SESSION : 2025-26

DPP
DAILY PRACTICE PRODUCTS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :1

Topic :-VECTOR ALGEBRA

1 **(d)**

We have,

$|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$

Let θ be the angle between $\vec{a}$ and $\vec{b}$

Now, $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow |\vec{c}|^2 = |\vec{a} + \vec{b}|$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 = 2|\vec{a}||\vec{b}| \cos\theta$

$\Rightarrow 49 = 9 + 25 + 2 \times 3 \times 5 \cos\theta$

$\Rightarrow 15 = 30 \cos\theta \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = \pi/3$

2 **(c)**

$\therefore\ [\vec{\mathbf{a}}\, \vec{\mathbf{b}}\, \vec{\mathbf{c}}] = \vec{\mathbf{a}} \cdot \left(|\vec{\mathbf{b}}||\vec{\mathbf{c}}| \sin\frac{2\pi}{3} \hat{\mathbf{n}}\right)$

$= |\vec{\mathbf{a}}||\vec{\mathbf{b}}||\vec{\mathbf{c}}| \left(\sin\frac{2\pi}{3}\right)$

$[\because\ \vec{\mathbf{a}} \cdot \hat{\mathbf{n}} = |\vec{\mathbf{a}}||\hat{\mathbf{n}}| \cos 0° = |\vec{\mathbf{a}}|]$

$= 2 \times 3 \times 4 \times \frac{\sqrt{3}}{2} = 12\sqrt{3}$

3 **(a)**

Given that, $\overrightarrow{\mathbf{OA}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}, \overrightarrow{\mathbf{OB}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{OC}} = \hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{OB}} - \overrightarrow{\mathbf{OA}}$

$= (3-2)\hat{\mathbf{i}} + (-2-1)\hat{\mathbf{j}} + (1+1)\hat{\mathbf{k}}$

$= \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$|\overrightarrow{\mathbf{AB}}| = \sqrt{1^2 + (-3)^2 + 2^2}$

$= \sqrt{1+9+4} = \sqrt{14}$

$\overrightarrow{\mathbf{BC}} = \overrightarrow{\mathbf{OC}} - \overrightarrow{\mathbf{OB}}$

$= (1-3)\hat{\mathbf{i}} + (4+2)\hat{\mathbf{j}} + (-3-1)\hat{\mathbf{k}}$

$= -2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

$|\overrightarrow{\mathbf{BC}}| = \sqrt{(-2)^2 + 6^2 + (-4)^2}$

$= \sqrt{4+36+16} = \sqrt{56}$

$\overrightarrow{\mathbf{CA}} = \overrightarrow{\mathbf{OA}} - \overrightarrow{\mathbf{OC}}$

$= (2-1)\hat{\mathbf{i}} + (1-4)\hat{\mathbf{j}} + (-1+3)\hat{\mathbf{k}}$
$= \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$
$|\overrightarrow{\mathbf{CA}}| = \sqrt{1^2 + (-3)^2 + (2)^2}$
$= \sqrt{1+9+4} = \sqrt{14}$
It is clear that two sides of a triangle are equal.
$\therefore$ Points A, B, C from an isosceles triangle.

4 **(b)**

The component of $\vec{a}$ along $\vec{b}$ is given by

$$\left\{\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right\} = \frac{18}{25}(3\hat{j} + 4\hat{k})$$

5 **(a)**

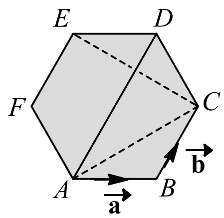
It is given that $\vec{c}$ and $\vec{d}$ are collinear vectors
$\therefore \vec{c} = \lambda\,\vec{d}$ for some scalar λ
$\Rightarrow (x-2)\vec{a} + \vec{b} = \lambda\{(2x+1)\vec{a} - \vec{b}\}$
$\Rightarrow \{x - 2 - \lambda(2x+1)\}\vec{a} + (\lambda+1)\vec{b} = \vec{0}$
$\Rightarrow \lambda + 1 = 0$ and $x - 2 - \lambda(2x+1) = 0$ [$\because \vec{a}, \vec{b}$ are non-collinear]
$\Rightarrow \lambda = -1$ and $x = \frac{1}{3}$

6 **(a)**

Equation of plane is $\vec{\mathbf{r}}\cdot\hat{\mathbf{n}} = d$,
where d is the perpendicular distance of the plane from origin
$\therefore$ Required plane is $(\alpha x + \beta y + \gamma z) = p$

7 **(c)**

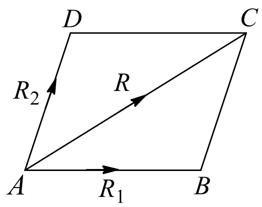
In $\Delta A\,BC$, $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} + \overrightarrow{\mathbf{AC}}$
$\Longrightarrow\ \overrightarrow{\mathbf{AC}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$
AD is parallel to BC and $AD = 2\,BC$



$\therefore\ \overrightarrow{\mathbf{AD}} = 2\vec{\mathbf{b}}$
In ΔACD, $\overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{CD}} = \overrightarrow{\mathbf{AD}}$
$\Longrightarrow\ \overrightarrow{\mathbf{CD}} = 2\vec{\mathbf{b}} - (\vec{\mathbf{a}} + \vec{\mathbf{b}}) = \vec{\mathbf{b}} - \vec{\mathbf{a}}$
Now, $\overrightarrow{\mathbf{CE}} = \overrightarrow{\mathbf{CD}} + \overrightarrow{\mathbf{DE}} = \vec{\mathbf{b}} - 2\vec{\mathbf{a}}$

9 **(d)**

Let $\vec{\mathbf{R}}_1 = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$
and $\vec{\mathbf{R}}_2 = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$



$\therefore\ \vec{\mathbf{R}}$ (along $\overrightarrow{\mathbf{AC}}$) $= \vec{\mathbf{R}}_1 + \vec{\mathbf{R}}_2 = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\therefore\ \vec{\mathbf{a}}$ $\left(\text{unit vector angle } \overrightarrow{\mathrm{AC}}\right) = \dfrac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|} = \dfrac{3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{\sqrt{9+36+4}}$

$= \dfrac{1}{7}\left(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}\right)$

11 **(b)**

Since $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors. Therefore, $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors

$\therefore x\vec{a} + y\,\vec{b} + z\,\vec{c} = \vec{0} \Rightarrow x = y = z = 0$

12 **(a)**

Suppose point $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ divides the join of points $-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ and $7\hat{\imath} - \hat{k}$ in the ratio $\lambda : 1$.

Then,

$\hat{\imath} + 2\hat{\jmath} + 3\hat{k} = \dfrac{\lambda\left(7\hat{\imath} - \hat{k}\right) + \left(-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}\right)}{\lambda + 1}$

$\Rightarrow (\lambda+1)\hat{\imath} + 2(\lambda+1)\hat{\jmath} + 3(\lambda+1)\hat{k} = (7\lambda - 2)\hat{\imath} + 3\hat{\jmath} + (-\lambda + 5)\hat{k}$

$\Rightarrow \lambda + 1 = 7\lambda - 2, 2(\lambda+1) = 3$ and $3(\lambda + 1) = -\lambda + 5$

$\Rightarrow \lambda = \dfrac{1}{2}$

Hence, required ratio is 1 : 2

13 **(d)**

Clearly,

$\vec{a} - \vec{b} + \vec{b} - \vec{c} + \vec{c} - \vec{a} = \vec{0}$

$\therefore \vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}$ are coplanar

$\Rightarrow \left(\vec{a} - \vec{b}\right) \cdot \left\{\left(\vec{b} \cdot \vec{c}\right) \times (\vec{c} - \vec{a})\right\} = 0$

314 **(d)**

Two given lines intersect, if

$7\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 13\hat{\mathbf{k}} + s\left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}\right)$

$= 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}} + t(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$\Longrightarrow (7 + 2s)\hat{\mathbf{i}} + (10 + 3s)\hat{\mathbf{j}} + (13 + 4s)\hat{\mathbf{k}}$

$= (3 + t)\hat{\mathbf{i}} + (5 + 2t)\hat{\mathbf{j}} + (7 + 3t)\hat{\mathbf{k}}$

$\Longrightarrow 7 + 2s = 3 + t$

$\Longrightarrow 2s - t = -4$ ...(i)

$10 + 3s = 5 + 2t$

$\Longrightarrow 3s - 2t = -5$ ...(ii)

and $13 + 4s = 7 + 3t$

$\Longrightarrow 4s - 3t = -6$ ...(iii)

On solving Eqs. (i) and (iii), we get

$s = -3, t = -2$

$\therefore$ Required line is

$7\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 13\hat{\mathbf{k}} + (-3)[2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}]$

$\Longrightarrow$ $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ is the required line.

16 **(c)**

Given that, $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\vec{\mathbf{b}} = 2\hat{\mathbf{i}} - \hat{\mathbf{k}}$

Let $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$, then

$\vec{\mathbf{r}} \times \vec{\mathbf{a}} = \vec{\mathbf{b}} \times \vec{\mathbf{a}} \Rightarrow (\vec{\mathbf{r}} - \vec{\mathbf{b}}) \times \vec{\mathbf{a}} = \vec{\mathbf{0}}$

Now, $\vec{\mathbf{r}} - \vec{\mathbf{b}} = (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} - \hat{\mathbf{k}})$

$= (x-2)\hat{\mathbf{i}} + y\hat{\mathbf{j}} + (z+1)\hat{\mathbf{k}}$

$$\therefore\ (\vec{\mathbf{r}} - \vec{\mathbf{b}}) \times \vec{\mathbf{a}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x-2 & y & z+1 \\ 1 & 1 & 0 \end{vmatrix} = \vec{\mathbf{0}}$$

$\Rightarrow\ -(z+1)\hat{\mathbf{i}} + (z+1)\hat{\mathbf{j}} + (x-2-y)\hat{\mathbf{k}} = \vec{\mathbf{0}}$

On equating the coefficient of $\hat{\mathrm{i}}, \hat{\mathrm{j}}$ and $\hat{\mathrm{k}}$, we get

$z = -1, x - y = 2$ ....(i)

Now, $\vec{\mathbf{r}} \times \vec{\mathbf{b}} = \vec{\mathbf{a}} \times \vec{\mathbf{b}} \Rightarrow (\vec{\mathbf{r}} - \vec{\mathbf{a}}) \times \vec{\mathbf{b}} = \vec{\mathbf{0}}$

And $\vec{\mathbf{r}} - \vec{\mathbf{a}} = (x-1)\hat{\mathbf{i}} + (y-1)\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$$\therefore (\vec{\mathbf{r}} - \vec{\mathbf{a}}) \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x-1 & y-1 & z \\ 2 & 0 & -1 \end{vmatrix} = \vec{\mathbf{0}}$$

$\Rightarrow (-y+1)\hat{\mathbf{i}} - \hat{\mathbf{j}}(-x+1-2z) + (-2y+2)\hat{\mathbf{k}} = \vec{\mathbf{0}}$

$\Rightarrow y = 1, x + 2z = 1$ ...(ii)

From Eqs. (i) and (ii), we get

$x = 3, y = 1\ z = -1$

$\therefore \vec{\mathbf{r}} =\ 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

17 **(a)**

Given, $\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = \vec{\mathbf{B}} \times (\vec{\mathbf{C}} \times \vec{\mathbf{A}})$ ....(i)

Also, $[\vec{\mathbf{A}}\, \vec{\mathbf{B}}\, \vec{\mathbf{C}}] \neq 0 ie. \vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{C}}$ are not coplanar.

$\therefore$From Eq. (i)

$(\vec{\mathbf{A}} \cdot \vec{\mathbf{C}}) - (\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})\vec{\mathbf{C}} = (\vec{\mathbf{B}} \cdot \vec{\mathbf{A}})\vec{\mathbf{C}} - (\vec{\mathbf{B}} \cdot \vec{\mathbf{C}})\vec{\mathbf{A}}$

$\Longrightarrow$ $(\vec{\mathbf{B}} \cdot \vec{\mathbf{C}})\vec{\mathbf{A}} + (\vec{\mathbf{A}} \cdot \vec{\mathbf{C}})\vec{\mathbf{B}} - [(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}) + (\vec{\mathbf{B}} \cdot \vec{\mathbf{C}})]\vec{\mathbf{C}} = \vec{\mathbf{0}}$

$\Longrightarrow$ $\vec{\mathbf{B}} \cdot \vec{\mathbf{C}} = \vec{\mathbf{A}} \cdot \vec{\mathbf{C}} = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{0}}$

$[\because [\vec{\mathbf{A}}\, \vec{\mathbf{B}}\, \vec{\mathbf{C}}] \neq 0]$

Now, consider

$\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = (\vec{\mathbf{A}} \cdot \vec{\mathbf{C}})\vec{\mathbf{B}} - (\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})\vec{\mathbf{C}}$

$= 0 \cdot \vec{\mathbf{B}} - 0 \cdot \vec{\mathbf{C}} = \vec{\mathbf{0}}$

319 **(a)**

$$[\vec{\mathbf{a}}\, \vec{\mathbf{b}}\, \vec{\mathbf{c}}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ y & x & 1+x \end{vmatrix} = 1[1 + x - x] = 1$$

Hence, $[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]$ does not depend upon neither x nor y.

