**SESSION : 2025-26**

# DPP
DAILY PRACTICE PROBLEMS

**CLASS : XIIth**
**DATE :**

**SOLUTIONS**

**SUBJECT : MATHS**
**DPP NO. :1**

## Topic :-DIFFERENTITATION

1 **(c)**

Since, $y=(1+x)(1+x^2)(1+x^4)\dots(1+x^{2^n})$

$$\Rightarrow (1-x)y=(1-x^2)(1+x^2)(1+x^4)\dots(1+x^{2^n})$$

$$=(1-x^4)(1+x^4)\dots\left(1+x^{2^n}\right)$$

$$\dots\ \dots\ \dots\ \dots\ \dots$$

$$\dots\ \dots\ \dots\ \dots\ \dots$$

$$=\left(1-x^{2^n}\right)\left(1+x^{2^n}\right)=1-x^{2^{n+1}}$$

$$\therefore\ y=\frac{1-x^{2^{n+1}}}{(1-x)}$$

$$\frac{dy}{dx}=\frac{(1-x)(-2^{n+1}).x^{2^{n+1-1}}-\left(1-x^{2^{n+1}}\right)(-1)}{(1-x)^2}$$

$$\therefore \frac{dy}{dx}|_{x=0}=\frac{(1-0)(-2^{n+1}.0)-(1-0)(-1)}{1}=1$$

2 **(b)**

We have,

$$\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$$

Putting $x=sin\,sin\,A$ , $y=sin\,sin\,B$ , it reduces to

$$\Rightarrow x\ -y\ =2(a)$$

Differentiating w.r.t. $x$, we get

$$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$$

3 **(c)**

Let $y = e^{x^3}, z = log\ log\ x$

On differentiating w.r.t. $x$, we get

$\frac{dy}{dx} = e^{x^3}(3x^2) = 3x^2 e^{x^3}$ and $\frac{dz}{dx} = \frac{1}{x}$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{3x^2 e^{x^3}}{\left(\frac{1}{x}\right)} = 3x^3 e^{x^3}$$

4 **(c)**

Let $y = \sqrt{x^2 + 16}\ and\ z = \frac{x}{x-1}$

On differentiating w.r.t., $x$, we get

$$\frac{dy}{dx} = \frac{1}{2}(x^2 + 16)^{-1/2}(2x)$$

and $\frac{dz}{dx} = \frac{x-1-x}{(x-1)^2} = \frac{-1}{(x-1)^2}$

$$\therefore \frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{-x}{\sqrt{x^2+16}} . \frac{1}{\frac{1}{(x-1)^2}}$$

$$\left(\frac{dy}{dz}\right)_{x=3} = \frac{-3(2)^2}{\sqrt{25}} = \frac{-12}{5}$$

5 **(c)**

Given, $x = log\ log\ (1 + t^2)$ $and$ $y = t - t$

$$\frac{dx}{dt} = \frac{1}{1+t^2} . 2t$$

and $\frac{dy}{dt} = 1 - \frac{1}{1+t^2} = \frac{t^2}{1+t^2}$

$\therefore$ $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{t^2/(1+t^2)}{2t/(1+t^2)} = \frac{t}{2}$ ...(i)

Also, $x = log\ log\ (1 + t^2) \Rightarrow t^2 = e^x - 1$ ...(ii)

From Eqs. (i) and (ii), we get

$\frac{dy}{dx} = \frac{\sqrt{e^x - 1}}{2}$

6 **(c)**

$$y = \frac{(1-x)(1+x)(1+x^2)(1+x^4) \dots \dots . (1+x^{2n})}{(1-x)}$$

$$= \frac{1 - x^{4n}}{1 - x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(1-x)(-4nx^{4n-1}) - (1-x^{4n})(-1)}{(1-x)^2}$$

$$= \frac{-4n(1-x)x^{4n-1} + (1-x^{4n})}{(1-x)^2}$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=0} = 1$$

7 **(b)**

We have,

$$f(x) = (1-x)^n$$

$\therefore f'(x) = -n(1-x)^{n-1}, f''(x) = n(n-1)x^{n-2},$
$f'''(x) = -n(n-1)(n-2)x^{n-3}$ and so on
$\Rightarrow f(0) = 1, f'(0) = -n, f''(0) = n(n-1),$
$f'''(0) = -n(n-1)(n-2)$ and so on

$$\therefore f(0) + f'(0) + \frac{f''(0)}{2!} + \cdots + \frac{f^n(0)}{n!}$$

$$= 1 - n + \frac{n(n-1)}{n!} + \cdots + (-1)^n \frac{n(n-1)\ldots 3.2.1}{n!}$$

$$= {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \cdots + (-1)^n \, {}^nC_n = (1-1)^n = 0$$

8 **(a)**

$$f(x) = (tan\, tan\, x)(cot\, cot\, x)^{-1} + tan^{-1}\frac{4x}{4-x^2}$$

$$= \frac{log\, log\,\ tan\, tan\, x}{log\, log\,\ cot\, cot\, x} . \frac{log\, log\,\ tan\, tan\, x}{log\, log\,\ cot\, cot\, x} + \left(\frac{4x}{4-x^2}\right)$$

$$= \frac{(log\, log\,\ tan\, tan\, x)^2}{(-\, log\, log\,\ tan\, tan\, x)^2} + \left(\frac{4x}{4-x^2}\right)$$

$$= 1 + \left(\frac{4x}{4-x^2}\right)$$

$$\therefore f'(x) = \frac{1}{1+\left(\frac{4x}{4-x^2}\right)^2} . \frac{(4-x^2)4 - 4x(-2x)}{(4-x^2)^2}$$

$$= \frac{16 - 4x^2 + 8x^2}{(4-x)^2 + 16x^2} = \frac{4(4+x^2)}{(4-x^2)^2 + (4x)^2}$$

