Rightarrow \text{Area of the square} = \frac{a^2}{6}\end{aligned}$$

12 (a)

$$\begin{aligned}\cos 1^\circ \cdot \cos 2^\circ \dots \cos 179^\circ \\ = \cos 1^\circ \cdot \cos 2^\circ \dots \cos 90^\circ \cdot \cos 179^\circ \\ = 0 \quad [\because \cos 90^\circ = 0]\end{aligned}$$

13 (a)

Given equation is

$$2^{\cos 2x} + 1 = 3 \cdot 2^{-\sin x}$$

By taking option (a)

Let $x = n\pi$

When, $n = 1, x = \pi$

$$\begin{aligned}\therefore 2^{\cos 2\pi} + 1 &= 3 \cdot 2^{-\sin \pi} \\ \Rightarrow 2 + 1 &= 3 \cdot 2^0 \Rightarrow 3 = 3\end{aligned}$$

When $n = 2, x = 2\pi$

$$\begin{aligned}\therefore 2^{\cos 4\pi} + 1 &= 3 \cdot 2^{-\sin 2\pi} \\ \Rightarrow 2^1 + 1 &= 3 \cdot 2^0 \\ \Rightarrow 3 &= 3\end{aligned}$$

14 (b)

On squaring given equation, we get

$$\begin{aligned}\sin^2 A + 6 \cos^2 A - 2\sqrt{6} \sin A \cos A &= 7 \cos^2 A \\ \Rightarrow \sin^2 A + 6(1 - \sin^2 A) &= \cos^2 A + 6 \cos^2 A + 2\sqrt{6} \sin A \cos A \\ \Rightarrow \sin^2 A - 6 \cos^2 A + 6 &= \cos^2 A + 6 \sin^2 A + 2\sqrt{6} \sin A \cos A \\ \Rightarrow 7 \sin^2 A &= (\cos A + \sqrt{6} \sin A)^2 \\ \Rightarrow \pm \sqrt{7} \sin A &= \cos A + \sqrt{6} \sin A\end{aligned}$$

Alternate

Given, $\sin A - \sqrt{6} \cos A = \sqrt{7} \cos A$

Replacing A by $90^\circ + A$, we get

$$\sin(90^\circ + A) - \sqrt{6} \cos(90^\circ + A)$$

$$= \sqrt{7} \cos(90^\circ + A)$$

$$\Rightarrow \cos A + \sqrt{6} \sin A = -\sqrt{7} \sin A$$

15 **(b)**

We have,

$$y = \frac{\tan x}{\tan 3x}$$

$$\Rightarrow y = \frac{1 - 3 \tan^2 x}{3 - \tan^2 x}$$

$$\Rightarrow 3y - y \tan^2 x = 1 - 3 \tan^2 x$$

$$\Rightarrow \tan^2 x (y - 3) = 1 - 3y$$

$$\Rightarrow \tan^2 x = \frac{y - 3}{1 - 3y}$$

$$\Rightarrow -\frac{y - 3}{3y - 1} \geq 0 \quad [\because \tan^2 x \geq 0]$$

$$\Rightarrow \frac{y - 3}{3y - 1} \leq 0 \Rightarrow \frac{1}{3} \leq y < 3 \Rightarrow y \in [1/3, 3]$$

16 **(c)**

We have, $\sqrt{\operatorname{cosec}^2 \alpha + 2 \cot \alpha}$

$$= \sqrt{1 + \cot^2 \alpha + 2 \cot \alpha} = |1 + \cot \alpha|$$

$$\text{But } \frac{3\pi}{4} < \alpha < \pi$$

$$\Rightarrow \cot \alpha < -1 \Rightarrow 1 + \cot \alpha < 0$$

$$\text{Hence, } |1 + \cot \alpha| = -(1 + \cot \alpha)$$

17 **(c)**

Since $-\sqrt{a^2 + b^2} \leq a \sin x + b \cos x \leq \sqrt{a^2 + b^2}$. Therefore, $a \sin x + b \cos x = c$ has no solution for $|c| > \sqrt{a^2 + b^2}$