20 **(b)**

The required vector is given by

$$\hat{n} = \frac{A\vec{B} \times A\vec{C}}{|A\vec{B} \times A\vec{C}|} = \frac{\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	A	B	A	A	C	C	D	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	D	D	A	C	A	C	A	B

SESSION : 2025-26

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DPP NO. :2

Topic :-VECTOR ALGEBRA

1 **(d)**

$(\vec{\mathbf{a}}-\vec{\mathbf{b}})\cdot(\vec{\mathbf{b}}+\vec{\mathbf{c}})\times(\vec{\mathbf{c}}+\vec{\mathbf{a}})$

$=(\vec{\mathbf{a}}-\vec{\mathbf{b}})\cdot[\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{b}}\times\vec{\mathbf{a}}+\vec{\mathbf{c}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}}]$

$=\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})+\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{a}})+\vec{\mathbf{a}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{a}})-\vec{\mathbf{b}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})-\vec{\mathbf{b}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{a}})-\vec{\mathbf{b}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{a}})$

$=\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})-\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$

$=[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]-[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]=0$

2 **(b)**

$\because\ \vec{\mathbf{a}},\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are coplanar vectors, so $2\,\vec{\mathbf{a}}-\vec{\mathbf{b}}\ 2\vec{\mathbf{b}}\ -\vec{\mathbf{c}}$ and $2\vec{\mathbf{c}}-\vec{\mathbf{a}}$ are also coplanar. Thus

$[2\,\vec{\mathbf{a}}-\vec{\mathbf{b}}\ \ 2\vec{\mathbf{b}}\ -\vec{\mathbf{c}}\ \ 2\vec{\mathbf{c}}-\vec{\mathbf{a}}]=0$

3 **(b)**

Clearly, angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}=\frac{\pi}{2}$

$\Rightarrow\ \vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=0$

$\therefore\ |\vec{\mathbf{a}}+\vec{\mathbf{b}}|^2=a^2+b^2+2\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$

$=1+1+0=2$

$\Rightarrow\ |\vec{\mathbf{a}}+\vec{\mathbf{b}}|=\sqrt{2}$

5 **(d)**

Given, $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\times\vec{\mathbf{c}}=-\frac{1}{4}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}}$

$\Longrightarrow(\vec{\mathbf{c}}\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}-(\vec{\mathbf{c}}\cdot\vec{\mathbf{b}})\vec{\mathbf{a}}=-\frac{1}{4}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}}$

On comparing both sides, we get

$(\vec{\mathbf{c}}\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}=0$

$|\vec{\mathbf{c}}||\vec{\mathbf{a}}|\cos\theta=0$

$\Longrightarrow\ \theta=\frac{\pi}{2}$

6 **(c)**

Now, $(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\times(\hat{\mathbf{i}}+\hat{\mathbf{j}})=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & 1 & 1\\ 1 & 1 & 0\end{vmatrix}$

$=\hat{\mathbf{i}}(-1)+\hat{\mathbf{j}}(1)+\hat{\mathbf{k}}(0)=-\hat{\mathbf{i}}+\hat{\mathbf{j}}$

and $|(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\times(\hat{\mathbf{i}}+\hat{\mathbf{j}})| = \sqrt{1^2+1^2} = \sqrt{2}$

Vector perpendicular to both of the vectors $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+\hat{\mathbf{j}}$

$= \dfrac{(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\times(\hat{\mathbf{i}}+\hat{\mathbf{j}})}{|(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\times(\hat{\mathbf{i}}+\hat{\mathbf{j}})|}$

$= \dfrac{-\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}} = \dfrac{-1}{\sqrt{2}}(\hat{\mathbf{i}}-\hat{\mathbf{j}})$

$= c(\hat{\mathbf{i}}-\hat{\mathbf{j}})$, c is a scalar.

7 **(b)**

It is given that $(\vec{a}+\vec{b})||\vec{c}$ and $(\vec{c}+\vec{a})||\vec{b}$

$\therefore (\vec{a}+\vec{b})\times\vec{c} = 0$ and $(\vec{c}+\vec{a})\times\vec{b} = 0$

$\Rightarrow \vec{a}\times\vec{c}+\vec{b}\times\vec{c} = 0$ and $\vec{c}\times\vec{b}+\vec{a}\times\vec{b} = 0$

$\Rightarrow \vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

Hence, $\vec{a},\vec{b},\vec{c}$ form the sides of a triangle

8 **(a)**

$\because$ Displacement, $\overrightarrow{\mathbf{AB}} = (3-2)\hat{\mathbf{i}}+(1+1)\hat{\mathbf{j}}+(2-1)\hat{\mathbf{k}}$

$= \hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

and force, $\vec{\mathbf{F}} = \dfrac{\sqrt{6}(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})}{\sqrt{6}}$

$= (\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})$

$\therefore$ Work done $= \vec{\mathbf{F}}\cdot\overrightarrow{\mathbf{AB}} = (1+2\hat{\mathbf{j}}+\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}) = 6$

9 **(c)**

let $\vec{\mathbf{a}} = l\hat{\mathbf{i}}+m\hat{\mathbf{j}}+n\hat{\mathbf{k}}$ makes an angle $\frac{\pi}{4}$ with z-axis

Also, $l^2+m^2+n^2 = 1$

Here, $n = \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}, \qquad l^2+m^2 = \dfrac{1}{2} \qquad \dots\dots(i)$

$\therefore \quad \vec{\mathbf{a}} = l\hat{\mathbf{i}}+m\hat{\mathbf{j}}+\frac{\hat{\mathbf{k}}}{\sqrt{2}}$

$\Longrightarrow \vec{\mathbf{a}}+\hat{\mathbf{i}}+\hat{\mathbf{j}} = (l+1)\hat{\mathbf{i}}(m+1)\hat{\mathbf{j}}+\dfrac{\hat{\mathbf{k}}}{\sqrt{2}}$

$\Longrightarrow |\vec{\mathbf{a}}+\hat{\mathbf{i}}+\hat{\mathbf{j}}| = \sqrt{(l+1)^2+(m+1)^2+\left(\dfrac{1}{\sqrt{2}}\right)^2}$

$\Longrightarrow 1 = l^2+m^2+2+2l+2m+\dfrac{1}{2}$

$\Longrightarrow l+m = -1$ (From Eq. (i)

$\Longrightarrow l^2+m^2+2lm = 1$

$\Longrightarrow 2lm = \dfrac{1}{2}$

$\Longrightarrow l = m = -\dfrac{1}{2}$

$\left(\because l = m = \dfrac{1}{2}\text{, is not satisfied the given equition}\right)$

$\therefore\ \vec{\mathbf{a}} = -\frac{\hat{\mathbf{i}}}{2} - \frac{\hat{\mathbf{j}}}{2} + \frac{\hat{\mathbf{k}}}{\sqrt{2}}$

10 **(b)**

Given, $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|^2 + |\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}|^2 = 144$

$\Longrightarrow |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 (\sin^2\theta + \cos^2\theta) = 144$

$\Longrightarrow 16|\vec{\mathbf{b}}|^2 = 144$

$\Longrightarrow |\vec{\mathbf{b}}| = 3$

11 **(c)**

Since, $m\,\vec{\mathbf{a}}$ is a unit vector, if and only, if

$|m\,\vec{\mathbf{a}}| = 1 \Rightarrow |m\,||\vec{\mathbf{a}}| = 1 \Rightarrow m|\vec{\mathbf{a}}| = 1$

$\Rightarrow m = \frac{1}{|\vec{\mathbf{a}}|}$

12 **(b)**

Resultant force $\vec{F}$ is given by

$\vec{F} = (2\hat{\imath} - 5\hat{\jmath} + 6\hat{k}) - (-\hat{\imath} + 2\hat{\jmath} - \hat{k}) = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$

Let $\vec{d}$ be the displacement vector. Then,

$\vec{d} = A\vec{B}$

$\Rightarrow \vec{d} = (6\hat{\imath} + \hat{\jmath} - 3\hat{k}) - (4\hat{\imath} - 3\hat{\jmath} - 2\hat{k}) = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$

$\therefore W$ = Work done

$\Rightarrow W = \vec{F} \cdot \vec{d}$

$\Rightarrow W = (\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) \cdot (2\hat{\imath} + 4\hat{\jmath} - \hat{k})$

$\Rightarrow W = 2 - 12 - 5 = -15$ units

13 **(d)**

Since, P, Q, R are collinear. Therefore,

$\vec{P}Q = m\,Q\vec{R}$ for same scalar m

$\Rightarrow -2\hat{\jmath} = m[(a-1)\hat{\imath} + (\vec{b}+1)\hat{\jmath} + c\,\hat{k}]$ for some non-zero scalar m

$\Rightarrow (a-1)m = 0, (b+1)m = -2, cm = 0$

$\Rightarrow a = 1, c = 0, b \in R$

14 **(b)**

The direction cosines of a vector making equal angles with the coordinate axes are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Therefore, the unit vector along the vector making equal angles with the coordinate axes is

$\vec{b} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b} = \vec{a} \cdot \vec{b}$

$= (4\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) \cdot \left(\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}\right) = \frac{4 - 3 + 2}{\sqrt{3}} = \sqrt{3}$

15 **(a)**

$[2\,\hat{\mathbf{i}}\ 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}\,]$

$= -30\,[\,\hat{\mathbf{i}}\ \hat{\mathbf{j}}\,\hat{\mathbf{k}}\,]$

$= -30 \quad (\because [\,\hat{\mathbf{i}}\ \hat{\mathbf{j}}\ \hat{\mathbf{k}}\,] = 1\,)$

16 **(b)**

We have,

$(\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{c}) \cdot \vec{d}$

$= \left\{\left((\vec{a} \times \vec{b}) \cdot \vec{c}\right)\vec{a} - \left((\vec{a} \times \vec{b}) \cdot \vec{a}\right)\vec{c}\right\} \cdot \vec{d}$

$= \{[\vec{a}\,\vec{b}\,\vec{c}]\vec{a} - 0\} \cdot \vec{d} = [\vec{a}\,\vec{b}\,\vec{c}](\vec{a} \cdot \vec{d})$

17 **(d)**

Resultant force $\vec{\mathbf{F}} = (2\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) + (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

and displacement, $\vec{\mathbf{d}} = (6\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

$= 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\therefore$ work done $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$

$= (\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= -15$

$=$ 15 units [neglecting – ve sign]

18 **(a)**

The resultant force is given by

$$\vec{F} = 6\frac{(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})}{\sqrt{1+4+4}} + 7\frac{(2\hat{\imath} - 3\hat{\jmath} - 6\hat{k})}{\sqrt{4+9+36}} = 4\hat{\imath} - 7\hat{\jmath} - 2\hat{k}$$

$\vec{d} =$ Displacement $= \vec{P}Q$

$\vec{d} = (5\hat{\imath} - \hat{\jmath} + \hat{k}) - (2\hat{\imath} - \hat{\jmath} - 3\hat{k}) = 3\hat{\imath} + 4\hat{k}$

$\therefore$ Work done $= \vec{F} \cdot \vec{d} = 12 + 0 - 8 = 4$ units

19 **(c)**

We know, $[\vec{\mathbf{b}} \times \vec{\mathbf{c}}\ \ \vec{\mathbf{c}} \times \vec{\mathbf{a}}\ \ \vec{\mathbf{a}} \times \vec{\mathbf{b}}]$

$= (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) \cdot [(\vec{\mathbf{c}} \times \vec{\mathbf{a}}) \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{b}})]$

$= (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) \cdot \left[\left((\vec{\mathbf{c}} \times \vec{\mathbf{a}}) \cdot \vec{\mathbf{b}}\right)\vec{\mathbf{a}} - ((\vec{\mathbf{c}} \times \vec{\mathbf{a}}) \cdot \vec{\mathbf{a}})\vec{\mathbf{b}}\right]$

$= (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) \cdot ([\,\vec{\mathbf{c}}\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,]\vec{\mathbf{a}} - [\,\vec{\mathbf{c}}\,\vec{\mathbf{a}}\,\vec{\mathbf{a}}\,]\,\vec{\mathbf{b}})$

$= (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) \cdot \vec{\mathbf{a}}[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\vec{\mathbf{c}}\,] - 0$

$= [\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,][\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,]$

$= [\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,]^2$

20 **(d)**

$\because$ $\overrightarrow{\mathbf{QP}}$ is parallel to $\overrightarrow{\mathbf{AB}}$ and $\overrightarrow{\mathbf{DC}}$.

$\therefore$ $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{DC}} = \overrightarrow{\mathbf{QP}} + \overrightarrow{\mathbf{QP}} = 2\overrightarrow{\mathbf{QP}}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	B	B	B	D	C	B	A	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	D	B	A	B	D	A	C	D

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :3

Topic :-VECTOR ALGEBRA

1 **(a)**

Taking A as the origin, let the position vectors of B and C be $\vec{b}$ and $\vec{c}$ respectively

$\therefore \vec{B}E + \vec{A}F = \left(\frac{\vec{c}}{2} - \vec{b}\right) + \left(\frac{\vec{b}+\vec{c}}{2} - \vec{0}\right) = \vec{c} - \frac{\vec{b}}{2} = \vec{D}C$

2 **(a)**

Since, $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors.

$\Rightarrow |\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = |\vec{\mathbf{c}}| = 1$

and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = 0$ .....(i)

Now, $|\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|^2 = (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}) \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})$

$= |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}})$

$= 1 + 1 + 1 + 0 = 3$ [from Eq. (i)]

$\Rightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}| = \sqrt{3}$

3 **(c)**

Any vector lying in the plane of $\vec{a}$ and $\vec{b}$ is of the from $x\vec{a} + y\vec{b}$

It is given that $\vec{c}$ is parallel to the plane of $\vec{a}$ and $\vec{b}$

$\therefore \vec{c} = \lambda(x\vec{a} + y\vec{b})$ for some scalar λ

$\Rightarrow d\hat{\imath} + \hat{\jmath} + (2d-1)\hat{k} = \lambda\{x(\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + y(3\hat{\imath} + 3\hat{\jmath} - \hat{k})\}$

$\Rightarrow d\hat{\imath} + \hat{\jmath} + (2d-1)\hat{k} = \lambda\{(x+3y)\hat{\imath} + (-2x+3y)\hat{\jmath} + (3x-y)\hat{k}\}$

$\Rightarrow \lambda(x+3y) = d, \lambda(-2x+3y) = 1$ and $\lambda(3x-y) = (2d-1)$

$[\because \hat{\imath}, \hat{\jmath}, \hat{k}$ are non $-$ coplanar$]$

Solving $\lambda(x+3y) = d$ and $3x - y = 2d - 1$, we get

$x = \frac{7d-3}{10\lambda}$ and $y = \frac{d+1}{10\lambda}$

Substituting these values in $\lambda(x+3y) = d$, we get $11d = -1$

<u>ALTER</u> *clearly*, $\vec{c}$ *is perpendicular to* $\vec{a} \times \vec{b}$

$\therefore \vec{c} \cdot (\vec{a} \times \vec{b}) = 0$

$$\Rightarrow [\vec{c}\vec{a}\vec{b}] = 0 \Rightarrow \begin{vmatrix} d & 1 & 2d-1 \\ 1 & -2 & 3 \\ 3 & 3 & -1 \end{vmatrix} = 0 \Rightarrow 11d = -1$$

4 **(c)**

$\because$ $\vec{\mathbf{p}}, \vec{\mathbf{q}}, \vec{\mathbf{r}}$ are reciprocal vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ respectively.

$\therefore \vec{\mathbf{p}} \cdot \vec{\mathbf{a}} = 1, \vec{\mathbf{p}} \cdot \vec{\mathbf{b}} = 0, \vec{\mathbf{p}} \cdot \vec{\mathbf{c}}$ etc.