Hence, $f(2) = \frac{4(4+4)}{0+(8)^2} = \frac{32}{64} = \frac{1}{2}$

9 **(c)**

Given, $y = \frac{3x}{2} - \frac{3x}{2} = 2\frac{3x}{2} - 1$

$$\Rightarrow \frac{dy}{dx} = 2.2\, cos\, cos\, \frac{3x}{2}\left(-\, sin\, sin\, \frac{3x}{2}\right)\left(\frac{3}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = -6\, cos\, cos\, \frac{3x}{2}\, sin\, sin\, \frac{3x}{2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -6\left[cos\, cos\, \frac{3x}{2}\left(cos\, cos\, \frac{3x}{2}\right).\frac{3}{2} - sin\, sin\, \frac{3x}{2}\, sin\, sin\, \frac{3x}{2}.\frac{3}{2}\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = -9\left[\frac{3x}{2} - \frac{3x}{2}\right] = -9y$$

**Alternate**

$$y = \frac{3x}{2} - \frac{3x}{2}$$

$$\Rightarrow y = cos\, cos\, 3x$$

$$\Rightarrow \frac{dy}{dx} = -3\, sin\, sin\, 3x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -9\, cos\, cos\, 3x = -9y$$

10 **(d)**

Given,

$$f(x) = (x) = \frac{x}{x}$$

On differentiating w.r.t. $x$, we get

$$f'(x) = \frac{x.\frac{1}{x}.\frac{1}{x} - x.\ \frac{1}{x}}{(x\ )^2}$$

$$\Rightarrow f'(x) = \frac{1-x}{x(x\ )^2}$$

$$\Rightarrow f'(e) = \frac{1-e}{e(e\ )^2} = \frac{1-1}{e} = \frac{1}{e}$$

11 **(c)**

We have,

$$f(x) = \left\{\frac{1+(x\ )^2}{1+(x\ )^2}\right\}$$

$$\Rightarrow f(x) = 2(x\ ) \quad [\because x\ > 0\ in\ the\ nbd\ of\ x = e]$$

$$\Rightarrow f'(x) = \frac{2}{1+(x\ )^2} \times \frac{1}{x} \Rightarrow f'(e) = \frac{1}{e}$$

12 **(c)**

Given, $f(x) = 1 + nx + \frac{n(n-1)}{2!}x^2$

$$+\frac{n(n-1)(n-2)}{3!}x^3+\ldots+x^n$$

$\Rightarrow f(x)=(1+x)^n$

$$\Rightarrow f'(x)=n(1+x)^{n-1}$$

$$\Rightarrow f''(x)=n(n-1)(1+x)^{n-2}$$

$$\Rightarrow f''(1)=n(n-1)2^{n-2}$$

13 **(c)**

On differentiating w.r.t. $x$, we get

$$2^x\, log\, log\, 2+2^y\, log\, log\, 2\frac{dy}{dx}=2^{x+y}\, log\, log\, 2\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow 2^x+2^y\frac{dy}{dx}=\ 2^{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow\ 2^x-2^{x+y}=\frac{dy}{dx}(2^{x+y}-2^y)$$

$$\Rightarrow\ 2^{x-y}\frac{(1-2^y)}{(2^x-1)}=\frac{dy}{dx}$$

14 **(a)**

We have,

$$2x^2-3xy+y^2+x+2y-8=0$$

Differentiating w.r.t. to $x$, we get

$$4x-3\left(x\frac{dy}{dx}+y\right)+2y\frac{dy}{dx}+1+2\frac{dy}{dx}=0$$

$$\Rightarrow 4x-3y+1=\frac{dy}{dx}(3x-2y-2)$$

$$\Rightarrow\frac{dy}{dx}=\frac{3y-4x-1}{2y-3x+2}$$

15 **(c)**

We have, $sin\, sin\, y\ +e^{-xcoscos\, y}\ =e$

Differentiating w.r.t. $x$, we get

$$cos\, cos\, y\ \frac{dy}{dx}-e^{-xcoscos\, y}\left(cos\, cos\, y\ -x\, sin\, sin\, y\ \frac{dy}{dx}\right)=0$$

Putting $x=1, y=\pi$, we get

$$-\frac{dy}{dx}-e(-1)=0\Rightarrow\frac{dy}{dx}=e$$

16 **(a)**

$$\frac{d}{dx}\left[\left(\frac{a-x}{1+ax}\right)\right]=\frac{d}{dx}[a-x]$$

$$=0-\frac{1}{1+x^2}=-\frac{1}{1+x^2}$$

17 **(b)**

$$\because y=\left(\frac{a\cos\cos x-b\sin\sin x}{b\cos\cos x+a\sin\sin x}\right)$$

$$=\left(\frac{\frac{a}{b}-\tan\tan x}{1+\frac{a}{b}\tan\tan x}\right)$$

$$=\left[\tan\tan\left(\left(\frac{a}{b}\right)-x\right)\right]$$

$$\Rightarrow y=\left(\frac{a}{b}\right)-x$$

$$\therefore\frac{dy}{dx}=0-1=-1$$

18 **(b)**

Given, $y=x^y$

$$\Rightarrow y=y\log\log x$$

$$\Rightarrow\frac{1}{y}.\frac{dy}{dx}=y.\frac{1}{x}+\frac{dy}{dx}.\log\log x$$

$$\Rightarrow\frac{dy}{dx}\left[\frac{1}{y}-\log\log x\right]=\frac{y}{x}$$

$\Rightarrow \frac{dy}{dx}=\frac{y^2}{x(1-y\log\log x)}$

19 **(b)**

$$\because x^x y^y z^z=c$$

$$\Rightarrow\quad x\log\log x+y\log\log y+z\log\log z=\log\log c$$

On differentiating partially w.r.t. $x$, we get

$$x.\frac{1}{x}+\log\log x+z.\frac{1}{z}\ \frac{\partial z}{\partial x}+\log\log z\frac{\partial z}{\partial x}=0$$

$$\Rightarrow(1+\log\log z)\frac{\partial z}{\partial x}=-(1+\log\log x)$$

$$\Rightarrow\frac{\partial z}{\partial x}=-\left(\frac{1+\log\log x}{1+\log\log z}\right)$$