18 **(c)**

We have,

$$\tan \theta + \sec \theta = 2 \cos \theta$$

$$\Rightarrow 1 + \sin \theta = 2 \cos^2 \theta$$

$$\Rightarrow 1 + \sin \theta = 2 - 2 \sin^2 \theta$$

$$\Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0$$

$$\Rightarrow (2 \sin \theta - 1)(\sin \theta + 1) = 0$$

$$\Rightarrow \sin \theta = \frac{1}{2}, \sin \theta = -1$$

$$\Rightarrow \sin \theta = \frac{1}{2} \left[\begin{array}{l} \because \sin \theta = -1 \Rightarrow \theta = \frac{3\pi}{2} \\ \text{for which the equation is not defined} \end{array} \right]$$

$$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad [\because \theta \in [0, 2\pi]]$$

Hence, the given equation has two solutions in $[0, 2\pi]$

19 **(c)**

Given, $\sin \pi(x^2 + x) - \sin \pi x^2 = 0$

$$\Rightarrow 2 \cos \pi \left(\frac{2x^2 + x}{2} \right) \sin \frac{\pi x}{2} = 0$$

$$\Rightarrow \pi \left(\frac{2x^2 + x}{2} \right) = n\pi + \frac{\pi}{2}$$

$$\Rightarrow 2x^2 + x = 2n + 1$$

$$\Rightarrow 2x^2 + x - p' = 0, \text{ where } p' = 2n + 1, \text{ is an odd integer}$$

$$\therefore x = \frac{-1 \pm \sqrt{1+8p'}}{4} \quad [\text{put } 1 + 8p' = p]$$

$$\therefore x = \frac{-1 \pm \sqrt{p}}{4} \Rightarrow x = \frac{\sqrt{p} - 1}{4} \quad \left[\text{neglect } x = \frac{-1 - \sqrt{p}}{4} \right]$$

20 **(b)**

Given that, $3 \sin^2 A + 2 \sin^2 B = 1$... (i)

and $3 \sin 2A - 2 \sin 2B = 0$... (ii)

From Eq. (i)

$$3 \left(\frac{1 - \cos 2A}{2} \right) + 2 \left(\frac{1 - \cos 2B}{2} \right) = 1$$

$$\Rightarrow 3 \cos 2A + 2 \cos 2B = 3 \quad \dots \text{(iii)}$$

$$\Rightarrow 3 \cos 2A = 3 - 2 \cos 2B$$

$$\Rightarrow 9 \cos^2 2A = 9 + 4 \cos^2 2B - 12 \cos 2B$$

$$\Rightarrow 9(1 - \sin^2 2A) = 9 + 4 \cos^2 2B - 12 \cos 2B$$

$$\Rightarrow 9 - 4 \sin^2 2B = 9 + 4 \cos^2 2B - 12 \cos 2B \quad [\text{from Eq. (ii)}]$$

$$\Rightarrow -4(1 - \cos^2 2B) = 4 \cos^2 2B - 12 \cos 2B$$

$$\Rightarrow -4 = -12 \cos 2B$$

$$\Rightarrow \cos 2B = \frac{1}{3}$$

Now, from Eq. (iii)

$$\cos 2A = \frac{7}{9} \Rightarrow 2 \cos^2 A - 1 = \frac{7}{9}$$

$$\Rightarrow \cos A = \frac{2\sqrt{2}}{3}$$

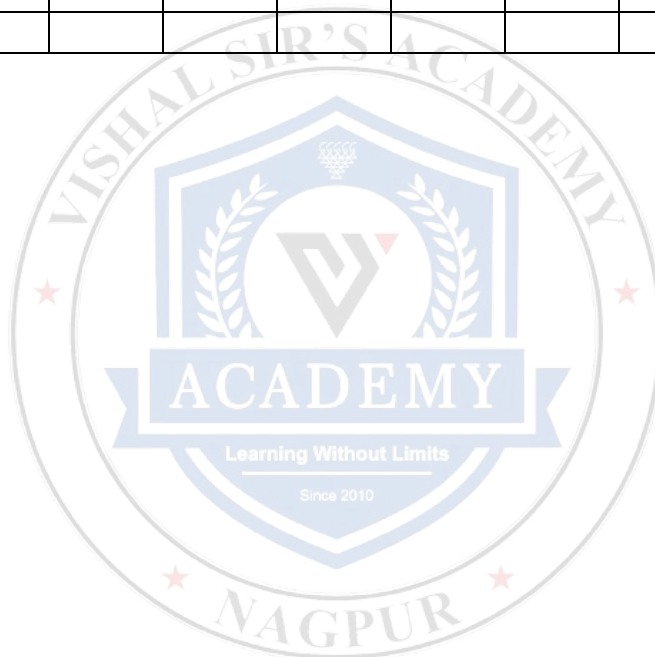
$$\therefore A + 2B = \cos^{-1} \frac{2\sqrt{2}}{3} + \cos^{-1} \frac{1}{3}$$