$\therefore (l\vec{\mathbf{a}} + m\vec{\mathbf{b}} + n\vec{\mathbf{c}}) \cdot (l\vec{\mathbf{p}} + m\vec{\mathbf{q}} + n\vec{\mathbf{r}}) = l^2 + m^2 + n^2$

5 **(b)**

Given expression $= 2(1+1+1) - 2\sum(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})$

$= 6 - 2\sum(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})$ ...(i)

But $(\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})^2 \geq 0$

$\therefore (1+1+1) + 2\sum \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \geq 0$

$\therefore 3 \geq -2\sum \vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$ ...(ii)

From relations (i) and (ii), we get

Given expression $\leq 6 + 3 = 9$

6 **(a)**

Let $\overrightarrow{\mathbf{OA}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{OB}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

$\therefore \overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$ work don, $W = \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{AB}}$

$= (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= 4 - 6 + 4 = 2$

7 **(d)**

$\overrightarrow{\mathbf{AC}} = (a\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) - (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) = (a-2)\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

and $\overrightarrow{\mathbf{BC}} = (a\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) - (\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}) = (a-1)\hat{\mathbf{i}} + 6\hat{\mathbf{k}}$

Since, the ΔABC is right angled at C, then

$\overrightarrow{\mathbf{AC}} \cdot \overrightarrow{\mathbf{BC}} = 0$

$\Rightarrow \{(a-2)\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\} \cdot \{(a-1)\hat{\mathbf{i}} + 6\hat{\mathbf{k}}\} = 0$

$\Rightarrow (a-2)(a-1) = 0 \Rightarrow a = 1$and 2

8 **(a)**

We have,

$(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$

$\Leftrightarrow -\vec{c} \times (\vec{a} \times \vec{b}) = \vec{a} \times (\vec{b} \times \vec{c})$

$\Leftrightarrow -\{(\vec{c} \cdot \vec{b})\vec{a} - (\vec{c} \cdot \vec{a})\vec{b}\} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$\Leftrightarrow (\vec{a} \cdot \vec{b})\vec{c} - (\vec{c} \cdot \vec{b})\vec{a} = 0$

$\Leftrightarrow (\vec{b} \cdot \vec{a})\vec{c} - (\vec{b} \cdot \vec{c})\vec{a} = 0$

$\Leftrightarrow \vec{b} \times (\vec{c} \times \vec{a}) = 0$

9 **(b)**

Clearly,

$(\vec{a} + \vec{b}) \times \{\vec{c} - (\vec{a} + \vec{b})\}$

$= (\vec{a} + \vec{b}) \times \vec{c} - (\vec{a} + \vec{b}) \times (\vec{a} + \vec{b}) = (\vec{a} + \vec{b}) \times \vec{c}$

10 **(a)**

$\overrightarrow{\mathbf{PQ}} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= \hat{\mathbf{i}} + \hat{\mathbf{k}}$

and $\vec{\mathbf{F}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

$\therefore$ Moment $= \left|\overrightarrow{\mathbf{PQ}} \times \vec{\mathbf{F}}\right|$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 1 \\ 3 & 2 & -4 \end{vmatrix}$$

$= -2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$Magnitude of moment$= \sqrt{4 + 49 + 4} = \sqrt{57}$

11 **(b)**

Since, $\left|\vec{\mathbf{a}} + \vec{\mathbf{b}}\right| = \sqrt{3}$

$\Longrightarrow |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 3$

$\Longrightarrow \ \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \dfrac{1}{2} \ldots\ldots \text{(i)}$

$\because [|\vec{\mathbf{a}}| = \left|\vec{\mathbf{b}}\right| = 1, \text{given}]$

$\therefore \left(3\vec{\mathbf{a}} - 4\vec{\mathbf{b}}\right) \cdot \left(2\vec{\mathbf{a}} + 5\vec{\mathbf{b}}\right) = 6 + 7\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} - 20$

$= 6 + \dfrac{7}{2} - 20$

$= -\dfrac{21}{2} \quad$ [from Eq. (i)]

12 **(c)**

We have,

$\hat{a} \times \left(\hat{b} \times \hat{c}\right) = \dfrac{1}{2}\hat{b}$

$\Rightarrow (\hat{a} \cdot \hat{c})\hat{b} - \left(\hat{a} \cdot \hat{b}\right)\hat{c} = \dfrac{1}{2}\hat{b}$

$\Rightarrow \left\{(\hat{a} \cdot \hat{c}) - \dfrac{1}{2}\right\}\hat{b} - \left(\hat{a} \cdot \hat{b}\right)\hat{c} = 0$

$\Rightarrow \hat{a} \cdot \hat{c} - \dfrac{1}{2} = 0 \text{ and } \hat{a} \cdot \hat{b} = 0 \ \begin{bmatrix} \because \hat{b}, \hat{c} \\ \text{are non} - \text{collinear vectors} \end{bmatrix}$

$\Rightarrow \cos\theta = \frac{1}{2}$, where θ is the angle between $\hat{a}$ and $\hat{c}$

$\Rightarrow \theta = \pi/3$

14 **(b)**

The given line is parallel to the vector $\vec{\mathbf{n}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. The required plane passing through the point $(2, 3, 1)$ *ie*, $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and is perpendicular to the vector $\vec{\mathbf{n}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$ Its equation is

$[\left(\vec{\mathbf{r}} - \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)\right] \cdot \left(\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right) = 0$

$\Longrightarrow \vec{\mathbf{r}} \cdot \left(\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right) = 1$

15 **(c)**

$\left(\vec{\mathbf{a}} - \vec{\mathbf{b}}\right) \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}} - \vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}})$

$= \vec{\mathbf{a}} \cdot \left(\vec{\mathbf{b}} \times \vec{\mathbf{c}}\right) - \vec{\mathbf{b}} \times (\vec{\mathbf{c}} \times \vec{\mathbf{a}})$

$= \left[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\right] - \left[\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,\vec{\mathbf{a}}\right] = 0$

16 **(a)**

We have,

$|\hat{n}_1 + \hat{n}_2|^2 = |\hat{n}_1| + |\hat{n}_2| + 2\hat{n}_1 \cdot \hat{n}_2$

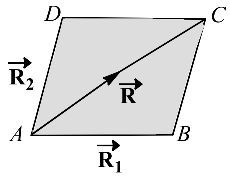
$\Rightarrow |\hat{n}_1 + \hat{n}_2|^2 = |\hat{n}_1|^2 + |\hat{n}_2|^2 + 2|\hat{n}_1| + |\hat{n}_2| \cos\theta$

$\Rightarrow |\hat{n}_1 + \hat{n}_2|^2 = 1 + 1 + 2\cos\theta = 4\cos^2\dfrac{\theta}{2}$

$\therefore \cos\dfrac{\theta}{2} = \dfrac{1}{2}|\hat{n}_1 + \hat{n}_2|$

17 **(d)**

Let $\vec{\mathbf{R}}_1 = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$



and $\vec{\mathbf{R}}_2 = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\therefore \vec{\mathbf{R}}$ (along $\overrightarrow{\mathbf{AC}}$) $= \vec{\mathbf{R}}_1 + \vec{\mathbf{R}}_2$

$= 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\therefore \vec{\mathbf{a}}$ (unit vector along AC) $= \dfrac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|}$

$= \dfrac{3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{\sqrt{9 + 36 + 4}}$

$= \dfrac{1}{7}(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

18 **(a)**

Let $P(60\hat{\mathbf{i}} + 3\hat{\mathbf{j}}), Q(40\hat{\mathbf{i}} - 8\hat{\mathbf{j}})$and $R(a\hat{\mathbf{i}} - 52\hat{\mathbf{j}})$ be the collinear points. Then $\overrightarrow{\mathbf{PQ}} = \lambda\overrightarrow{\mathbf{QR}}$ for some scalar λ

$\Longrightarrow (-20\hat{\mathbf{i}} - 11\hat{\mathbf{j}}) = \lambda[(a - 40)\hat{\mathbf{i}} - 44\hat{\mathbf{j}}]$

$\Longrightarrow \lambda(a - 40) = -20, -44\lambda = -11$

$\Longrightarrow \lambda(a - 40) = -20, \lambda = \dfrac{1}{4}$

$\therefore a - 40 = -20 \times 4 \Longrightarrow a = -40$

19 **(a)**

We have,

$\vec{a} + \vec{b} + \vec{c} = \alpha\vec{d}$ and $\vec{b} + \vec{c} + \vec{d} = \beta\vec{a}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} + \vec{d} = (\alpha + 1)\vec{d}$ and $\vec{a} + \vec{b} + \vec{c} + \vec{d} = (\beta + 1)\vec{a}$

$\Rightarrow (\alpha + 1)\vec{d} = (\beta + 1)\vec{a}$

If $\alpha \neq -1$, then

$(\alpha + 1)\vec{d} = (\beta + 1)\vec{a} \Rightarrow \vec{d} = \dfrac{\beta + 1}{\alpha + 1}\vec{a}$

$\therefore \vec{a} + \vec{b} + \vec{c} = \alpha\vec{d}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = \alpha\left(\dfrac{\beta + 1}{\alpha + 1}\right)\vec{a}$

$\Rightarrow \left\{1-\frac{\alpha(\beta+1)}{\alpha+1}\right\}\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{a},\vec{b},\vec{c}$ are coplanar

It is a contradiction to the given condition

$\therefore \alpha=-1 \Rightarrow \vec{a}+\vec{b}+\vec{c}=0$

20 **(c)**

Let the unit vector $\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$ is perpendicular to $\hat{\mathbf{i}}-\hat{\mathbf{j}}$, then we get

$\frac{(\hat{\mathbf{i}}+\hat{\mathbf{j}})\cdot(\hat{\mathbf{i}}-\hat{\mathbf{j}})}{\sqrt{2}}=\frac{1-1}{\sqrt{2}}=0$

$\therefore \frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$ is the unit vector

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	C	C	B	A	D	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	C	B	C	A	D	A	A	C

Topic :-VECTOR ALGEBRA

1 **(c)**

We have,

$$\left.\begin{array}{l}\vec{r}\cdot\vec{a}=0\Rightarrow\vec{r}\perp\vec{a}\\ \vec{r}\cdot\vec{b}=0\Rightarrow\vec{r}\perp\vec{b}\\ \vec{r}\cdot\vec{c}=0\Rightarrow\vec{r}\perp\vec{c}\end{array}\right\}\Rightarrow\vec{a},\vec{b},\vec{c}\text{ are coplanar}$$

Hence, $[\vec{a}\vec{b}\vec{c}]=0$

2 **(b)**

$$\cos\frac{\pi}{3}=\frac{(\hat{\mathbf{i}}+\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}}+\hat{\mathbf{j}}+a\hat{\mathbf{k}})}{\sqrt{2}\sqrt{1+1+a^2}}$$

$$\Rightarrow\frac{1}{2}=\frac{1+a}{\sqrt{2}\sqrt{2+a^2}}$$

$$\Rightarrow\frac{1}{4}=\frac{(1+a)^2}{2(2+a^2)}$$

$\Rightarrow 2+a^2=2(1+a^2+2a)$

$\Rightarrow a^2+4a=0$

$\Rightarrow a=0,-4$

3 **(b)**

Let the required vector be $\vec{a}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$

It makes equal angles with the unit vectors

$\vec{b}=\frac{1}{3}(\hat{\imath}-2\hat{\jmath}+2\hat{k}),\vec{c}=\frac{1}{5}(-4\hat{\imath}-3\hat{k})$ and $\vec{d}=\hat{\jmath}$

$\therefore\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=\vec{a}\cdot\vec{d}$ $[\because\vec{b},\vec{c},\vec{d}$ are unit vectors$]$

$$\Rightarrow\frac{1}{3}(x-2y+2z)=\frac{1}{5}(-4x-3z)=y$$

$\Rightarrow x-2y+2z=3y$ and $-4x-5y-3z=0$

$\Rightarrow x-5y+2z=0$ and $4x+5y+3z=0$

$$\Rightarrow\frac{x}{-5}=\frac{y}{1}=\frac{z}{5}=\lambda\text{ (say)}$$

$\Rightarrow x=-5\lambda,y=\lambda,z=5\lambda$ for some scalar λ

$\Rightarrow\vec{a}=\lambda(-5\hat{\imath}+\hat{\jmath}+5\hat{k})$

Clearly, option (b) is true for $\lambda=1$

4 **(d)**

$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{vmatrix}$

$= \hat{\mathbf{i}}(4+2) - \hat{\mathbf{j}}(4-1) + \hat{\mathbf{k}}(-4-2)$

$= 6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$

$\Longrightarrow |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = \sqrt{36+9+36} = \sqrt{81} = 9$

$\therefore$ Required vectors are

$\pm 6 \left| \frac{\vec{\mathbf{a}} \times \vec{\mathbf{b}}}{|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|} \right|$

$= \pm \frac{6}{9}(6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}})$

$= \pm 2(2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

6 **(d)**

(a) Let $\vec{\mathbf{p}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ where at least one of x, y, z is non-zero. Let

$\vec{\mathbf{a}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{l}} + a_3\hat{\mathbf{k}}$

$\vec{\mathbf{b}} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{l}} + b_3\hat{\mathbf{k}}$

$\vec{\mathbf{c}} = c_1\hat{\mathbf{i}} + c_2\hat{\mathbf{l}} + c_3\hat{\mathbf{k}}$

$\therefore$ By given conditions

$a_1x + a_2y + a_3z = 0$

$b_1x + b_2y + b_3z = 0$

$c_1x + c_2y + c_3z = 0$

$\Rightarrow \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_3 & c_3 \end{vmatrix} = 0$

$\Rightarrow [\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,] = 0$

$\Rightarrow$ $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar.

(b) Vectors are coplanar, if

$\begin{vmatrix} 1 & 3 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$

$ie, -7 = 0$

Which is not possible.

(c) $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})\vec{\mathbf{b}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{c}}$

$\Rightarrow$ $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ is coplanar with $\vec{\mathrm{b}}$ and $\vec{\mathrm{c}}$.