1 **(d)**

We have,

$$f(x) = \sqrt{(x-1)^2} = |x-1| = \{x-1, \quad if\ x \geq 1\ -(x-1), \quad if\ x < 1$$

$$\therefore f'(x) = \{1, \quad if\ x > 1\ -1, \quad if\ x < 1$$

2 **(a)**

$$\because u = \left(\frac{x}{y}\right) + \left(\frac{y}{x}\right)$$

$$\therefore \frac{\partial u}{\partial x} = \frac{1}{\sqrt{\left\{1-\left(\frac{x}{y}\right)^2\right\}}}\frac{1}{y} + \frac{1}{1+\left(\frac{y}{x}\right)^2}.\left(-\frac{y}{x^2}\right)$$

$\Rightarrow x\frac{\partial u}{\partial x} = \frac{x}{\sqrt{(y^2-x^2)}} - \frac{xy}{(x^2+y^2)}$ ...(i)

and $\frac{\partial u}{\partial y} = \frac{1}{\sqrt{\left\{1-\left(\frac{x}{y}\right)^2\right\}}}\left(-\frac{x}{y^2}\right) + \frac{1}{1+\left(\frac{y}{x}\right)^2}.\left(\frac{1}{x}\right)$

$\Rightarrow\ y\frac{\partial u}{\partial y} = -\frac{x}{\sqrt{(y^2-x^2)}} + \frac{xy}{(x^2+y^2)}$ ...(ii)

On adding Eqs. (i)and (ii), we get

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

3 **(c)**

On differentiating the given equation partially w.r.t. $x$ and $y$ respectively

$$u_x = \frac{y}{x} + log\ log\ y, \quad u_y = log\ log\ x + \frac{x}{y}$$

Now, $u_x u_y - u_x\ log\ log\ x - u_y\ log\ log\ y + log\ log\ x\ log\ log\ y$

$$= \left(\frac{y}{x} + log\ log\ y\right)\left(log\ log\ x + \frac{x}{y}\right) - \left(\frac{y}{x} + log\ log\ y\ log\ log\ x\right)$$

$$-\left(log\ log\ x + \frac{x}{y}\right) log\ log\ y + log\ log\ x\ log\ log\ y = 1$$

4 **(a)**

Here, $x = A\ cos\ 4t + B\ sin\ 4t$

$$\Rightarrow \frac{dx}{dt} = -4A\ sin\ sin\ 4t + 4B\ cos\ cos\ 4t$$

$$\Rightarrow \frac{d^2x}{dt^2} = -16\ Acos\ 4t - 16\ B\ sin\ sin\ 4t$$

$$\Rightarrow \frac{d^2x}{dt^2} = -16x$$

5 **(a)**

$$\frac{xf(a) - af(x)}{x - a}$$

$$= \frac{xf(a) - af(a) - af(x) + af(a)}{x - a}$$

$$= \frac{f(a)(x - a) - a[f(x) - f(a)]}{x - a}$$

$$= \frac{f(a)(x - a)}{x - a} - a\frac{f(x) - f(a)}{(x - a)}$$

$$= f(a) - af'(a)$$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | C | B | C | C | C | C | B | A | C | D |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | C | C | C | A | C | A | B | B | B | B |
| | | | | | | | | | | |

SESSION : 2025-26

**DPP**
DAILY PRACTICE PRODUCTS

CLASS : XIIth
DATE :

**SOLUTIONS**

SUBJECT : MATHS
DPP NO. :2

# Topic :-DIFFERENTITATION

1 **(d)**

We have,

$$f(x)=\sqrt{(x-1)^2}=|x-1|=\begin{cases} x-1, & \text{if } x\geq 1 \\ -(x-1), & \text{if } x<1 \end{cases}$$

$$\therefore f'(x)=\begin{cases} 1, & \text{if } x>1 \\ -1, & \text{if } x<1 \end{cases}$$

2 **(a)**

$$\because u=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$$

$$\therefore \frac{\partial u}{\partial x}=\frac{1}{\sqrt{\left\{1-\left(\frac{x}{y}\right)^2\right\}}}\frac{1}{y}+\frac{1}{1+\left(\frac{y}{x}\right)^2}.\left(-\frac{y}{x^2}\right)$$

$$\Rightarrow x\frac{\partial u}{\partial x}=\frac{x}{\sqrt{(y^2-x^2)}}-\frac{xy}{(x^2+y^2)} \quad \text{...(i)}$$

and $\frac{\partial u}{\partial y}=\frac{1}{\sqrt{\left\{1-\left(\frac{x}{y}\right)^2\right\}}}\left(-\frac{x}{y^2}\right)+\frac{1}{1+\left(\frac{y}{x}\right)^2}.\left(\frac{1}{x}\right)$

$$\Rightarrow \ y\frac{\partial u}{\partial y}= \ -\frac{x}{\sqrt{(y^2-x^2)}}+\frac{xy}{(x^2+y^2)} \quad \text{...(ii)}$$