$$= \cos^{-1} \left(\frac{2\sqrt{2}}{3} \cdot \frac{1}{3} - \sqrt{1 - \frac{8}{9}} \sqrt{1 - \frac{1}{9}} \right)$$

$$= \cos^{-1} \left(\frac{2\sqrt{2}}{9} - \frac{2\sqrt{2}}{9} \right)$$

$$= \cos^{-1}(0) = \frac{\pi}{2}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	D	A	C	B	C	D	A	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	A	B	B	C	C	C	C	B



Topic :-TRIGONOMETRIC FUNCTIONS

1 (b)

$$\text{Let } f(x) = \sin x \cos x = \frac{1}{2} \sin 2x$$

We know that, $-1 \leq \sin 2x \leq 1$

$$\Rightarrow -\frac{1}{2} \leq \frac{1}{2} \sin 2x \leq \frac{1}{2}$$

Thus, the greatest and least value of $f(x)$ are $\frac{1}{2}$ and $\frac{1}{2}$ respectively

2 (b)

We have,

$$\begin{aligned} x^2 + 4xy + y^2 &= (X \cos \theta - Y \sin \theta)^2 + 4(X \cos \theta - Y \sin \theta)(X \sin \theta + Y \cos \theta) + (X \sin \theta + Y \cos \theta)^2 \\ &= (1 + 4 \sin \theta \cos \theta)X^2 + 4(\cos^2 \theta - \sin^2 \theta)XY + (1 - 4 \sin \theta \cos \theta)Y^2 \\ \therefore x^2 + 4xy + y^2 &= AX^2 + BY^2 \\ \Rightarrow (1 + 2 \sin 2\theta)X^2 + 4 \cos 2\theta XY + (1 - 2 \sin 2\theta)Y^2 &= AX^2 + BY^2 \\ \Rightarrow \cos 2\theta &= 0, A = 1 + 2 \sin 2\theta, B = 1 - 2 \sin 2\theta \\ \Rightarrow \theta &= \frac{\pi}{4} \text{ and } A = 1 + 2 = 3, B = 1 - 2 = -1 \end{aligned}$$

3 (d)

$$\text{Given, } 3 \cos 2x - 10 \cos x + 7 = 0$$

$$\Rightarrow 6 \cos^2 x - 10 \cos x + 4 = 0$$

$$[\because \cos 2x = 2 \cos^2 x - 1]$$

$$\Rightarrow 2(3 \cos x - 2)(\cos x - 1) = 0$$

$$\Rightarrow \cos x = 1 \text{ or } \cos x = \frac{2}{3}$$

Since, $\cos x$ is positive in Ist and IIIrd quadrant.

Hence, total number of solutions are 4

4 (a)

$$\begin{aligned} \cos \alpha \sin(\beta - \gamma) + \cos \beta \sin(\gamma - \alpha) + \cos \gamma \sin(\alpha - \beta) \\ = \cos \alpha [\sin \beta \cos \gamma - \cos \beta \sin \gamma] + \cos \beta [\sin \gamma \cos \alpha - \cos \gamma \sin \alpha] + \cos \gamma [\sin \alpha \cos \beta - \cos \alpha \sin \beta] \\ = 0 \end{aligned}$$

5 (d)

Given, $A + B = 45^\circ$

$$\Rightarrow \cot(A + B) = 1$$

$$\Rightarrow \frac{\cot A \cot B - 1}{\cot A + \cot B} = 1$$

$$\Rightarrow \cot A \cot B - (\cot A + \cot B) = 1 \quad \dots(i)$$

$$\text{Now, } (\cot A - 1)(\cot B - 1) = \cot A \cot B - (\cot A + \cot B) + 1$$

$$= 1 + 1 = 2 \quad [\text{from Eq. (i)}]$$

6 **(c)**

$$\text{Let } I = [\sin x + \cos x]^{1+\sin 2x}$$

$$= \left[\sqrt{2} \sin \left(\frac{\pi}{4} + x \right) \right]^{1+\sin 2x}$$

$$\text{At } x = \frac{\pi}{4},$$

$$I = \left[\sqrt{2} \sin \left(\frac{\pi}{4} + \frac{\pi}{4} \right) \right]^{1+\sin \frac{2\pi}{4}}$$