(d) $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = 1$

$\therefore |\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = (\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}})$

$= |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$

$= 1 + 1 = 2 \cdot 1 \cdot 1 \cos\frac{\pi}{3} = 3$

$\Rightarrow$ $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = \sqrt{3} > 1$

7 **(d)**

Here, $\vec{\mathbf{a}}_1 = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}}, \quad \vec{\mathbf{a}}_2 = -2\hat{\mathbf{i}} + 7\hat{\mathbf{k}}$
$\vec{\mathbf{b}}_1 = -4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}_2 = -4\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$
Now, $\vec{\mathbf{a}}_2 - \vec{\mathbf{a}}_1 = \hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$
and

$$\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -4 & 3 & 2 \\ -4 & 1 & 1 \end{vmatrix} = \hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

$$\Rightarrow \left|\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2\right| = \sqrt{1 + 16 + 64} = 9$$

Now,

$$(\vec{\mathbf{a}}_2 - \vec{\mathbf{a}}_1) \cdot (\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2) = (\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 8\hat{\mathbf{k}})$$
$$= 1 + 24 + 56 = 81$$

∴ Shortest distance,

$$d = \left|\frac{(\vec{\mathbf{a}}_2 - \vec{\mathbf{a}}_1) \cdot (\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2)}{\left|\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2\right|}\right|$$
$$= \frac{81}{9} = 9 \text{ unit}$$

8 **(b)**

We know that a vector perpendicular to the plane containing the points $\vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{C}}$ is given by $\vec{\mathbf{A}} \times \vec{\mathbf{B}} + \vec{\mathbf{B}} \times \vec{\mathbf{C}} + \vec{\mathbf{C}} \times \vec{\mathbf{A}}$.
Given, $\vec{\mathbf{A}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}, \vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \hat{\mathbf{k}}$
and $\vec{\mathbf{C}} = 0\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$
Now,

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} = \hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} \times \vec{\mathbf{C}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 0 & -1 \\ 0 & 2 & 1 \end{vmatrix} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

$$\vec{\mathbf{C}} \times \vec{\mathbf{A}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 5\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

Thus,

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} + \vec{\mathbf{B}} \times \vec{\mathbf{C}} + \vec{\mathbf{C}} \times \vec{\mathbf{A}}$$
$$= (\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) + (5\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$
$$= 8\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

9 **(c)**

Given,

$$(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) = \frac{1}{4}$$
$$\Rightarrow (|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sin\theta)^2 = \frac{1}{4}$$
$$\Rightarrow \sin^2\theta = \frac{1}{4}$$

$\Longrightarrow\ \theta=\frac{\pi}{6}$

10 **(b)**

Given that, $|\vec{\mathbf{a}}|=3, |\vec{\mathbf{b}}|=4$ and $\vec{\mathbf{a}}+\lambda\vec{\mathbf{b}}$ is perpendicular to $\vec{\mathbf{a}}-\lambda\vec{\mathbf{b}}$.

$\therefore\ (\vec{\mathbf{a}}+\lambda\vec{\mathbf{b}})\cdot(\vec{\mathbf{a}}-\lambda\vec{\mathbf{b}})=0$

$\Rightarrow \vec{\mathbf{a}}\cdot\vec{\mathbf{a}}-\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}\lambda+\lambda\vec{\mathbf{b}}\cdot\vec{\mathbf{a}}-\lambda^2\vec{\mathbf{b}}\cdot\vec{\mathbf{b}}=0$

$\Rightarrow |\vec{\mathbf{a}}|^2-\lambda^2|\vec{\mathbf{b}}|^2=0$

$\Rightarrow\ \lambda^2=\frac{|\vec{\mathbf{a}}|^2}{|\vec{\mathbf{b}}|^2}\ \Rightarrow\ \lambda=\frac{|\vec{\mathbf{a}}|}{|\vec{\mathbf{b}}|}=\frac{3}{4}$

11 **(a)**

$(\vec{\mathbf{x}}-\vec{\mathbf{y}})\times(\vec{\mathbf{x}}+\vec{\mathbf{y}})$

$=\vec{\mathbf{x}}\times\vec{\mathbf{x}}+\vec{\mathbf{x}}\times\vec{\mathbf{y}}-\vec{\mathbf{y}}\times\vec{\mathbf{x}}-\vec{\mathbf{y}}\times\vec{\mathbf{y}}$

$=\vec{\mathbf{0}}+\vec{\mathbf{x}}\times\vec{\mathbf{y}}+\vec{\mathbf{x}}\times\vec{\mathbf{y}}-\vec{\mathbf{0}}$

$=2(\vec{\mathbf{x}}\times\vec{\mathbf{y}})$

12 **(a)**

$\vec{\mathbf{a}}\cdot\vec{\mathbf{c}}=0$

$\Longrightarrow \lambda-1+2\mu=0$

$\Longrightarrow \lambda+2\mu=1$ ...(i)

$\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}=0$

$\Longrightarrow 2\lambda+4+\mu=0$

$\Longrightarrow 2\lambda+\mu=-4$....(ii)

On solving Eqs. (i) and (ii), we get

$\lambda-3, \mu=2$

15 **(b)**

The projection $\vec{x}\times\vec{y}$ on $\vec{z}$ is given by

$$\frac{(\vec{x}\times\vec{y})\cdot\vec{z}}{|\vec{z}|}=\frac{1}{|\vec{z}|}[\vec{x}\ \vec{y}\ \vec{z}]=\frac{1}{13}\begin{vmatrix}3 & -6 & -1\\ 1 & 4 & -3\\ 3 & -4 & -12\end{vmatrix}=-14$$

16 **(c)**

We have,

$\vec{a}\times\left\{\vec{a}\times\{\vec{a}\times(\vec{a}\times\vec{b})\}\right\}$

$=\vec{a}\times\left\{\vec{a}\times\{(\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}\}\right\}=\vec{a}\times\{\vec{0}-|\vec{a}|^2(\vec{a}\times\vec{b})\}$

$=-|\vec{a}|^2\{\vec{a}\times(\vec{a}\times\vec{b})\}=-|\vec{a}|^2\{(\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}\}$

$=-|\vec{a}|^2\{0-|\vec{a}|^2\vec{b}\}=|\vec{a}|^4\vec{b}$

19 **(c)**

For an abtuse angle

$(cx\hat{\mathbf{i}}-6\hat{\mathbf{j}}+3\hat{\mathbf{k}})\cdot(x\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2cx\hat{\mathbf{k}})<0$

$\Longrightarrow\ cx^2-12+6cx<0$

$\Longrightarrow cx^2+6cx-12<0$

$\therefore (6c)^2-4c(-12)<0\quad [\because f(x)<0\Longrightarrow D<0]$

$\Rightarrow 36c\left(c+\frac{4}{3}\right)<0$

$\Rightarrow -\frac{4}{3}<c<0$

20 **(a)**

$\cos\theta=\frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|}$

$=\frac{(2\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}})\cdot(6\hat{\mathbf{i}}-3\hat{\mathbf{j}}+2\hat{\mathbf{k}})}{\sqrt{2^2+2^2+(-1)^2}\sqrt{6^2+(-3)^2+2^2}}$

$=\frac{12-6-2}{\sqrt{4+4+1}\sqrt{36+9+4}}=\frac{4}{21}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	B	D	C	D	D	B	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	C	C	B	C	A	B	C	A

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :5

Topic :-VECTOR ALGEBRA

1 **(b)**

Given vectors $2\hat{\imath}+3\hat{\jmath}-4\hat{k}$ and $a\hat{\imath}+b\hat{\jmath}+c\hat{k}$ will be perpendicular, if

$(2\hat{\imath}+3\hat{\jmath}-4\hat{k}).(a\hat{\imath}+b\hat{\jmath}+c\hat{k})=0 \Rightarrow 2a+3b-c=0$

Clearly, $a=4, b=4, c=5$ satisfy the above equation

2 **(a)**

We have ,$\vec{\alpha}=x(\vec{\mathbf{a}}\times\vec{\mathbf{b}})+y(\vec{\mathbf{b}}\times\vec{\mathbf{c}})+z(\vec{\mathbf{c}}\times\vec{\mathbf{a}})$

Taking dot product with $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ respectively, we get

$\vec{\alpha}\cdot\vec{\mathbf{a}}=y[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] \implies y=8(\vec{\alpha}\cdot\vec{\mathbf{a}})$

$\vec{\alpha}\cdot\vec{\mathbf{b}}=z\left((\vec{\mathbf{c}}\times\vec{\mathbf{a}})\cdot\vec{\mathbf{b}}\right)$

$\implies \vec{\alpha}\cdot\vec{\mathbf{b}}=z[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] \implies z=8(\vec{\alpha}\cdot\vec{\mathbf{b}})$

and $\vec{\alpha}\cdot\vec{\mathbf{c}}=x(\vec{\mathbf{a}}\times\vec{\mathbf{b}}.\vec{\mathbf{c}})$

$\vec{\alpha}\cdot\vec{\mathbf{c}}=x[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] \implies x=8(\vec{\alpha}\cdot\vec{\mathbf{c}})$

$\therefore x+y+z=8\vec{\alpha}\cdot(\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}})$

3 **(d)**

Let $\vec{\mathbf{c}}=3\hat{\mathbf{i}}+\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ and $\vec{\mathbf{d}}=a\hat{\mathbf{i}}+b\hat{\mathbf{j}}-15\hat{\mathbf{k}}$

For collinears, $\vec{\mathbf{c}}\times\vec{\mathbf{d}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 3 & 1 & -5\\ a & b & -15\end{vmatrix}=\vec{\mathbf{0}}$

$\implies \hat{\mathbf{i}}(-15+5b)-\hat{\mathbf{j}}(-45+5a)+\hat{\mathbf{k}}(3b-a)=\vec{\mathbf{0}}$

$\implies -15+5b=0, \quad -45+5a=0, \quad 3b-a=0$

$\implies b=3, a=9$

4 **(d)**

$|\vec{\mathbf{a}}-\vec{\mathbf{b}}|^2=|\vec{\mathbf{a}}|^2+|\vec{\mathbf{b}}|^2-2|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$

$=1^2+1^2 2\cdot1\cdot1\cdot\cos 60° \quad [\because |\vec{\mathbf{a}}|=|\vec{\mathbf{b}}|=1]$

$=2-2\cdot\frac{1}{2}=1$

5 **(c)**

Let $\vec{\mathbf{a}}=\hat{\mathbf{i}}-2\hat{\mathbf{j}}-3\hat{\mathbf{k}}, \ \vec{\mathbf{b}}=2\hat{\mathbf{i}}+0\hat{\mathbf{j}}+0\hat{\mathbf{k}}$

Now take option (c).

Let $\vec{\mathbf{c}} = 0\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$

Now, $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \begin{vmatrix} 1 & -2 & -3 \\ 2 & 0 & 0 \\ 0 & -4 & -6 \end{vmatrix}$

$= 1(0) + 2(-12) - 3(-8) = 0$

6 **(a)**

$(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})$

$= \vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + \vec{\mathbf{b}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})$

$= (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}} + (\vec{\mathbf{b}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{b}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}}$

$= (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - \vec{\mathbf{b}} + \vec{\mathbf{a}} - (\vec{\mathbf{b}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}}$

$= (\vec{\mathbf{a}} - \vec{\mathbf{b}})(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} - 1)$

$\therefore$ Given vector is parallel to $(\vec{\mathbf{a}} - \vec{\mathbf{b}})$.

7 **(a)**

$\overrightarrow{\mathbf{AB}} = (2-1)\hat{\mathbf{i}} + (0-2)\hat{\mathbf{j}} + (3+1)\hat{\mathbf{k}}$

$= \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

and

$\overrightarrow{\mathbf{AC}} = (3-1)\hat{\mathbf{i}} + (-1-2)\hat{\mathbf{j}} + (2+1)\hat{\mathbf{k}}$

$= 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\cos\theta = \dfrac{(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}})}{\sqrt{1+4+16}\sqrt{4+9+9}}$

$= \dfrac{2+6+12}{\sqrt{21}\sqrt{22}} = \dfrac{20}{\sqrt{462}}$

$\Rightarrow \sqrt{462}\cos\theta = 20$

8 **(c)**

$[\,\vec{\mathbf{u}}\,\vec{\mathbf{v}}\,\vec{\mathbf{w}}\,] = |\vec{\mathbf{u}} \cdot (\vec{\mathbf{v}} \times \vec{\mathbf{w}})|$

$= |\vec{\mathbf{u}} \cdot (3\hat{\mathbf{i}} - 7\hat{\mathbf{j}} - \hat{\mathbf{k}})|$

$= |\,\vec{\mathbf{u}}\,|\sqrt{59}\cos\theta$

$\therefore$ Maximum value of $[\,\vec{\mathbf{u}}\,\vec{\mathbf{v}}\,\vec{\mathbf{w}}\,] = \sqrt{59}$ $(\because |\vec{\mathbf{u}}| = 1, \cos\theta \le 1)$

10 **(b)**

Given, force $= 5\left(\frac{2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}}{|2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}|}\right) = \frac{5}{3}\,(2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$

Displacement $= (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) - (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$= (4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

$\therefore$ Required work done = Force $\cdot$ Displacement

$= \frac{5}{3}[(2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\hat{\mathbf{k}})]$

$= \frac{5}{3}[\,8 - 2 + 4] = \frac{50}{3}$ unit

11 **(b)**

We know that the equation of the plane passing through three non-collinear points $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ is

$\vec{\mathbf{r}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}}) = [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

12 **(a)**

We have,

Required vector $\vec{r} = \lambda(\hat{a} + \hat{b}), \lambda$ is a scalar

$\Rightarrow \vec{r} = \lambda\left\{\frac{1}{9}(7\hat{\imath} - 4\hat{\jmath} - 4\hat{k}) + \frac{1}{3}(-2\hat{\imath} - \hat{\jmath} + 2\hat{k})\right\} = \frac{\lambda}{9}(\hat{\imath} - 7\hat{\jmath} + 2\hat{k})$

Now,

$|\vec{r}| = 3\sqrt{6} \Rightarrow |\vec{r}|^2 = 54 \Rightarrow \frac{\lambda^2}{81}(1 + 49 + 4) = 54 \Rightarrow \lambda = \pm 9$

Hence, required vector $\vec{r} = \pm(\hat{\imath} - 7\hat{\jmath} + 2\hat{k})$

Clearly, option (a) is true for $\lambda = 1$

13 **(b)**

Given vectors are collinear, if $\begin{vmatrix} 2 & 1 & 1 \\ 6 & -1 & 2 \\ 14 & -5 & p \end{vmatrix} = 0$

$\Longrightarrow 2[-p + 10] - 1[6p - 28] + 1[-30 + 14] = 0$

$\Longrightarrow -8p + 32 = 0$

$\Longrightarrow p = 4$

14 **(d)**

Given,

$\frac{1}{3}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}} = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}}$

$\therefore \frac{1}{3}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}} = (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})\vec{\mathbf{b}} - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})\vec{\mathbf{a}}$

On comparing the coefficient of $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, we get

$\frac{1}{2}|\vec{\mathbf{b}}||\vec{\mathbf{c}}| = -\vec{\mathbf{b}} \cdot \vec{\mathbf{c}}$ and $\vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = 0$

$\Longrightarrow \frac{1}{3}|\vec{\mathbf{b}}||\vec{\mathbf{c}}| = -|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\cos\theta \Longrightarrow \cos\theta = -\frac{1}{3}$

$\Longrightarrow 1 - \sin^2\theta = \frac{1}{9} \Longrightarrow \sin\theta = \frac{2\sqrt{2}}{3}$

15 **(c)**

Let $\vec{\mathbf{A}} = 7\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$, $\vec{\mathbf{B}} = -\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and $\vec{\mathbf{C}} = -4\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

Now, $\overrightarrow{\mathbf{AB}} = -\hat{\mathbf{i}} - \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, $\quad \overrightarrow{\mathbf{BC}} = -3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

and $\overrightarrow{\mathbf{CA}} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

Here, $|\overrightarrow{\mathbf{AB}}| = |\overrightarrow{\mathbf{BC}}| = 3\sqrt{2}$ and $|\overrightarrow{\mathbf{CA}}| = 6$

Now, $|\overrightarrow{\mathbf{AB}}|^2 + |\overrightarrow{\mathbf{BC}}|^2 = |\overrightarrow{\mathbf{AC}}|^2$

Hence, the triangle is right angled isosceles triangle.