On adding Eqs. (i)and (ii), we get

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$$

3 **(c)**

On differentiating the given equation partially w.r.t. $x$ and $y$ respectively

$$u_x=\frac{y}{x}+\log y, \quad u_y=\log x+\frac{x}{y}$$

Now, $u_x u_y-u_x\log x-u_y\log y+\log x\log y$

$$=\left(\frac{y}{x}+\log y\right)\left(\log x+\frac{x}{y}\right)-\left(\frac{y}{x}+\log y\log x\right)$$

$$-\left(\log x+\frac{x}{y}\right)\log y+\log x\log y=1$$

4 **(a)**

Here, $\quad x=A\,cos\,4t+B\,sin\,4t$

$\Rightarrow \dfrac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$

$\Rightarrow \dfrac{d^2x}{dt^2} = -16\,A\cos 4t - 16\,B\sin 4t$

$\Rightarrow \dfrac{d^2x}{dt^2} = -16x$

5 **(a)**

$$\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$$
$$=\lim_{x\to a}\frac{xf(a)-af(a)-af(x)+af(a)}{x-a}$$
$$=\lim_{x\to a}\frac{f(a)(x-a)-a[f(x)-f(a)]}{x-a}$$
$$=\lim_{x\to a}\frac{f(a)(x-a)}{x-a}-a\lim_{x\to a}\frac{f(x)-f(a)}{(x-a)}$$
$$=f(a)-af'(a)$$

6 **(b)**

$\dfrac{d}{dx}[x^x + x^a + a^x + a^a]$

$= x^x(1+\log x) + ax^{a-1} + a^x\log a + 0$

$= x^x(1+\log x) + ax^{a-1} + a^x\log a$

7 **(d)**

Given, $y = a^x.b^{2x-1}$

$\Rightarrow \log y = x\log a + (2x-1)\log b$

$\Rightarrow \dfrac{1}{y}.\dfrac{dy}{dx} = \log a + 2\log b$

$\Rightarrow \dfrac{dy}{dx} = y\log ab^2$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{dy}{dx}\log ab^2 = y\,(\log ab^2)^2$

9 **(d)**

Differentiating $ax^2 + 2\,h\,xy + by^2 = 1$ w.r.t. $x$,we get

$2\,ax + 2\,hy + 2\,hx\dfrac{dy}{dx} + 2\,by\dfrac{dy}{dx} = 0$

$\Rightarrow \dfrac{dy}{dx} = -\left(\dfrac{ax+hy}{hx+by}\right)$

$\Rightarrow \dfrac{d^2y}{dx^2} = \left\{\dfrac{(hx+by)\left(a+h\dfrac{dy}{dx}\right)-(ax+hy)\left(h+b\dfrac{dy}{dx}\right)}{(hx+by)^2}\right\}$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{h^2-ab}{(hx+by)^3}$

10 **(d)**

$[hog](x) = h(x^2) = 2\log_e x$

$\Rightarrow\ (hogof)(x) = hog(\sin x) = 2\log_e \sin x$

$\Rightarrow F(x) = \ 2\log_e \sin x$

$\Rightarrow F'(x) = 2\cot x$

$\Rightarrow F''(x) = -2\,\text{cosec}^2 x$

11 **(b)**

Since, $\sin^{-1} x = \frac{\pi}{2} - \sin^{-1} y$

$$\Rightarrow \sin^{-1} x = \cos^{-1} y$$

$\Rightarrow \; y = \sqrt{1-x^2} \;\; (\because \sin^{-1} x = \cos^{-1}\sqrt{1-x^2})$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}(-2x) = -\frac{x}{y}$$

12 **(d)**

$y = \tan^{-1}(\sec x - \tan x)$

$\frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{1-\sin x}{\cos x}\right)$

$\frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}\right)$

$= \frac{d}{dx}\tan^{-1}\left(\frac{1-\tan x/2}{1+\tan x/2}\right)$

$= \frac{d}{dx}\tan^{-1}\left\{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right\}$

$= \frac{d}{dx}\left(\frac{\pi}{4} - \frac{x}{2}\right) = -\frac{1}{2}$

13 **(d)**

We have,

$x = e^t \sin t$ and $y = e^t \cos t$

$\Rightarrow \frac{dx}{dt} = e^t(\sin t + \cos t)$ and $\frac{dy}{dt} = e^t(\cos t - \sin t)$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t - \sin t}{\cos t + \sin t}$

Now, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{\cos t - \sin t}{\cos t + \sin t}\right) \times \frac{1}{e^t(\sin t + \cos t)}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-(\cos t + \sin t)^2 - (\cos t - \sin t)^2}{(\cos t + \sin t)^2} \times \frac{1}{e^t(\cos t + \sin t)}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-2}{(\cos t + \sin t)^3 e^t} \Rightarrow \left(\frac{d^2y}{dx^2}\right)_{t=\pi} = \frac{-2}{-e^\pi} = \frac{2}{e^\pi}$