$$= (\sqrt{2})^2 = 2$$

7 **(d)**

The given expression can be written as

$$\cos^6 x (\cos^6 x + 3 \cos^4 x + 3 \cos^2 x + 1) + 2 \cos^4 x + \cos^2 x - 2$$

$$= \sin^3 x (\cos^2 x + 1)^3 + 2 \cos^4 x + \cos^2 x - 2$$

$$= \sin^3 x (\sin x + 1)^3 + 2 \sin^2 x + \cos^2 x - 2$$

$$[\because \sin x + \sin^2 x = 1 \Rightarrow \sin x = \cos^2 x]$$

$$= (\sin x + \sin^2 x)^3 + \sin^2 x + (\sin^2 x + \cos^2 x) - 2$$

$$= 1^3 + \sin^2 x + 1 - 2 = \sin^2 x$$

8 **(b)**

$$\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8}$$

$$= \frac{1}{4} \left[\left(2 \sin^2 \frac{\pi}{8} \right)^2 + \left(2 \sin^2 \frac{3\pi}{8} \right)^2 \right] + \frac{1}{4} \left[\left(2 \sin^2 \frac{\pi}{8} \right)^2 + \left(2 \sin^2 \frac{3\pi}{8} \right)^2 \right]$$

$$= \frac{1}{4} \left[\left(1 - \cos \frac{\pi}{4} \right)^2 + \left(1 - \cos \frac{3\pi}{4} \right)^2 \right] + \frac{1}{4} \left[\left(1 - \cos \frac{\pi}{4} \right)^2 + \left(1 - \cos \frac{3\pi}{4} \right)^2 \right]$$

$$= \frac{1}{4} \left[\left(1 - \frac{1}{\sqrt{2}} \right)^2 + \left(1 + \frac{1}{\sqrt{2}} \right)^2 \right] + \frac{1}{4} \left[\left(1 - \frac{1}{\sqrt{2}} \right)^2 + \left(1 + \frac{1}{\sqrt{2}} \right)^2 \right]$$

$$= \frac{1}{4} (3) + \frac{1}{4} (3) = \frac{3}{2}$$

9 **(c)**

$$\text{Given, } \frac{\sin(x+3\alpha)}{\sin(\alpha-x)} = 3$$

Applying componendo and dividendo, we get

$$\begin{aligned}
\frac{\sin(x+3\alpha) + \sin(\alpha-x)}{\sin(x+3\alpha) - \sin(\alpha-x)} &= \frac{3+1}{3-1} \\
\Rightarrow \frac{2\sin 2\alpha \cos(\alpha+x)}{2\cos 2\alpha \sin(\alpha+x)} &= 2 \\
\Rightarrow \frac{\tan 2\alpha}{\tan(\alpha+x)} &= 2 \\
\Rightarrow \frac{2\tan \alpha}{1-\tan^2 \alpha} \times \frac{(1-\tan \alpha \tan x)}{(\tan \alpha + \tan x)} &= 2 \\
\Rightarrow \tan \alpha - \tan^2 \alpha \tan x &= \tan \alpha + \tan x - \tan^3 \alpha - \tan^2 \alpha \tan x \\
\Rightarrow \tan x &= \tan^3 \alpha
\end{aligned}$$

10 **(a)**

We have,

$$\begin{aligned}
&\cos \alpha \sin(\beta - \gamma) + \cos \beta \sin(\gamma - \alpha) + \cos \gamma \sin(\alpha - \beta) \\
&= \frac{1}{2} \{ \sin(\alpha + \beta - \gamma) + \sin(\beta - \gamma - \alpha) + \sin(\gamma - \alpha + \beta) + \sin(\gamma - \alpha - \beta) + \sin(\alpha - \beta + \gamma) \\
&\quad + \sin(\alpha - \beta - \gamma) \} \\
&= \frac{1}{2} \{ \sin(\alpha + \beta - \gamma) - \sin(\alpha - \beta + \gamma) - \sin(\alpha - \beta - \gamma) - \sin(\alpha + \beta - \gamma) + \sin(\alpha - \beta + \gamma) \\
&\quad + \sin(\alpha - \beta - \gamma) \} \\
&= \frac{1}{2} \times 0 = 0
\end{aligned}$$