16 **(b)**

We know that if A and B are two points and P is any point on AB. Then,

$m\,P\vec{A} + n\,P\vec{B} = (m + n)P\vec{C}$, where C divides AB in the ratio $n:m$

Here, $m = n = 1$

$\therefore P\vec{A} + P\vec{B} = 2P\vec{C}$

17 **(a)**

$(2\vec{\mathbf{a}} + 3\vec{\mathbf{b}}) \times (5\vec{\mathbf{a}} + 7\vec{\mathbf{b}}) + \vec{\mathbf{a}} \times \vec{\mathbf{b}}$

$= \vec{\mathbf{0}} + 14(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) - 15(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + \vec{\mathbf{0}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}}$

$= \vec{\mathbf{0}}$

19 **(c)**

Let $\overrightarrow{\mathbf{OA}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}, \overrightarrow{\mathbf{OB}} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

and $\overrightarrow{\mathbf{OC}} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

$\therefore a = |\overrightarrow{\mathbf{OA}}| = \sqrt{6}, b = |\overrightarrow{\mathbf{OB}}| = \sqrt{35}$

and $\vec{\mathbf{c}}|\overrightarrow{\mathbf{OC}}| = \sqrt{41}$

$\therefore \cos A = \dfrac{b^2 + c^2 + a^2}{2bc}$

$= \dfrac{(\sqrt{35})^2 + (\sqrt{41})^2 - (\sqrt{6})^2}{2\sqrt{35}\sqrt{41}}$

$\Rightarrow \cos A = \sqrt{\dfrac{35}{41}}$

$\Rightarrow \sin^2 A = \dfrac{35}{41}$

20 **(d)**

Let $\vec{p} \neq \vec{0}$. Then,

$\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar, which is a contradiction

Hence, $\vec{r} = \vec{0}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	D	D	C	A	A	C	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	B	D	C	B	A	A	C	D

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :6

Topic :-VECTOR ALGEBRA

1 **(c)**

Let $\vec{\alpha} = \lambda \vec{\mathbf{a}} + \mu\vec{\mathbf{b}} + t\vec{\mathbf{c}}$ ...(i)

Now, $\vec{\mathbf{a}} \cdot \vec{\mathbf{p}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{q}} = \vec{\mathbf{c}} \cdot \vec{\mathbf{r}} = 1$

$\Rightarrow \vec{\alpha} \cdot \vec{\mathbf{p}} = \lambda (\vec{\mathbf{a}} \cdot \vec{\mathbf{p}}) + 0 + 0$

$\Rightarrow \lambda = \vec{\alpha} \cdot \vec{\mathbf{p}}$

Similarly, $\mu = \vec{\alpha} \cdot \vec{\mathbf{q}}$

and $t = \vec{\alpha} \cdot \vec{\mathbf{r}}$

From Eq. (i), we get

$\vec{\alpha} = (\vec{\alpha} \cdot \vec{\mathbf{p}})\vec{\mathbf{a}} + (\vec{\alpha} \cdot \vec{\mathbf{q}})\vec{\mathbf{b}} + (\vec{\alpha} \cdot \vec{\mathbf{r}})\vec{\mathbf{c}}$

2 **(a)**

Since, $\vec{\mathbf{b}} \times \vec{\mathbf{c}}$ is a vector perpendicular to $\vec{\mathbf{b}}, \vec{\mathbf{c}}$. Therefore $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ is a vector again in plane of $\vec{\mathbf{b}}, \vec{\mathbf{c}}$.

3 **(c)**

$(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{b}} + \vec{\mathbf{b}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})$

$= (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{b}} + (\vec{\mathbf{b}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{b}}$

$= \vec{\mathbf{a}} \quad [\because |\vec{\mathbf{b}}| = 1]$

4 **(d)**

$$\because \sum_{i=1}^{n} \vec{\mathbf{a}}_i = \vec{\mathbf{0}}$$

$$\therefore \left(\sum_{i=1}^{n} \vec{\mathbf{a}}_i\right)\left(\sum_{i=1}^{n} \vec{\mathbf{a}}_j\right) = \sum_{i=1}^{n} |\vec{\mathbf{a}}_i|^2 + 2 \sum_{1\le i<} \sum_{j\le n} \vec{\mathbf{a}}_i \cdot \vec{\mathbf{a}}_j$$

$$\Rightarrow 0 = n + 2 \sum_{1\le i<} \sum_{j\le n} \vec{\mathbf{a}}_i \cdot \vec{\mathbf{a}}_j$$

$$\therefore \sum_{1\le i<} \sum_{j\le n} \vec{\mathbf{a}}_i \cdot \vec{\mathbf{a}}_j = -\frac{n}{2}$$

5 **(b)**

Since, given vectors are perpendicular.

$\therefore (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}}) \cdot (6\hat{\mathbf{i}} - \hat{\mathbf{j}} + c\hat{\mathbf{k}}) = 0$

$\Longrightarrow 18 + 2 - 5c = 0 \Longrightarrow c = 4$

6 **(d)**

Given, $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{0}}$ and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$

$\Rightarrow$ $\vec{\mathbf{a}}$ is parallel to $\vec{\mathbf{b}}$ and $\vec{\mathbf{a}}$ is perpendicular to $\vec{\mathbf{b}}$ which is possible only if $\vec{\mathbf{a}} = \vec{\mathbf{0}}$ or $\vec{\mathbf{b}} = \vec{\mathbf{0}}$

7 **(a)**

Let $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}},\ \vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

First diagonal, $\vec{\mathbf{a}} + \vec{\mathbf{b}} = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\Rightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}}| = 7$

Second diagonal, $\vec{\mathbf{a}} - \vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

$\Rightarrow |\vec{\mathbf{a}} - \vec{\mathbf{b}}| = \sqrt{69}$

8 **(b)**

Given $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$

$\Rightarrow$ $\vec{\mathbf{a}} \times \vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{a}} \times \vec{\mathbf{c}} = 0$

$\Rightarrow$ $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}}$

Similarly, $\vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}}$

$\therefore$ $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}} \neq \vec{\mathbf{0}}$

Alternate: Since, $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are unit vectors and $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{a}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$, so $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ represent an equilateral triangle.

$\therefore$ $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}} \neq \vec{\mathbf{0}}$

9 **(c)**

We have,

$\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$

$= \vec{ED} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{CD}$ $[\because \vec{AB} = \vec{ED}$ and $\vec{AF} = \vec{CD}]$

$= (\vec{AC} + \vec{CD}) + (\vec{AE} + \vec{ED}) + \vec{AD}$

$= 3\vec{AD} = 6\vec{AG}$ $[\because \vec{AD} = 2\vec{AG}]$

10 **(c)**

I. It is true that non-zero, non-collinear vectors are linearly independent.

II. It is also true that any three coplanar vectors are linearly dependent.

$\therefore$ Both I and II are true.

11 **(a)**

Let $\vec{\alpha} = 2\vec{a} - 3\vec{b}, \vec{\beta} = 7\vec{b} - 9\vec{c}$ and $\vec{\gamma} = 12\vec{c} - 23\vec{a}$

Then,

$$[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = \begin{vmatrix} 2 & -3 & 0 \\ 0 & 7 & -9 \\ -23 & 0 & 12 \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}]$$

$\Rightarrow [\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = (168 + 3 \times -207)[\vec{a}\ \vec{b}\ \vec{c}]$

$\Rightarrow [\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = 0$ $[\because [\vec{a}\ \vec{b}\ \vec{c}] = 0]$

$\Rightarrow \vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are coplanar vectors

12 **(b)**

Given, $[\vec{\mathbf{a}} + \vec{\mathbf{b}}\ \vec{\mathbf{b}} + \vec{\mathbf{c}}\ \vec{\mathbf{c}} + \vec{\mathbf{a}}] = [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]$

$\Rightarrow 2[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}] = [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]$

$= [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] = 0$

Hence, $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are coplanar.

13 **(c)**

Given, $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$ and $|\vec{\mathbf{a}}| = \sqrt{37}, |\vec{\mathbf{b}}| = 3,$ and $|\vec{\mathbf{c}}| = 4$

Now, $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$

$\Rightarrow \vec{\mathbf{a}} = -(\vec{\mathbf{b}} + \vec{\mathbf{c}})$

$\Rightarrow |\vec{\mathbf{a}}|^2 = |-(\vec{\mathbf{b}} + \vec{\mathbf{c}})|^2$

$\Rightarrow |\vec{\mathbf{a}}|^2 = |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2|\vec{\mathbf{b}}||\vec{\mathbf{c}}| \cos\theta$

$= 9 + 16 + 24\cos\theta$

$\Rightarrow 37 = 25 + 24\cos\theta$

$\Rightarrow 24\cos\theta = 12 \Rightarrow \theta = 60°$

14 **(a)**

Let unit vector be $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$

$\therefore$ $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ is perpendicular to $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$,

Then $a + b + c = 0$ .....(i)

Since, $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}, (\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}), (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$ are coplanar

$$\therefore \begin{vmatrix} a & b & c \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Longrightarrow -3a + b + c = 0$ ....(ii)

From Eqs. (i) and (ii), we get

$a = 0$ and $c = -b$

Also, $a^2 + b^2 + c^2 = 1$

$\Longrightarrow 0 + b^2 + b^2 = 1$

$$\Longrightarrow b = \frac{1}{\sqrt{2}}$$

$$\therefore a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}} = \frac{1}{\sqrt{2}}\hat{\mathbf{j}} - \frac{1}{\sqrt{2}}\hat{\mathbf{k}}$$

16 **(b)**

Given, $\overrightarrow{\mathbf{OA}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\overrightarrow{\mathbf{OB}} = 5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

and $\overrightarrow{\mathbf{OC}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

volume of parallelopiped

$= [\overrightarrow{\mathbf{OA}}\,\overrightarrow{\mathbf{OB}}\,\overrightarrow{\mathbf{OC}}]$

$$= \begin{vmatrix} 2 & -2 & 1 \\ 5 & -4 & 4 \\ 1 & -2 & 4 \end{vmatrix}$$

$= 2(-16 + 8) + 2(20 - 4) + 1(-10 + 4)$

$= 10$ cu units

18 **(a)**

We have,

$$\vec{a} = \lambda(\vec{b} \times \vec{c}) = \lambda \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ -2 & 4 & 1 \end{vmatrix} = \lambda(-10\hat{\imath} - 7\hat{k} + 8\hat{k})$$

Now,

$\vec{a} \cdot (\hat{\imath} - 2\hat{\jmath} + \hat{k}) = -6$

$\Rightarrow \lambda(-10 + 14 + 8) = -6 \Rightarrow \lambda = -\frac{1}{2}$

Hence, $\vec{a} = -\frac{1}{2}(-10\hat{\imath} - 7\hat{k} + 8\hat{k}) = 5\hat{\imath} + \frac{7}{2}\hat{\jmath} - 4\hat{k}$

19 **(c)**

The projection of

$\vec{\mathbf{a}}$ on $\vec{\mathbf{b}} = \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{b}}|}$

$= \frac{(3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})}{\sqrt{2^2 + 3^2 + 1^2}} = \frac{8}{\sqrt{14}}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	C	D	B	D	A	B	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	C	A	A	B	A	A	C	B

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :7

Topic :-VECTOR ALGEBRA

1 **(d)**

$$\begin{vmatrix} 7 & -11 & 1 \\ 5 & 3 & -2 \\ 12 & -8 & -1 \end{vmatrix}$$

$= 7(-3-16)+11(-5+24)+1(-40-36)$

$= -133+209-76=0$

$\therefore$ Vector are collinear.

2 **(c)**

Let the position vectors of the points A, B, C are $\vec{\mathbf{0}}, \vec{\mathbf{a}}+\vec{\mathbf{b}}, \vec{\mathbf{a}}-\vec{\mathbf{b}}$ respectively and θ= 90°

$\therefore$ Area of triangle $= \frac{1}{2}|\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}}|$

$= \frac{1}{2}|(\vec{\mathbf{a}}+\vec{\mathbf{b}}) \times (\vec{\mathbf{a}}-\vec{\mathbf{b}})|$

$= \frac{1}{2}|2\vec{\mathbf{b}} \times \vec{\mathbf{a}}|$

$= |\vec{\mathbf{b}}|\,|\vec{\mathbf{a}}| \sin\theta = 3 \times 2 \sin 90° = 6$

3 **(a)**

We have, $|[\vec{a}\,\vec{b}\,\vec{c}]| = V$

Let V_1 be the volume of the parallelopiped formed by the vectors $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$. Then,

$V_1 = |[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]|$

Now,

$$[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix} [\vec{a}\,\vec{b}\,\vec{c}]$$

$\Rightarrow [\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = [\vec{a}\,\vec{b}\,\vec{c}]^2 [\vec{a}\vec{b}\,\vec{c}]$

$\Rightarrow [\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = [\vec{a}\,\vec{b}\,\vec{c}]^3$

$\therefore V_1 = |[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]| = \left|[\vec{a}\,\vec{b}\,\vec{c}]^3\right| = V^3$

4 **(a)**

Let l, m, n be the direction cosines of the required vector. As it makes equal angles with X and Y axes

$\therefore l = m$

$\therefore$ Required vector $\vec{r} = l\hat{\imath} + m\hat{\jmath} + n\hat{k} = l\hat{\imath} + l\hat{\jmath} + n\hat{k}$

Now, $l^2 + m^2 + n^2 = 1 \Rightarrow 2l^2 + n^2 = 1$ ...(i)

Since, $\vec{r}$ is perpendicular to $-\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

$\therefore \vec{r} \cdot \left(-\hat{\imath} + 2\hat{\jmath} + 2\hat{k}\right) = 0 \Rightarrow -l + 2l + 2n = 0 \Rightarrow l + 2n = 0$ ....(ii)

From (i) and (ii), we get $n \mp \frac{1}{3}, l = \mp \frac{2}{3}$

Hence, $\vec{r} = \frac{1}{3}\left(\pm 2\hat{\imath} \pm 2\hat{\jmath} \mp \hat{k}\right) = \pm \frac{1}{3}\left(2\hat{\imath} + 2\hat{\jmath} - \hat{k}\right)$

5 **(a)**

Let the required vector be $\vec{a}$. Then, $\hat{\imath} - \hat{\jmath}, \hat{\imath} + \hat{\jmath}$ and $\vec{a}$ form a right handed system

$\therefore \vec{a} = (\hat{\imath} - \hat{\jmath}) \times (\hat{\imath} + \hat{\jmath}) = \hat{k} + \hat{k} = 2\hat{k}$

Hence, the required unit vector $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \hat{k}$