14 **(b)**

Given, $f(x) = 3|2+x|$

$$f(x) = \begin{cases} 3(2+x), & x \geq -2 \\ -3(2+x), & x \leq -2 \end{cases}$$

On differentiating w. r. t, $x$, we get

$$f'(x) = \begin{cases} 3, & x \geq 2 \\ -3, & x \leq -2 \end{cases}$$

at $x=-3, f'(3)=-3$

15 **(c)**

We know that be Newton's Leibnitz formula

If $I=\int_u^v f(t)dx$,

Then $\frac{dI}{dx}=f(v)\frac{dv}{dx}-f(u)\frac{du}{dx}$

Where $u$ and $v$ are function of $x$

$$\therefore \frac{dx}{dy}=\frac{1}{\sqrt{1+9y^2}}$$

$$\Rightarrow \frac{dy}{dx}=\sqrt{1+9y^2}$$

$$\therefore \frac{d^2y}{dx^2}=\frac{9y}{\sqrt{1+9y^2}}.\frac{dy}{dx}$$

$$=\frac{9y}{\sqrt{1+9y^2}}\sqrt{1+9y^2}=9y$$

16 **(a)**

Given, $f(x)=x\tan^{-1}x$

$$\therefore f'(x)=\frac{x}{1+x^2}+\tan^{-1}x$$

$$\Rightarrow f'(1)=\frac{1}{1+1^2}+\tan^{-1}1=\frac{1}{2}+\frac{\pi}{4}$$

17 **(b)**

Given , $g(x)=[f(2f(x)+2)]^2$

$\therefore g'(x)=2.f(2f(x)+2).f'(2f(x)+2).2f'(x)$

$=4f(2f(x)+2)f'(2f(x)+2)f'(x)$

$\therefore g'(0)=4f(0)f'(0)f'(0)=-4$

18 **(b)**

Let $f(x)=|x-1|+|x-3|$

$$f(x)=\begin{cases}-(x-1)-(x-3), & x<1\\ (x-1)-(x-3), & 1\le x<3\\ (x-1)+(x-3), & x\ge 3\end{cases}$$

$$f(x)=\begin{cases}4-2x, & x<1\\ 2, & 1\le x<3\\ 2x-4, & x\ge 3\end{cases}$$

At $x=2$,

$$f(x)=2\Rightarrow f'(x)=0$$

19 **(d)**

We have, $y=\tan^{-1}(\sec x-\tan x)$

$$\Rightarrow \frac{dy}{dx}=\frac{d}{dx}\tan^{-1}\left(\frac{1-\sin x}{\cos x}\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{d}{dx}\tan^{-1}\left(\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right)$$

$$= \frac{d}{dx}\tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$= \frac{d}{dx}\left(\frac{\pi}{4} - \frac{x}{2}\right) = -\frac{1}{2}$$

20 **(a)**

Given that, $x = a\cos^4\theta$ and $y = a\sin^4\theta$

On differentiating w.r.t. θ, we get

$$\frac{dx}{d\theta} = 4\,a\cos^3\theta(-\sin\theta)$$

and $\frac{dy}{d\theta} = 4\,a\sin^3\theta\cos\theta$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{4a\sin^3\theta\cos\theta}{4\,a\cos^3\theta\sin\theta}$$

$$= -\frac{\sin^2\theta}{\cos^2\theta} = -\tan^2\theta$$

Now, $\left(\frac{dy}{dx}\right)_{\theta=\frac{3\pi}{4}} = -\tan^2\left(\frac{3\pi}{4}\right) = -1$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | D | | A | C | A | A | D | B | D | D |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | B | D | D | B | C | A | B | B | D | A |
| | | | | | | | | | | |

SESSION : 2025-26

# DPP
DAILY PRACTICE PRODUCTS

CLASS : XIIth
DATE :

**SOLUTIONS**

SUBJECT : MATHS
DPP NO. :3

## Topic :-DIFFERENTITATION

1 **(b)**

Given, $\left(\frac{x^2-y^2}{x^2+y^2}\right) = \sec^{-1} e^a$

$\Rightarrow \frac{\left[\begin{array}{l}(x^2+y^2)\left(2x-2y\frac{dy}{dx}\right)\\-(x^2-y^2)\left(2x+2y\frac{dy}{dx}\right)\end{array}\right]}{(x^2+y^2)^2}=0$

$$\Rightarrow (2x^3+2xy^2-2x^3+2xy^2)-2y\frac{dy}{dx}$$

$(x^2+y^2+x^2-y^2)=0$

$$\Rightarrow 4xy^2-4x^2y\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}$$

2 **(a)**

$$\because y=\frac{a+bx^{3/2}}{x^{5/4}}$$

On differentiating w.r.t. $x$, we get

$$y'=\frac{\frac{3}{2}bx^{7/4}-\frac{5}{4}(a+bx^{3/2})x^{1/4}}{(x^{5/4})^2}$$

$$\because y'=0 \text{ at } x=5$$

$$\therefore \frac{3}{2}bx^{7/4}-\frac{5}{4}(a+bx^{3/2})x^{1/4}=0, \text{at } x=5$$

$$\Rightarrow 6bx^{3/2}-5(a+bx^{3/2})=0, \text{at } x=5$$

$$\Rightarrow bx^{3/2}=5a, \text{at } x=5 \Rightarrow b(5)^{3/2}=5a$$

$$\Rightarrow \frac{a}{b}=\frac{5^{3/2}}{5}\Rightarrow a:b=\sqrt{5}:1$$

3 **(c)**

Given, $f(x)=be^{ax}+ae^{bx}$

$\Rightarrow f'(x)= abe^{ax}+abe^{bx}$

$\Rightarrow f''(x)=a^2be^{ax}+ab^2e^{bx}$

$\Rightarrow f''(0)=a^2b+ab^2=ab(a+b)$

4 **(b)**

Given, $x^my^n=(x+y)^{m+n}$

$$m\log x+n\log y=(m+n)\log(x+y)$$

$$\frac{m}{x}+\frac{n}{y}\frac{dy}{dx}=\frac{(m+n)}{(x+y)}\left[1+\frac{dy}{dx}\right]$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{n}{y}-\frac{(m+n)}{(x+y)}\right]=\frac{m+n}{x+y}-\frac{m}{x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1,y=2}=2$$

5 **(a)**

Since, $f\left(x-\frac{1}{x}\right)=x^3-\frac{1}{x^3}$

$$f\left(x-\frac{1}{x}\right)=\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+1\right)$$