11 **(c)**

$$\sin A + \cos A = m \quad [\text{given}]$$

$$\Rightarrow \sin^3 A + \cos^3 A + 3\cos A \sin A$$

$$(\sin A + \cos A) = m^3$$

$$\Rightarrow n + 3m \sin A \cos A = m^3 \quad \dots(i)$$

$$[\because \sin^3 A + \cos^3 A = n]$$

$$\text{Again, } \sin A + \cos A = m$$

$$\Rightarrow \sin^2 A + \cos^2 A + 2\sin A \cos A = m^2$$

$$\Rightarrow \sin A \cos A = \frac{m^2 - 1}{2} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$n + 3m \frac{(m^2 - 1)}{2} = m^3$$

$$\Rightarrow 2n + 3m^3 - 3m = 2m^3$$

$$\Rightarrow m^3 - 3m + 2n = 0$$

12 **(b)**

We have,

$$(\sec \theta - 1) = (\sqrt{2} - 1) \tan \theta$$

$$\Rightarrow 1 - \cos \theta = (\sqrt{2} - 1) \sin \theta$$

$$\Rightarrow 2 \sin^2 \frac{\theta}{2} = 2(\sqrt{2} - 1) \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$\Rightarrow \sin \frac{\theta}{2} = 0 \text{ or, } \tan \frac{\theta}{2} = \sqrt{2} - 1 = \tan \frac{\pi}{8}$$

$$\Rightarrow \frac{\theta}{2} = n\pi \text{ or } \frac{\theta}{2} = n\pi + \frac{\pi}{8}, n \in Z$$

$$\Rightarrow \theta = 2n\pi, \theta = 2n\pi + \frac{\pi}{4}, n \in Z$$

13 (d)

$$\text{Given, } \cos \theta + \sin 2\theta = 0$$

$$\Rightarrow \cos \theta + 2 \sin \theta \cos \theta = 0$$

$$\Rightarrow \cos \theta (1 + 2 \sin \theta) = 0$$

$$\Rightarrow \cos \theta = 0 \text{ or } \sin \theta = -\frac{1}{2}$$

For $\theta \in [-\pi, \pi]$

$$\theta = \frac{\pi}{2}, -\frac{\pi}{2}$$

$$\text{Or } \theta = -\frac{\pi}{6}, -\frac{5\pi}{6}$$

14 (b)

We have,

$$\tan\left(\frac{\pi}{2} \sin \theta\right) = \cot\left(\frac{\pi}{2} \cos \theta\right)$$

$$\Rightarrow \tan\left(\frac{\pi}{2} \sin \theta\right) = \tan\left(\frac{\pi}{2} - \frac{\pi}{2} \cos \theta\right)$$

$$\Rightarrow \frac{\pi}{2} \sin \theta = r\pi + \frac{\pi}{2} - \frac{\pi}{2} \cos \theta, r \in Z$$

$$\Rightarrow \sin \theta + \cos \theta = (2r + 1), r \in Z$$

$$\Rightarrow \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta = \frac{2r + 1}{\sqrt{2}}, r \in Z$$

$$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{2r + 1}{\sqrt{2}}, r \in Z$$

$$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \text{ or } -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta - \frac{\pi}{4} = 2r\pi \pm \frac{\pi}{4}, r \in Z$$

$$\Rightarrow \theta = 2r\pi \pm \frac{\pi}{4} + \frac{\pi}{4}, r \in Z$$

$$\Rightarrow \theta = 2r\pi, 2r\pi + \frac{\pi}{2}, r \in Z$$

But, $\theta = 2r\pi + \frac{\pi}{2}, r \in Z$ gives extraneous roots as it does not satisfy the given equation. Therefore,
 $\theta = 2r\pi, r \in Z$