6 **(b)**

$\vec{\mathbf{p}} = x\vec{\mathbf{a}} + y\vec{\mathbf{b}} + z\vec{\mathbf{c}}$

$\Longrightarrow 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}} = x\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right) + y\left(\hat{\mathbf{j}} + \hat{\mathbf{k}}\right) + z\left(\hat{\mathbf{i}} + \hat{\mathbf{k}}\right)$

$\Longrightarrow 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}} = (x + z)\hat{\mathbf{i}} + (x + y)\hat{\mathbf{j}} + (y + z)\hat{\mathbf{k}}$

On comparing both sides the coefficients of $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$, we get

$x + z = 3$ ....(i)

$x + y = 2$ ....(ii)

and $y + z = 4$ (iii)

on solving Eqs. (i), (ii) and (iii), we get

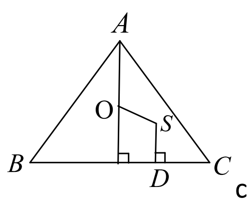
$x = \frac{1}{2}, y = \frac{3}{2}, z = \frac{5}{2}$

7 **(a)**

From geometry

$\overrightarrow{\mathbf{AO}} = 2\overrightarrow{\mathbf{SD}}$

Where D is the mind point of BC



$\therefore \overrightarrow{\mathbf{SA}} + \overrightarrow{\mathbf{SB}} + \overrightarrow{\mathbf{SC}}$

$= \overrightarrow{\mathbf{SA}} + 2\overrightarrow{\mathbf{SD}}$ $(\because \overrightarrow{\mathbf{SB}} + \overrightarrow{\mathbf{SC}} = 2\overrightarrow{\mathbf{SD}})$

$= \overrightarrow{\mathbf{SA}} + \overrightarrow{\mathbf{AO}}$

$= \overrightarrow{\mathbf{SO}}$

8 **(c)**

We have,

$\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$

$\Rightarrow |\vec{a}||\vec{b}| \cos\theta = 0$ and $|\vec{a}||\vec{b}| \sin\theta = 0$

$\Rightarrow \left(|\vec{a}| = 0 \text{ or}, |\vec{b}| = 0 \text{ or}, \cos\theta = 0\right)$

And,

$(|\vec{a}| = 0 \text{ or}, |\vec{b}| = 0 \text{ or}, \sin\theta = 0)$

$\Rightarrow |\vec{a}| = 0 \text{ or}, |\vec{b}| = 0 \quad \begin{bmatrix} \because \cos\theta \text{ and} \sin\theta \\ \text{are not zero zimultaneously} \end{bmatrix}$

10 **(c)**

Given $|\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}} - \vec{\mathbf{b}}|^2$

$\Longrightarrow 4\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0 \Longrightarrow \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$

So, angle between them is 90°

11 **(c)**

We have,

$\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$

$\Rightarrow (\vec{r} - \vec{b}) \times \vec{a} = 0$

$\Rightarrow \vec{r} - \vec{b}$ is parallel to $\vec{a}$

$\Rightarrow \vec{r} - \vec{b} = \lambda\vec{a}$ for some scalar λ

$\Rightarrow \vec{r} - \vec{b} + \lambda\vec{a}$ ....(i)

Now,

$\vec{r} \perp \vec{c}$

$\Rightarrow \vec{r} \cdot \vec{c} \cdot \vec{c} = 0$

$\Rightarrow \vec{b} \cdot \vec{c} + \lambda(\vec{a} \cdot \vec{c}) = 0 \Rightarrow \lambda = -\dfrac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{c}}$

Putting the value of λ in (i), we get

$\vec{r} = \vec{b} - \left(\dfrac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{c}}\right)\vec{a}$

12 **(d)**

We have, $|\vec{\alpha}| = 1 = |\vec{\beta}|$ and $\vec{\alpha} \cdot \vec{\beta} = 0$

Now,

$\vec{\gamma} = x\vec{\alpha} + y\vec{\beta} + z(\vec{\alpha} \times \vec{\beta})$

$\Rightarrow \vec{\alpha} \cdot \vec{\gamma} = x(\vec{\alpha} \cdot \vec{\alpha}) + y(\vec{\alpha} \cdot \vec{\beta}) + z\{\vec{\alpha} \cdot (\vec{\alpha} \times \vec{\beta})\}$

$\vec{\beta} \cdot \vec{\gamma} = x(\vec{\beta} \cdot \vec{\alpha}) + y(\vec{\beta} \cdot \vec{\beta}) + z\{\vec{\beta} \cdot (\vec{\alpha} \times \vec{\beta})\}$

And,

$(\vec{\alpha} \times \vec{\beta}) \cdot \vec{\gamma} = x\{\vec{\alpha} \cdot (\vec{\alpha} \times \vec{\beta}) + y\{\vec{\beta} \cdot (\vec{\alpha} \times \vec{\beta})\} + z\{(\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\alpha} \times \vec{\beta})\}$

$\Rightarrow \cos\theta = x, \cos\theta = y$ and $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = z|\vec{\alpha} \times \vec{\beta}|^2$

$\Rightarrow x = \cos\theta, y = \cos\theta$ and $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = z$

$[\because |\vec{\alpha} \times \vec{\beta}| = |\vec{\alpha}|\,|\vec{\beta}| \sin 90° = 1]$

Now,

$$[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]^2 = \begin{vmatrix} \vec{\alpha} \cdot \vec{\alpha} & \vec{\alpha} \cdot \vec{\beta} & \vec{\alpha} \cdot \vec{\gamma} \\ \vec{\beta} \cdot \vec{\alpha} & \vec{\beta} \cdot \vec{\beta} & \vec{\beta} \cdot \vec{\gamma} \\ \vec{\gamma} \cdot \vec{\alpha} & \vec{\gamma} \cdot \vec{\beta} & \vec{\gamma} \cdot \vec{\gamma} \end{vmatrix}$$

$$\Rightarrow [\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]^2 = \begin{vmatrix} 1 & 0 & \cos\theta \\ 0 & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix} = 1 - 2\cos^2\theta$$

$\Rightarrow z^2 = 1 - 2x^2$
Also, $z^2 = 1 - 2y^2$ and $z^2 = 1 - x^2 - y^2$
13 **(a)**
$(\vec{\mathbf{a}} - \vec{\mathbf{d}}) \times (\vec{\mathbf{b}} - \vec{\mathbf{c}})$
$= \vec{\mathbf{a}} \times \vec{\mathbf{b}} - \vec{\mathbf{a}} \times \vec{\mathbf{c}} - \vec{\mathbf{d}} \times \vec{\mathbf{b}} + \vec{\mathbf{d}} \times \vec{\mathbf{c}}$
$= \vec{\mathbf{c}} \times \vec{\mathbf{d}} - \vec{\mathbf{b}} \times \vec{\mathbf{d}} - \vec{\mathbf{d}} \times \vec{\mathbf{b}} + \vec{\mathbf{d}} \times \vec{\mathbf{c}} = \vec{\mathbf{0}}$
$[\because\ \vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}} \times \vec{\mathbf{d}}, \vec{\mathbf{a}} \times \vec{\mathbf{c}} = \vec{\mathbf{b}} \times \vec{\mathbf{d}}$, given$]$
$\Longrightarrow (\vec{\mathbf{a}} - \vec{\mathbf{d}}) || (\vec{\mathbf{b}} - \vec{\mathbf{c}})$
$\Longrightarrow\ \vec{\mathbf{a}} - \vec{\mathbf{d}} = \lambda(\vec{\mathbf{b}} - \vec{\mathbf{c}})$
14 **(a)**
Since $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar unit vectors
$\therefore [\vec{a}\vec{b}\vec{c}] =$ Volume of a parallelopiped whose each edge is of one unit length
$\Rightarrow [\vec{a}\vec{b}\vec{c}] = \pm 1$
16 **(d)** Let D be the mid-point of BC. Then,
$\vec{A}B + \vec{A}C = 2\vec{A}D$
$\Rightarrow 2\,\vec{A}D = 8\hat{\imath} + 2\hat{\jmath} + 8\hat{k}$
$\Rightarrow \vec{A}D = 4\hat{\imath} + \hat{\jmath} + 4\hat{k}$
$\Rightarrow |\vec{A}D| = \sqrt{16 + 1 + 16} = \sqrt{33}$
17 **(c)**
$\therefore$ Median vector through $\vec{\mathbf{A}} = \frac{1}{2}(\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{AC}})$
$= \frac{1}{2}[(3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) + (5\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}})]$
$= 4\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$
$\therefore$ Length of the median $= \sqrt{4^2 + 3^2} = 5$ units
18 **(d)**
Given, $(\vec{\mathbf{a}} - \lambda\,\vec{\mathbf{b}}) \cdot (\vec{\mathbf{b}} - 2\vec{\mathbf{c}}) \times (\vec{\mathbf{c}} + 2\vec{\mathbf{a}}) = 0$
$\Longrightarrow (\vec{\mathbf{a}} - \lambda\,\vec{\mathbf{b}}) \cdot \{\vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{b}} \times 2\vec{\mathbf{a}} - 4(\vec{\mathbf{c}} \times \vec{\mathbf{a}})\} = 0$
$\Longrightarrow\ \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) + \vec{\mathbf{a}}.(\vec{\mathbf{b}} \times 2\vec{\mathbf{a}}) - \vec{\mathbf{a}}.4(\vec{\mathbf{c}} \times \vec{\mathbf{a}})$
$-\lambda\vec{\mathbf{b}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) - \lambda\vec{\mathbf{b}} \cdot (\vec{\mathbf{b}} \times 2\vec{\mathbf{a}}) + 4\lambda\,\vec{\mathbf{b}} \cdot (\vec{\mathbf{c}} \times \vec{\mathbf{a}}) = 0$
$\Longrightarrow \vec{\mathbf{a}}(\vec{\mathbf{b}} \times \vec{\mathbf{c}}) + 4\lambda\vec{\mathbf{b}}.(\vec{\mathbf{c}} \times \vec{\mathbf{a}}) = 0$
$\Longrightarrow \{\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})\}(1 + 4\,\lambda) = 0$
$\Longrightarrow\ \lambda = -\frac{1}{4}\ [\because\ \vec{\mathbf{a}}.(\vec{\mathbf{b}} \times \vec{\mathbf{c}}) \neq 0, \text{given}]$
20 **(d)** $\therefore$ Total force $\vec{\mathbf{P}} = \vec{\mathbf{P}}_1 + \vec{\mathbf{P}}_2 + \vec{\mathbf{P}}_3$
$= \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}} - \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}} + \hat{\mathbf{j}} - \hat{\mathbf{k}} = 2\hat{\mathbf{j}}$
and displacement $\overrightarrow{\mathbf{AB}} = 6\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}} - (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$
$= 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$
$\therefore$Work done $= \vec{\mathbf{P}} \cdot \overrightarrow{\mathbf{AB}}$

$= 2\hat{\mathbf{j}}\left(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}\right) = 8$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	A	A	A	B	A	C	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	A	A	C	D	C	D	A	D

SESSION : 2025-26

DPP

DAILY PRACTICE PRODUCTS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :8

Topic :-VECTOR ALGEBRA

1 **(a)**

The point of intersection of $\vec{\mathbf{r}} \times \vec{\mathbf{a}} = \vec{\mathbf{b}} \times \vec{\mathbf{a}}$ and $\vec{\mathbf{r}} \times \vec{\mathbf{b}} = \vec{\mathbf{a}} \times \vec{\mathbf{b}}$ is $\vec{\mathbf{r}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$

$\therefore\ \vec{\mathbf{r}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}}) + (2\hat{\mathbf{i}} - \hat{\mathbf{k}}) = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

2 **(a)**

Since $\vec{a}, \vec{b}$ and $a \times \vec{b}$ are non-coplanar vectors

$\therefore \vec{r} = x\vec{a} + y\vec{b} + z(\vec{a} \times \vec{b})$ for some scalars x, y, z ...(i)

Now,

$\vec{b} = \vec{r} \times \vec{a}$

$\Rightarrow \vec{b} = \{x\vec{a} + y\vec{b} + z(\vec{a} \times \vec{b})\} \times \vec{a}$

$\Rightarrow \vec{b} = y(\vec{b} \times \vec{a}) + z\left((\vec{a} \times \vec{b}) \times \vec{a}\right)$

$\Rightarrow \vec{b} = y(\vec{b} \times \vec{a}) - z\left(\vec{a} \times (\vec{a} \times \vec{b})\right)$

$\Rightarrow \vec{b} = y(\vec{b} \times \vec{a}) - z\{(\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}\}$

$\Rightarrow \vec{b} = y(\vec{b} \times \vec{a}) + z(\vec{a} \cdot \vec{a})\vec{b}\quad [\because \vec{a} \cdot \vec{b} = 0]$

Comparing the coefficients, we get

$y = 0, z = \dfrac{1}{\vec{a} \cdot \vec{a}} = \dfrac{1}{|\vec{a}|^2}$

Putting the values of y and z in (i), we get

$\vec{r} = x\vec{a} + \dfrac{1}{|\vec{a}|^2}(\vec{a} \times \vec{b})$

4 **(b)**

$(\vec{\mathbf{u}} + \vec{\mathbf{v}} - \vec{\mathbf{w}}).[(\vec{\mathbf{u}} - \vec{\mathbf{v}}) \times (\vec{\mathbf{v}} - \vec{\mathbf{w}})]$

$= (\vec{\mathbf{u}} + \vec{\mathbf{v}} - \vec{\mathbf{w}}).[\vec{\mathbf{u}} \times \vec{\mathbf{v}} - \vec{\mathbf{u}} \times \vec{\mathbf{w}} + \vec{\mathbf{v}} \times \vec{\mathbf{w}}]$

$= \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}} - \vec{\mathbf{v}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{w}} - \vec{\mathbf{w}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}}$

$= \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}} + \vec{\mathbf{w}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}} - \vec{\mathbf{w}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}}$

$= \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}}$

5 **(d)**

$\therefore\ \vec{\mathbf{p}} - 2\,\vec{\mathbf{q}} = 7\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} - 2(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}})$

$= \hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$

$\Longrightarrow |\vec{\mathbf{p}} - 2\vec{\mathbf{q}}| = \sqrt{1^2 + (-4)^2 + (-7)^2} = \sqrt{66}$

7 **(a)**

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$= -\hat{\mathbf{i}} + \hat{\mathbf{j}}$

$$\Rightarrow (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = -\hat{\mathbf{k}}$$

Now, $\lambda \vec{\mathbf{a}} + \mu \vec{\mathbf{b}} = \lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + \mu(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

$= (\lambda + \mu)\hat{\mathbf{i}} + (\lambda + \mu)\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$

$\because \lambda \vec{\mathbf{a}} + \mu \vec{\mathbf{b}} = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}}$

$\Rightarrow (\lambda + \mu)\hat{\mathbf{i}} + (\lambda + \mu)\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}} = -\hat{\mathbf{k}}$

On equating the coefficient of $\hat{\mathbf{i}}$ we get $\lambda + \mu = 0$

13 **(d)**

We have,

$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$

$\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$

$\Rightarrow \vec{a} \perp (\vec{b} - \vec{c})$ or, $\vec{b} - \vec{c} = 0 \Rightarrow \vec{a} \perp (\vec{b} - \vec{c})$ or, $\vec{b} = \vec{c}$

14 **(c)**

Given that, $|\vec{\mathbf{a}}| = 2\sqrt{2}, |\vec{\mathbf{b}}| = 3$

The longer vectors is $5\vec{\mathbf{a}} + 2\vec{\mathbf{b}} + \vec{\mathbf{a}} - 3\vec{\mathbf{b}} = 6\vec{\mathbf{a}} - \vec{\mathbf{b}}$

Length of one diagonal

$= |6\vec{\mathbf{a}} - \vec{\mathbf{b}}|$

$= \sqrt{36\vec{\mathbf{a}}^2 + \vec{\mathbf{b}}^2 - 2 \times 6|\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}| \cos 45\circ}$

$= \sqrt{36 \times 8 + 9 - 12 \times 2\sqrt{2} \times 3 \times \frac{1}{\sqrt{2}}}$

$= \sqrt{288 + 9 - 12 \times 6} = \sqrt{225} = 15$

Other diagonal is $4\vec{\mathbf{a}} + 5\vec{\mathbf{b}}$.