$=\left(x-\frac{1}{x}\right)\left[\left(x-\frac{1}{x}\right)^2+3\right]$

$$\Rightarrow f(x)=x(x^2+3)=x^3+3x$$

$\Rightarrow f'(x)=3x^2+3$

6 **(b)**

We have,

$x=a\cos t+\frac{b}{2}\cos 2t\,, y=a\sin t+\frac{b}{2}\sin 2t$

$\Rightarrow \frac{dy}{dx}=-a\sin t-b\sin 2t\,, \frac{dy}{dt}=a\cos t+b\cos 2t$

$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{a\cos t+b\cos 2t}{-a\sin t-b\sin 2t}$

$\Rightarrow \frac{dy}{dx}=-\frac{(a\cos t+b\cos 2t)}{(a\sin t+b\sin 2t)}$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dt}\left(\frac{dy}{dt}\right)\cdot\frac{dt}{dx}=-\frac{d}{dt}\frac{(a\cos t+b\cos 2t)}{(a\sin t+b\sin 2t)}\cdot\frac{dt}{dx}$

$\Rightarrow \frac{d^2y}{dx^2}=-\left[\frac{a^2+2b^2+3ab\cos t}{(a\sin t+b\sin 2t)^3}\right]$

$\therefore \frac{d^2y}{dx^2}=0\Rightarrow a^2+2b^2+3ab\cos t=0\Rightarrow \cos t=-\left(\frac{a^2+2b^2}{3ab}\right)$

7 **(d)**

$h'(x)=[f(x)^2+g(x)^2]$

$\Rightarrow h''(x)=2f(x)f'(x)+2g(x)g'(x)$

$\left[\begin{array}{l}\because g(x)=f'(x)\\ \Rightarrow g'(x)=f''(x)\end{array}\right]$

$\therefore h''(x)=2f(x)g(x)+2g(x)(-f(x))$

$[\because f''(x)=-f(x)]$

$\Rightarrow h''(x)=0$

$\Rightarrow h'(x)=C$, a constant for all $x\in R$

$\Rightarrow h(x)=Cx+C_1$

$\Rightarrow h(0)=C_1$ and $h(1)=C+C_1$

$\Rightarrow 2 = C_1$ and $8 = C + C_1$
$\Rightarrow C_1 = 2$ and $C = 6$
$\therefore h(x) = 6x + 2$
$\Rightarrow h(2) = 6 \times 2 + 2 = 14$

8 **(b)**

Given, $y = x \log x$
On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = \frac{x}{x} + \log x$$
$$\Rightarrow \frac{dy}{dx} = \log e + \log x$$

$\Rightarrow \frac{dy}{dx} = \log(ex)$

9 **(b)**

Given, $f''(x) = -f(x)$
$\Rightarrow g'(x) = -f(x)$ and $f'(x) = g(x)$ ...(i)
Now, $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$
$\therefore F'(x) = 2\left(f\left(\frac{x}{2}\right)\right).f'\left(\frac{x}{2}\right).\frac{1}{2}$
$+2\left(g\left(\frac{x}{2}\right)\right).g'\left(\frac{x}{2}\right).\frac{1}{2} = 0$
[using Eq.(i)]
$\therefore$ $F(x)$ is a constant $\Rightarrow F(10) = F(5) = 5$

10 **(a)**

$$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$$
$$\Rightarrow y = \sqrt{\sin x + y}$$
$$\Rightarrow y^2 = \sin x + y$$

On differentiating w.r.t. $x$, we get

$$2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y - 1}$$

11 **(a)**

Given, $y = x^2 e^{mx} \Rightarrow \frac{dy}{dx} = 2xe^{mx} + mx^2 e^{mx}$
$\Rightarrow \frac{d^2y}{dx^2} = 2(e^{mx} + mxe^{mx}) + m(2xe^{mx} - x^2 me^{mx})$
$\Rightarrow \frac{d^2y}{dx^2} = e^{mx}(m^2x^2 + 4mx + 2)$
$\Rightarrow \frac{d^3y}{dx^3} = e^{mx}[m^3x^2 + 4m^2x + 2m + 2m^2x + 4m]$
$= e^{mx}[m^3x^2 + 6m^2x + 6m]$

12 **(d)**

Given, $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$

$\Rightarrow \quad x^4 + y^4 + 2x^2y^2 = t^2 + \dfrac{1}{t^2} - 2$

$\Rightarrow \quad x^4 + y^4 + 2x^2y^2 = x^4 + y^4 - 2$

$\Rightarrow \quad x^2y^2 + 1 = 0 \Rightarrow y^2 = \dfrac{-1}{x^2}$

On differentiating w.r.t. $x$, we get

$2y\,\dfrac{dy}{dx} = \dfrac{2}{x^3} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x^3y}$

13 **(d)**

Given that, $y = \cos^{-1}\sqrt{1-t^2} = \sin^{-1} t$

and $x = \sin^{-1}(3t - 4t^3) = 3\sin^{-1} t$

On differentiating both w.r.t. $t$ respectively, we get

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}} \text{ and } \frac{dx}{dt} = \frac{3}{\sqrt{1-t^2}}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\left(\frac{1}{\sqrt{1-t^2}}\right)}{3\left(\frac{1}{\sqrt{1-t^2}}\right)} \Rightarrow \frac{dy}{dx} = \frac{1}{3}$$

14 **(c)**

We have,

$f(x) = \sqrt{1 - \sin 2x} = \sqrt{(\cos x - \sin x)^2}$

$\Rightarrow f(x) = |\cos x - \sin x|$

$\Rightarrow f(x) = \begin{cases} \cos x - \sin x, & \text{for } 0 \le x \le \pi/4 \\ -(\cos x - \sin x), & \text{for } \pi/4 < x \le \pi/2 \end{cases}$

$\therefore f'(x) = \begin{cases} -(\cos x - \sin x), & \text{for } 0 < x < \pi/4 \\ (\cos x + \sin x), & \text{for } \pi/4 < x < \pi/2 \end{cases}$