15 (b)

$$\tan \theta + \tan\left(\frac{3\pi}{4} + \theta\right) = 2$$

$$\Rightarrow \tan \theta + \tan\left(\frac{\pi}{2} + \left(\frac{\pi}{4} + \theta\right)\right) = 2$$

$$\Rightarrow \tan \theta - \cot\left(\frac{\pi}{4} + \theta\right) = 2$$

$$\Rightarrow \tan \theta - \frac{\cot \frac{\pi}{4} \cot \theta - 1}{\cot \frac{\pi}{4} + \cot \theta} = 2$$

$$\begin{aligned} \Rightarrow \tan \theta - \frac{\cot \theta - 1}{1 + \cot \theta} &= 2 \\ \Rightarrow \tan \theta - \frac{1 - \tan \theta}{1 + \tan \theta} &= 2 \\ \Rightarrow \tan \theta + \tan^2 \theta - 1 + \tan \theta &= 2 + 2 \tan \theta \\ \Rightarrow \tan^2 \theta &= 3 \\ \Rightarrow \tan \theta &= \pm \sqrt{3} = \pm \tan \frac{\pi}{3} \\ \Rightarrow \theta &= n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z} \end{aligned}$$

16 **(b)**

We have,

$$\begin{aligned} (1 + \tan \theta)(1 + \tan \phi) &= 2 \\ \Rightarrow 1 + \tan \theta + \tan \phi + \tan \theta \tan \phi &= 2 \\ \Rightarrow \tan \theta + \tan \phi &= 1 - \tan \theta \tan \phi \\ \Rightarrow \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi} &= 1 \\ \Rightarrow \tan(\theta + \phi) &= 1 \Rightarrow \theta + \phi = \frac{\pi}{4}, n \in \mathbb{Z} \end{aligned}$$

17 **(c)**

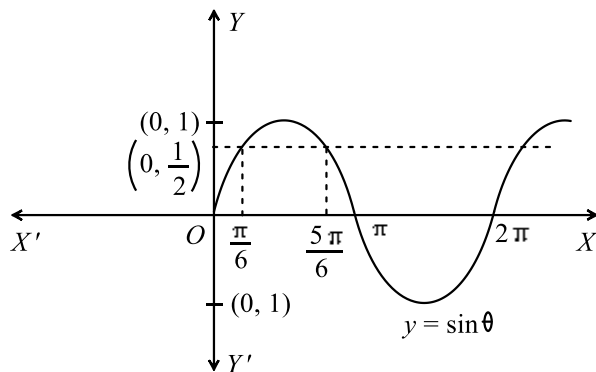
Given, $2 \sec 2\alpha = \tan \beta + \cot \beta$

$$\begin{aligned} \Rightarrow 2 \sec 2\alpha &= \frac{\sin^2 \beta + \cos^2 \beta}{\sin \beta \cos \beta} \\ \Rightarrow \frac{2}{\cos 2\alpha} &= \frac{1}{\sin \beta \cos \beta} \\ \Rightarrow \sin 2\beta &= \cos 2\alpha \\ \Rightarrow \alpha + \beta &= \frac{\pi}{4} \end{aligned}$$

18 **(a)**

We have,

$$\begin{aligned} 2 \sin^2 \theta - 5 \sin \theta + 2 &> 0 \\ \Rightarrow (\sin \theta - 2)(2 \sin \theta - 1) &> 0 \\ \Rightarrow 2 \sin \theta - 1 < 0 \quad [\because -1 \leq \sin \theta \leq 1 \therefore \sin \theta - 2 < 0] \\ \Rightarrow \sin \theta &< \frac{1}{2} \\ \Rightarrow \theta &\in (0, \pi/6) \cup (5\pi/6, \pi) \end{aligned}$$



19 (d)

Given that,

$$\tan A - \tan B = x \quad \dots (i)$$

$$\text{and } \cot B - \cot A = y \quad \dots (ii)$$

$$\text{Now, } \cot(A - B) = \frac{1}{\tan(A - B)}$$

$$= \frac{1 + \tan A \tan B}{\tan A - \tan B}$$

$$= \frac{1}{\tan A - \tan B} + \frac{\tan A \tan B}{\tan A - \tan B}$$

$$= \frac{1}{x} + \frac{1}{y} \quad [\text{from Eqs.(i) and (ii)}]$$

20 (b)

We have,

$$a \cos B - b \cos A$$

$$= \frac{a - b}{a - b} \left(\frac{c^2 + a^2 - b^2}{2ac} \right) - b \left(\frac{b^2 + c^2 - a^2}{2bc} \right)$$

$$= \frac{2(a^2 - b^2)}{2c(a - b)} = \frac{a + b}{c} = \frac{2c}{c} = 2 \quad \left[\because a, c, b \text{ are in A.P.} \right]$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	D	A	D	C	D	B	C	A

Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	D	B	B	B	C	A	D	B