Its length $= \sqrt{16 \times 8 + 25 \times 9 + 40 \times 6} = \sqrt{593}$

15 **(a)**

Given projection of $\vec{\mathbf{a}}$ on $\vec{\mathbf{b}} = |\vec{\mathbf{a}} \times \vec{\mathbf{b}}|$

$\Rightarrow \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{b}}|} = |\vec{\mathbf{a}} \times \vec{\mathbf{b}}|$

$\Rightarrow \frac{|\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta}{|\vec{\mathbf{b}}|} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta$

$\Rightarrow \tan\theta = \frac{1}{|\vec{\mathbf{b}}|}$

$\Rightarrow \tan\theta = \frac{1}{\frac{1}{3}\sqrt{1^2 + 1^2 + 1^2}}$

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \ \theta = \dfrac{\pi}{3}$

17 **(c)**

Since, $\vec{\mathbf{a}} + 2\vec{\mathbf{b}} = k\vec{\mathbf{c}}$

$\therefore \ \vec{\mathbf{a}} + 2\vec{\mathbf{b}} + 6\vec{\mathbf{c}} = k\vec{\mathbf{c}} + 6\vec{\mathbf{c}}$

$= (k+6)\vec{\mathbf{c}} = \lambda\vec{\mathbf{c}} \ (\because \lambda \neq 0)$

18 **(d)**

$$\vec{\mathbf{u}} \times \vec{\mathbf{v}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = -2\hat{\mathbf{k}}$$

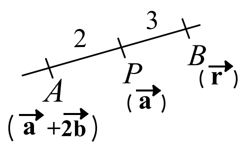
$$\therefore |\vec{\mathbf{w}} \cdot \hat{\mathbf{n}}| = \frac{|\vec{\mathbf{w}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}}|}{|\vec{\mathbf{u}} \times \vec{\mathbf{v}}|}$$

$$\Rightarrow |\vec{\mathbf{w}} \cdot \hat{\mathbf{n}}| = \frac{|-6\hat{\mathbf{k}}|}{|-2\hat{\mathbf{k}}|} = 3$$

19 **(c)**

Let the position of B is $\vec{\mathbf{r}}$.

$$\therefore \ \vec{\mathbf{a}} = \frac{2\vec{\mathbf{r}} + 3(\vec{\mathbf{a}} + 2\vec{\mathbf{b}})}{2+3}$$



$\Rightarrow 5\vec{\mathbf{a}} = 2\vec{\mathbf{r}} + 3\vec{\mathbf{a}} + 6\vec{\mathbf{b}}$

$\Rightarrow \ 2\vec{\mathbf{r}} = 2\vec{\mathbf{a}} - 6\vec{\mathbf{b}}$

$\therefore \ \vec{\mathbf{r}} = \vec{\mathbf{a}} - 3\vec{\mathbf{b}}$

20 **(a)**

Since, $(\vec{\mathbf{A}} + t\vec{\mathbf{B}}) \cdot \vec{\mathbf{C}} = 0$ [given]

$\Rightarrow \left[(1-t)\hat{\mathbf{i}} + (2+2t)\hat{\mathbf{j}} + (3+t)\hat{\mathbf{k}}\right] \cdot (3\hat{\mathbf{i}} + \hat{\mathbf{j}}) = 0$

$\Rightarrow 3(1-t) + (2+2t) = 0 \Rightarrow t = 5$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	B	B	D	A	A	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	D	C	A	C	C	D	C	A

SESSION : 2025-26

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :9

Topic :-VECTOR ALGEBRA

1 **(a)**

We have,

$|\vec{a}| = 1, |\vec{b}| = 1$ and $\vec{a}.\vec{b} = \cos\theta$

Now, $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$

$\Rightarrow |\vec{a}-\vec{b}|^2 = 1 + 1 - 2|\vec{a}|\,|\vec{b}|\cos\theta$

$\Rightarrow |\vec{a}-\vec{b}|^2 = 4\sin^2\frac{\theta}{2}$

$\Rightarrow \left|\frac{\vec{a}-\vec{b}}{2}\right|^2 = \sin^2\frac{\theta}{2} \Rightarrow \left|\frac{\vec{a}-\vec{b}}{2}\right| = \sin\frac{\theta}{2}$

2 **(c)**

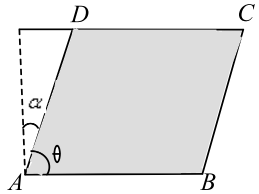
If $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ are two non-zero non-collinear vectors and x, y are two scalars such that $x\vec{\mathbf{a}} + y\vec{\mathbf{b}} = 0$, then $x = 0, y = 0$.

Because otherwise one will be a scalar multiple of the other and hence collinear, which is a contradiction

3 **(b)**

$\overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{AD}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$



$\overrightarrow{\mathbf{AB}} \cdot \overrightarrow{\mathbf{AD}} = -2 + 20 + 22 = 40$

$|\overrightarrow{\mathbf{AB}}| = \sqrt{4 + 100 + 120} = \sqrt{225} = 15$

$|\overrightarrow{\mathbf{AD}}| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$

$\therefore \cos\theta = \frac{40}{45} = \frac{8}{9}$

$\therefore \theta + \alpha = 90°$

$\Longrightarrow \alpha = 90° - \theta$

$$\Rightarrow \cos\alpha = \sin\theta = \sqrt{1-\frac{64}{81}} = \frac{\sqrt{17}}{9}$$

4 **(a)**

Let $\vec{\mathbf{a}} = x\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

Sience, $\frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{b}}|} = \frac{1}{\sqrt{30}}$

$$\Rightarrow \frac{(x\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}})}{|\sqrt{4+1+25}|} = \frac{1}{\sqrt{30}}$$

$\Rightarrow 2x - 1 + 5 = 1$

$\Rightarrow x = -\frac{3}{2}$

5 **(b)**

Now, $2\vec{\mathbf{a}} - \vec{\mathbf{c}} = 2(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$= \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

and $\vec{\mathbf{a}} + \vec{\mathbf{b}} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}} + 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$= \hat{\mathbf{i}} + \hat{\mathbf{k}}$

let θ be the angle between $2\vec{\mathbf{a}} - \vec{\mathbf{c}}$ and $\vec{\mathbf{a}} + \vec{\mathbf{b}}$.

$$\therefore \cos\theta = \frac{(\hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{\sqrt{1^2+1^2}\sqrt{1^2+1^2}}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

$\Rightarrow \theta = \frac{\pi}{3}$

6 **(d)**

Since $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ are collinear with $\vec{c}$ and $\vec{a}$ respectively. Therefore, there exist scalars x, y such that $\vec{a} + \vec{b} = x\vec{c}$ and $\vec{b} + \vec{c} = y\vec{a}$. Now,

$\vec{a} + \vec{b} = x\vec{c} \Rightarrow \vec{a} + \vec{b} + \vec{c} = (x+1)\vec{c}$ ...(i)

and,

$\vec{b} + c = y\vec{a} \Rightarrow \vec{a} + \vec{b} + \vec{c} = (y+1)\vec{a}$ ....(ii)

From (i) and (ii), we get

$(x+1)\vec{c} = (y+1)\vec{a}$

If $x \neq -1$, then

$(x+1)\vec{c} = (y+1)\vec{a} \Rightarrow \vec{c} = \frac{y+1}{x+1}\vec{a}$

$\Rightarrow \vec{c}$ and $\vec{a}$ are collinear

This is a contradiction to the given condition. Therefore, $x = -1$

Putting $x = -1$ in $\vec{a} + \vec{b} = x\vec{c}$, we get

$\vec{a} + \vec{b} + \vec{c} = (-1+1)\vec{c} = \vec{0}$

7 **(b)**

We have, $[\,\vec{\mathbf{a}}\;\vec{\mathbf{b}} + \vec{\mathbf{c}}\;\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}]$

$= \vec{\mathbf{a}} \cdot [\,(\vec{\mathbf{b}} + \vec{\mathbf{c}}) \times (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})]$
$= \vec{\mathbf{a}} \cdot \left(\vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{b}} \times \vec{\mathbf{b}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} + \vec{\mathbf{c}} \times \vec{\mathbf{b}} + \vec{\mathbf{c}} \times \vec{\mathbf{c}}\,\right)$
$= \vec{\mathbf{a}} \cdot \left(\vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} - \vec{\mathbf{b}} \times \vec{\mathbf{c}}\,\right)$
$= \vec{\mathbf{a}} \cdot \left(\vec{\mathbf{b}} \times \vec{\mathbf{a}}\right) + \vec{\mathbf{a}} \cdot (\,\vec{\mathbf{c}} \times \vec{\mathbf{a}})$
$= [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] + [\vec{\mathbf{a}}\,\vec{\mathbf{c}}\,\vec{\mathbf{a}}] = 0$

8 **(a)**

It is given that points P, Q and R with position vectors $60\hat{\imath} + 3\hat{\jmath}, 40\hat{\imath} - 8\hat{\jmath}$ and $a\hat{\imath} - 52\hat{\jmath}$ respectively are collinear
$\therefore \vec{PQ} = \lambda\,\vec{QR}$ for some scalar λ
$\Rightarrow -20\hat{\imath} - 11\hat{\jmath} = \lambda\{(a - 40)\hat{\imath} - 44\hat{\jmath}\}$
$\Rightarrow \lambda(a - 40) = -20, -11 = -44\,\lambda$
$\Rightarrow \lambda = \frac{1}{4}$ and $a = -40$

9 **(a)**

Required unit vector

$$\vec{\mathbf{c}} = \frac{\vec{\mathbf{a}} \times \left(\vec{\mathbf{a}} \times \vec{\mathbf{b}}\right)}{\left|\vec{\mathbf{a}} \times \left(\vec{\mathbf{a}} \times \vec{\mathbf{b}}\right)\right|}$$

Now,
$\vec{\mathbf{a}} \times \left(\vec{\mathbf{a}} \times \vec{\mathbf{b}}\right) = \left(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}\right)\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}}$
$= 3\left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right) - 6(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$
$= -9\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

$$\therefore\ \vec{\mathbf{c}} = \frac{-9\hat{\mathbf{j}} + 9\hat{\mathbf{k}}}{\sqrt{9^2 + 9^2}} = \pm\frac{1}{\sqrt{2}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

10 **(b)**

$$\begin{vmatrix} 2 & 1 & 4 \\ 4 & -2 & 3 \\ 2 & -3 & -\lambda \end{vmatrix} = 0$$

$\Longrightarrow 2(2\lambda + 9) - 1(-4\lambda - 6) + 4(-12 + 4) = 0$
$\Longrightarrow 4\lambda + 18 + 4\,\lambda + 6 - 48 + 16 = 0$
$\Longrightarrow 8\lambda = 8$
$\Longrightarrow\ \lambda = 1$

11 **(b)**

We have,

$$[\vec{u}\,\vec{v}\,\vec{w}] = \begin{vmatrix} al + a_1 l_1 & am + a_1 m_1 & an + a_1 n_1 \\ bl + b_1 l_1 & bm + b_1 m_1 & bn + b_1 n_1 \\ cl + c_1 l_1 & cm + c_1 m_1 & cn + a_1 n_1 \end{vmatrix}$$

$$\Rightarrow [\vec{u}\,\vec{v}\,\vec{w}] = \begin{vmatrix} a & a_1 & 0 \\ b & b_1 & 0 \\ c & c_1 & 0 \end{vmatrix} \begin{vmatrix} l & l_1 & 0 \\ m & m_1 & 0 \\ n & n_1 & 0 \end{vmatrix} = 0$$

Hence, the given vectors are coplanar

13 **(a)**

Given that $\vec{a}, \vec{b}, \vec{c}$ are coplanar

$\therefore \vec{a} \perp \vec{b} \times \vec{c} \Rightarrow \vec{a} \cdot (\vec{b} \times \vec{c}) = 0 \Rightarrow [\vec{a}\vec{b}\vec{c}] = 0$

14 **(c)**

$(\vec{\mathbf{d}} + \vec{\mathbf{a}}) \cdot [\vec{\mathbf{a}} \times \{\vec{\mathbf{b}} \times (\vec{\mathbf{c}} \times \vec{\mathbf{d}})\}]$

$= (\vec{\mathbf{d}} + \vec{\mathbf{a}}) \cdot [\vec{\mathbf{a}} \times \{\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})\vec{\mathbf{c}} - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})\vec{\mathbf{d}}\}]$

$= (\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})[\vec{\mathbf{d}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{c}})] - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})[\vec{\mathbf{d}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{d}})]$

$+(\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})[\vec{\mathbf{a}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{c}})] - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})[\vec{\mathbf{a}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{d}})]$

$= (\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})[\vec{\mathbf{d}}\,\vec{\mathbf{a}}\,\vec{\mathbf{c}}] = (\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})[\vec{\mathbf{a}}\,\vec{\mathbf{c}}\,\vec{\mathbf{d}}]$

16 **(a)**

Let $\vec{\mathbf{a}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \vec{\mathbf{b}} = -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

and $\vec{\mathbf{c}} = \lambda\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore \; [\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}] = 0$

$\Longrightarrow \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ \lambda & -1 & 2 \end{vmatrix} = 0$

$\Longrightarrow 1(6-4) + 2(-4+4\lambda) + 3(2-3\lambda) = 0$

$\Longrightarrow \; \lambda = 0$

17 **(b)**

Let $\vec{\mathbf{a}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

$|\vec{\mathbf{a}}|^2 = a_1^2 + a_2^2 + a_3^2$

and $\vec{\mathbf{a}} \times \hat{\mathbf{i}} = (a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}) \times \hat{\mathbf{i}}$

$= -a_2\hat{\mathbf{k}} + a_3\hat{\mathbf{j}}$

$(\vec{\mathbf{a}} \times \hat{\mathbf{i}})^2 = a_2^2 + a_3^2$

Similarly, $(\vec{\mathbf{a}} \times \hat{\mathbf{j}})^2 = a_3^2 + a_1^2$

and $(\vec{\mathbf{a}} \times \hat{\mathbf{k}})^2 = a_1^2 + a_2^2$

Now, $(\vec{\mathbf{a}} \times \hat{\mathbf{i}})^2 + (\vec{\mathbf{a}} \times \hat{\mathbf{j}})^2 + (\vec{\mathbf{a}} \times \hat{\mathbf{k}})^2$

$= a_2^2 + a_3^2 + a_3^2 + a_1^2 + a_1^2 + a_2^2$

$= 2(a_1^2 + a_2^2 + a_3^2) = 2(\vec{\mathbf{a}})^2$

18 **(d)**

Since, $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = 2\hat{\mathbf{i}} - 4\hat{\mathbf{k}}, \vec{\mathbf{c}} = \hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ are coplanar.