15 **(c)**

Let $u = \sin x$ and $v = \cos x$

On differentiating w.r.t. $x$ respectively, we get

$$\frac{du}{dx} = \cos x \text{ and } \frac{dv}{dx} = -\sin x$$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = -\cot x$$

16 **(d)**

Let $y = F\{f(\phi(x))\}$

On differentiating w.r.t. $x$, we get

$$y' = F'[f\{\phi(x)\}]\frac{d}{dx}f\{\phi(x)\}$$

$$= F'[f\{\phi(x)\}]f'\{\phi(x)\}\frac{d}{dx}\phi(x)$$

$$= F'[f\{\phi(x)\}]f'\{\phi(x)\}\phi'(x)$$

17 **(b)**

Given, $x\sqrt{1+y} = -y\sqrt{1+x}$ ...(i)

On squaring both sides, we get

$$x^2(1+y) = y^2(1+x)$$

$\Rightarrow (x-y)(x+y) + xy(x-y) = 0$

$$\Rightarrow (x-y)(x+y+xy) = 0$$

$x - y \neq 0$ because it does not satisfy the Eq. (i).

$$\therefore x + y + xy = 0 \Rightarrow y = -\frac{x}{1+x}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(1+x)(1) - x(1)}{(1+x)^2} = -\frac{1}{(1+x)^2}$$

18 **(b)**

$$\because \frac{x^2 - y^2}{x^2 + y^2} = \sec^{-1}(e^a)$$

On differentiating w.r.t. $x$, we get

$$\frac{(x^2+y^2)\left(2x - 2y\frac{dy}{dx}\right) - (x^2-y^2)\left(2x + 2y\frac{dy}{dx}\right)}{(x^2+y^2)^2} = 0$$

$$\Rightarrow x(x^2+y^2) - y(x^2+y^2)\frac{dy}{dx}$$

$$= (x^2-y^2)x + y(x^2-y^2)\frac{dy}{dx}$$

$$\Rightarrow (x^2y - y^3 + x^2y + y^3)\frac{dy}{dx} = (x^3 + xy^2 - x^3 + xy^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy^2}{2x^2y} = \frac{y}{x}$$

19 **(a)**

We have,

$f(x) = \log_a(\log_a x)$

$\Rightarrow f'(x) = \dfrac{1}{\log_a x \cdot \log_e a}\dfrac{d}{dx}(\log_a x)$

$\Rightarrow f'(x) = \dfrac{1}{\log_a x \log_e a} \times \dfrac{1}{x \log_e a} = \dfrac{\log_a e}{x \log_e x}$

20 **(b)**

$$\because y = \log^n x$$

On differentiating w.r.t. $x$, we get

$$x \log x \log^2 x \log^3 x \dots \log^{n-1} x \log^n x \frac{dy}{dx}$$

$$= \frac{x \log x \log^2 x \log^3 x \dots \log^{n-1} x \log^n x . 1}{x \log x \log^2 x \log^3 x \dots \log^{n-1} x}$$

$$= \log^n x$$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | B | A | C | B | A | B | D | B | B | A |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | A | D | D | C | C | D | B | B | A | B |
| | | | | | | | | | | |

**SESSION : 2025-26**

# DPP
DAILY PRACTICE PROBLEMS

**CLASS : XIIth** | **SOLUTIONS** | **SUBJECT : MATHS**

**DATE :** | **DPP NO. :4**

## Topic :-DIFFERENTITATION

1 **(b)**

Let $D=\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$

$$\Rightarrow D=\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -p^3\cos px & p^4\sin px & p^5\cos px \\ -p^6\sin px & -p^7\cos px & p^8\sin px \end{vmatrix}$$

Taking $p^3$ and $p^6$common from $R_2$ and $R_3$row

$$=p^9\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -\cos px & p\sin px & p^2\cos px \\ -\sin px & -p\cos px & p^2\sin px \end{vmatrix}$$

$$=-p^9\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -\cos px & p\sin px & p^2\cos px \\ \sin px & p\cos px & -p^2\sin px \end{vmatrix}$$

$=0$ ($R_1$and $R_3$rows are identical)

2 **(d)**

$$y=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots$$

$$\Rightarrow y=e^{-x}$$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx}=-e^{-x}$$

Again differentiating w.r.t. $x$, we get

$$\frac{d^2y}{dx^2}=-e^{-x}(-1)=e^{-x}=y$$

3 **(b)**

We have, $f(4)=4$ and $f'(4)=1$

$$\therefore \lim_{x\to 4}\frac{2-\sqrt{f(x)}}{2-\sqrt{x}}=\lim_{x\to 4}\frac{\frac{-f'(x)}{2\sqrt{f(x)}}}{-\frac{1}{2\sqrt{x}}}\ \text{[Using L'Hospital's Rule]}$$

$$\Rightarrow \lim_{x\to 4}\frac{2-\sqrt{f(x)}}{2-\sqrt{x}}=\lim_{x\to 4}\frac{\sqrt{x}\,f'(x)}{\sqrt{f(x)}}=\frac{2f'(4)}{\sqrt{f(4)}}=\frac{2}{2}=1$$

4 **(c)**

$\because\ x = \dfrac{2at}{1+t^3}$ and $y = \dfrac{2at^2}{(1+t^3)^2}$

$\therefore 2ay = x^2$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{x}{a}$

5 **(c)**

Given, $y = e^{a\sin^{-1}x}$

On differentiating w.r.t. $x$, we get

$y_1 = e^{a\sin^{-1}x} a.\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow\ y_1\sqrt{1-x^2} = ay$