$\therefore [\vec{\mathbf{a}}\;\vec{\mathbf{b}}\;\vec{\mathbf{c}}] = 0 \; \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & -4 \\ 1 & \lambda & 3 \end{vmatrix} = 0$

$\Rightarrow 4\lambda - 1(6+4) + 2\lambda = 0$

$\Rightarrow 6\lambda = 10 \; \Rightarrow \; \lambda = \dfrac{5}{3}$

20 **(c)**

$\vec{\mathbf{A}}, \vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$ are three vectors, then volume of parallelepiped

$V = [\vec{\mathbf{A}}\ \vec{\mathbf{B}}\ \vec{\mathbf{C}}]$

$= \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1 + a^3 - a$

$\Rightarrow V = 1 + a^3 - a$

On differentiating with respect to a, we get

$\frac{dV}{da} = 3a^2 - 1 = 0$

For maximum or minimum, put $\frac{dV}{a} = 0$

$\Rightarrow a = \pm\frac{1}{\sqrt{3}}$

$\frac{d^2V}{da^2} = 6a$, positive at $a = \frac{1}{\sqrt{3}}$.

$\therefore V$ is minimum at $a = \frac{1}{\sqrt{3}}$.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	A	B	D	B	A	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	A	C	B	A	B	D	C	C

SESSION : 2025-26

DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :10

Topic :-VECTOR ALGEBRA

1 **(c)**

By the properties of midpoint theorem,

$\overrightarrow{\mathbf{PA}} + \overrightarrow{\mathbf{PB}} = 2\overrightarrow{\mathbf{PC}}$

2 **(a)**

The vector equation of line passing through points $(3, 2, 1)$ and $(-2, 1, 3)$

$\vec{\mathbf{r}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda\left[(-2-3)\hat{\mathbf{i}} + (1-2)\hat{\mathbf{j}} + (3-1)\hat{\mathbf{k}}\right]$

$= 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(-5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

3 **(d)**

$\because \quad \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos \frac{5\pi}{6}$

$= -\frac{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sqrt{3}}{2}$

Since, the projection of $\vec{\mathbf{a}}$ in the direction of

$\vec{\mathbf{b}} = -\frac{6}{\sqrt{3}}$

$\Rightarrow -\frac{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sqrt{3}}{2|\vec{\mathbf{b}}|} = -\frac{6}{\sqrt{3}}$

$\Rightarrow |\vec{\mathbf{a}}| = \frac{6 \times 2}{3} = 4$

4 **(d)**

Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ in $OXYZ$ system

Also, let $\vec{r} = X\hat{\imath} + Y\hat{\jmath} + Z\hat{k}$ in the new coordinate system

Since the right handed rectangular system $OXYZ$ is rotated about z-axis through $\frac{\pi}{4}$ in anticlockwise direction. Therefore,

$x = X\cos\theta - Y\sin\theta$ and $y = X\sin\theta + Y\cos\theta$

$\Rightarrow x = X\cos\frac{\pi}{4} - Y\sin\frac{\pi}{4}, y = X\sin\frac{\pi}{4} + Y\cos\frac{\pi}{4}$

and, $z = Z$

It is given that $X = 2\sqrt{2}, Y = 3\sqrt{2}$ and $Z = 4$

$\therefore x = 2 - 3 = -1, y = 5$ and $z = 4$

Hence, $\vec{r} = -\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$

<u>ALITER</u> Let $l_1, m_1, n_1; l_2, m_2, n_2$ and l_3, m_3, n_3 be the direction cosines of the new axes with respect to the old axes. Then,

$l_1 = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, m_1 = \cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, n_1 = \cos\frac{\pi}{2} = 0$

$l_2 = \cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}, m_2 = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, n_2 = \cos\frac{\pi}{2} = 0$

$l_3 = \cos\frac{\pi}{2} = 0, m_3 = \cos\frac{\pi}{2} = 0, n_3 = \cos 0 = 1$

Let x', y', z' and x, y, z be the components of the given vector with respect to new and old axes.
Then,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2\sqrt{2} \\ 3\sqrt{2} \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -3 & +0 \\ 2 & +3 & +0 \\ 0 & 0 & +4 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 4 \end{bmatrix}$$

Hence, the components of $\vec{a}$ in the $Oxyz$ coordinate system are $-1,5,4$

5 **(d)**

$\because \vec{\mathbf{x}} \cdot \vec{\mathbf{a}} = \vec{\mathbf{x}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{x}} \cdot \vec{\mathbf{c}} = 0$

For non-zero vector $\vec{\mathbf{x}}$

$[\vec{\mathbf{a}}\, \vec{\mathbf{b}}\, \vec{\mathbf{c}}] = 0$ (three vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar)

and $[\vec{\mathbf{a}} \times \vec{\mathbf{b}}\ \ \vec{\mathbf{b}} \times \vec{\mathbf{c}}\ \ \vec{\mathbf{c}} \times \vec{\mathbf{a}}]$

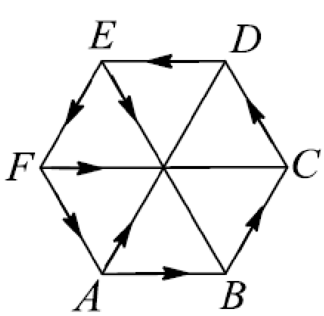
$= [\vec{\mathbf{a}}\, \vec{\mathbf{b}}\, \vec{\mathbf{c}}]^2 = 0$

6 **(d)**

$ABCDEF$ is a regular hexagon. We know from the hexagon that $\overrightarrow{\mathbf{AD}}$ is parallel to $\overrightarrow{\mathbf{BC}}$.

$\Rightarrow \overrightarrow{\mathbf{AD}} = 2\overrightarrow{\mathbf{BC}}$

Similarly, $\overrightarrow{\mathbf{EB}}$ is a parallel to $\overrightarrow{\mathbf{FA}}$



$\Rightarrow \overrightarrow{\mathbf{EB}} = 2\overrightarrow{\mathbf{FA}}$

and $\overrightarrow{\mathbf{FC}}$ is parallel to $\overrightarrow{\mathbf{AB}}$.

$\Rightarrow\ \overrightarrow{\mathbf{FC}} = 2\overrightarrow{\mathbf{AB}}$

Thus, $\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{EB}} + \overrightarrow{\mathbf{FC}} = 2\overrightarrow{\mathbf{BC}} + 2\overrightarrow{\mathbf{FA}} + 2\overrightarrow{\mathbf{AB}}$

$= 2\left(\overrightarrow{\mathbf{FA}} + \overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}}\right)$

$= 2\left(\overrightarrow{\mathbf{FC}}\right) = 2\left(2\overrightarrow{\mathbf{AB}}\right) = 4\,\overrightarrow{\mathbf{AB}}$

7 **(d)**

Here, $\overrightarrow{\mathbf{a_1}} = 6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}, \overrightarrow{\mathbf{a_2}} = -4\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \hat{\mathbf{k}}$,
$\overrightarrow{\mathbf{b_1}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{b_2}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

∴Shortest distance

$$= \left|\frac{(\overrightarrow{\mathbf{a_2}} - \overrightarrow{\mathbf{a_1}}) \cdot (\overrightarrow{\mathbf{b_1}} \times \overrightarrow{\mathbf{b_2}})}{|\overrightarrow{\mathbf{b_1}} \times \overrightarrow{\mathbf{b_2}}|}\right|$$

$$= \left|\frac{(-10\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) \cdot (8\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 4\hat{\mathbf{k}})}{\sqrt{64 + 64 + 16}}\right|$$

$$= \left|-\frac{108}{12}\right| = 9$$

8 **(c)**

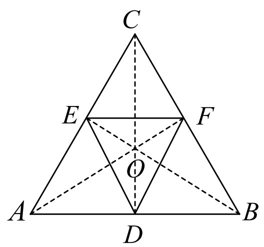
$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix} = 15\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 30\hat{\mathbf{k}}$$

and $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = \sqrt{15^2 + (-10)^2 + (30)^2} = 35$

$$\therefore \text{ Required vector} = \frac{3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}}{7}$$

10 **(a)**

Let O be the origin



$\therefore\ \overrightarrow{\mathbf{BE}} + \overrightarrow{\mathbf{AF}} = \overrightarrow{\mathbf{OE}} - \overrightarrow{\mathbf{OB}} + \overrightarrow{\mathbf{OF}} - \overrightarrow{\mathbf{OA}}$

$$= \frac{\overrightarrow{\mathbf{OA}} + \overrightarrow{\mathbf{OC}}}{2} - \overrightarrow{\mathbf{OB}} + \frac{\overrightarrow{\mathbf{OB}} + \overrightarrow{\mathbf{OC}}}{2} - \overrightarrow{\mathbf{OA}}$$

$$= \frac{\overrightarrow{\mathbf{OC}}}{2} + \frac{\overrightarrow{\mathbf{OC}}}{2} + \frac{\overrightarrow{\mathbf{OA}}}{2} - \overrightarrow{\mathbf{OA}} + \frac{\overrightarrow{\mathbf{OB}}}{2} - \overrightarrow{\mathbf{OB}}$$

$$= \overrightarrow{\mathbf{OC}} - \frac{\overrightarrow{\mathbf{OA}} + \overrightarrow{\mathbf{OB}}}{2} = \overrightarrow{\mathbf{OC}} - \overrightarrow{\mathbf{OD}} = \overrightarrow{\mathbf{DC}}$$

11 **(d)**

$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\,|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow\ |\vec{a}-\vec{b}|^2 = 1 + 1 - 2\cos 60° = 2 - 1$

$\Rightarrow |\vec{a}-\vec{b}| = 1$

12 **(b)**

Given, $2\vec{a} + 3\vec{b} + \vec{c} = \vec{0}$

$\Longrightarrow 2\vec{a} + 3\vec{b} = -\vec{c}$

Taking cross product with $\vec{a}$ and $\vec{b}$ respectively, we get

$2(\vec{a}\times\vec{a}) + 3(\vec{a}\times\vec{b}) = -\vec{a}\times\vec{c}$

$\Longrightarrow 3(\vec{a}\times\vec{b}) = -\vec{c}\times\vec{a}$ ...(i)

and $2(\vec{b}\times\vec{a}) + 3(\vec{b}\times\vec{b}) = -\vec{b}\times\vec{c}$

$\Longrightarrow 2(\vec{a}\times\vec{b}) = \vec{b}\times\vec{c}$ ....(ii)

Now, $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}$

$= \vec{a}\times\vec{b} + \vec{b}\times\vec{c} + 3(\vec{a}\times\vec{b})$ [using Eq. (i)]

$= 4(\vec{a}\times\vec{b}) + \vec{b}\times\vec{c}$

$= 2(\vec{b}\times\vec{c}) + \vec{b}\times\vec{c}$ [using Eq. (ii)]

$= 3(\vec{b}\times\vec{c})$

13 **(d)**

$[\vec{a}-2\vec{b}, \vec{b}-3\vec{c}, \vec{c}-4\vec{a}]$

$= (\vec{a}-2\vec{b})\cdot\{(\vec{b}-3\vec{c})\times(\vec{c}-4\vec{a})\}$

$= (\vec{a}-2\vec{b})\cdot\{\vec{b}\times\vec{c} - 4\vec{b}\times\vec{a} + 12\vec{c}\times\vec{a}\}$

$= (\vec{a}-2\vec{b})\cdot(\vec{a} + 4\vec{c} + 12\vec{b})$

$= \vec{a}\cdot\vec{a} - 24\,\vec{b}\cdot\vec{b}$

$= 1 - 24\times 9 = 1 - 216 = -215$

14 **(b)**

Given , area $= |\vec{a}\times\vec{b}| = 15$

If the sides are $(3\vec{a}+2\vec{b})$and $(\vec{a}+3\vec{b})$, then

Area of parallelogram

$= |(3\vec{a}+2\vec{b})\times(\vec{a}+3\vec{b})| = 7|\vec{a}\times\vec{b}|$

$= 7\times 15 = 105$ sq units

18 **(a)**

Given, $\vec{a}\cdot(\vec{b}+\vec{c}) = 0 \ \Longrightarrow \vec{a}\cdot\vec{b} + \vec{c}\cdot\vec{a} = 0$

$\vec{b}\cdot(\vec{c}+\vec{a}) = 0$

$\Longrightarrow\ \vec{b}\cdot\vec{c} + \vec{a}\cdot\vec{b} = 0$

and $\vec{c}\cdot(\vec{a}+\vec{b}) = 0$

$\Longrightarrow\ \vec{c}\cdot\vec{a} + \vec{b}\cdot\vec{c} = 0$

$\therefore\ \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = 0$

Now, $|\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}|^2=|\vec{\mathbf{a}}|^2+|\vec{\mathbf{b}}|^2+|\vec{\mathbf{c}}|^2+2(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}+\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}+\vec{\mathbf{c}}\cdot\vec{\mathbf{a}})$

$\Longrightarrow |\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}|^2=9+16+25+0=50$

$\Longrightarrow |\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}|=5\sqrt{2}$

19 **(b)**

We have,

$(\vec{b}\times\vec{c})\times\vec{a}=-\{\vec{a}\times(\vec{b}\times\vec{c})\}$

$\Rightarrow (\vec{b}\times\vec{c})\times\vec{a}=-\{(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}\}=(\vec{a}\cdot\vec{b})\vec{c}-(\vec{a}\cdot\vec{c})\vec{b}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	D	D	D	D	D	C	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	B	D	B	C	A	D	A	B	B