$\Rightarrow\ (1-x^2)y_1^2 = a^2y^2$

Again, differentiating w.r.t. $x$, we get

$(1-x^2)2y_1y_2 - 2xy_1^2 = a^2 2yy_1$

$\Rightarrow (1-x^2)y_2 - xy_1 - a^2y = 0$

Using Leibnitz's rule,

$(1-x^2)y_{n+2} + {}^nC_1 y_{n+1}(-2x) + {}^nC_2 y_n(-2)$

$-xy_{n+1} - {}^nc_1 y_n - a^2y_n = 0$

$\Rightarrow\ (1-x^2)y_{n+2} + xy_{n+1}(-2n-1)$

$+y_n[-n(n-1) - n - a^2] = 0$

$\Rightarrow\ (1-x^2)y_{n+2} - (2n+1)xy_{n+1} = (n^2+a^2)y_n$

6 **(a)**

Since, $\frac{x-y}{x+y} = \sec^{-1}a$

$\Rightarrow\ \frac{(x+y)\left(1-\frac{dy}{dx}\right)-(x-y)\left(1+\frac{dy}{dx}\right)}{(x+y)^2}=0$

$$\Rightarrow x + y - x + y - (x+y+x-y)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

7 **(c)**

Given, $y = x + x^2 + x^3 + \ldots \Rightarrow y = \frac{x}{1-x}$

$$\Rightarrow\ x = \frac{y}{1+y} = y - y^2 + y^3 - \ldots$$

On differentiating w.r.t. $y$, we get

$$\frac{dx}{dy} = 1 - 2y + 3y^2 - \ldots$$

8 **(b)**

Let $y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} = \frac{\cos x - \sin x}{\cos x + \sin x}$

$$= \frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4} - x\right)$$

$\Rightarrow\ \frac{dy}{dx} = -\sec^2\left(\frac{\pi}{4} - x\right)$

9 **(b)**

Let $u = \log_{10} x$ and $v = x^2$

$$\therefore \frac{du}{dx} = \frac{\log_{10} e}{x} \text{ and } \frac{dv}{dx} = 2x$$

$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\log_{10} e}{x}/2x$

$$= \frac{\log_{10} e}{2x^2}$$

10 **(d)**

$$\because y = x \text{ In}\left(\frac{x}{a+bx}\right) = x(\text{In } x - \text{In}(a+bx))$$

$$\text{or } \left(\frac{y}{x}\right) = \text{In } x - \text{In }(a+bx)$$

On differentiating both sides w.r.t. $x$, we get

$$\left(\frac{x\frac{dy}{dx} - y.1}{x^2}\right) = \frac{1}{x} - \frac{b}{a+bx} = \frac{a}{x(a+bx)} \quad \dots \text{(i)}$$

or $\left(x\frac{dy}{dx} - y\right) = \frac{ax}{a+bx}$

On taking log on both sides, we get

$$\text{In }\left(x\frac{dy}{dx} - y\right) = \text{In }(ax) - \text{In }(a+bx)$$

On differentiating both sides w.r.t. $x$, we get

$$\frac{x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx}}{\left(x\frac{dy}{dx} - y\right)} = \frac{1}{x} - \frac{b}{a+bx} = \frac{a}{x(a+bx)}$$

$$= \frac{\left(x\frac{dy}{dx} - y\right)}{x^2} \quad \text{[from Eq. (i)]}$$

or $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$

12 **(a)**

On differentiating given equation w.r.t. $x$, we get

$$4x - 3x\frac{dy}{dx} - 3y + 2y\frac{dy}{dx} + 1 + 2\frac{dy}{dx} - 0 = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{3y - 4x - 1}{2y - 3x + 2}$$

13 **(c)**

Given, $y = \sqrt{\sin x + y} \quad \Rightarrow \quad y^2 = \sin x + y$

$$\Rightarrow 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$$

$$\Rightarrow (2y-1)\frac{dy}{dx} = \cos x$$

14 **(b)**

We have,
$2^x + 2^y = 2^{x+y}$
Differentiating with respect to $x$, we get
$2^x \log 2 + 2^y \log 2 \dfrac{dy}{dx} = 2^{x+y} \log 2 \left(1 + \dfrac{dy}{dx}\right)$
$\Rightarrow 2^x + 2^y \dfrac{dy}{dx} = 2^{x+y}\left(1 + \dfrac{dy}{dx}\right)$
$\Rightarrow \dfrac{dy}{dx} = \dfrac{2^x - 2^{x+y}}{2^{x+y} - 2y} \Rightarrow \left(\dfrac{dy}{dx}\right)_{(1,0)} = \dfrac{2-4}{4-2} = -1$
15 **(a)**
Given, $y = \sec(\tan^{-1} x)$
$$\Rightarrow\ y = \sec(\sec^{-1}\sqrt{1+x^2}) = \sqrt{1+x^2}$$
On differentiating w.r.t. $x$,we get
$$\frac{dy}{dx} = \frac{1}{2\sqrt{1+x^2}}(2x) = \frac{x}{\sqrt{1+x^2}}$$
16 **(b)**
Given, $f(x) = (x-7)^2(x-2)^7$
$\Rightarrow \quad f(\theta) = (\theta-7)^2(\theta-2)^7$
On differentiating w.r.t. $\theta$, we get
$\Rightarrow \quad f'(\theta) = 2(\theta-7)(\theta-2)^7 + 7(\theta-2)^6(\theta-7)^2$
put $\quad f'(\theta) = 0$
$\Rightarrow \quad (\theta-7)(\theta-2)^6[2(\theta-2) + 7(\theta-7)] = 0$
$\Rightarrow \quad 9\theta = 53 \Rightarrow \theta = \frac{53}{9}$
17 **(b)**
We have,
$x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$
$\Rightarrow (x^2+y^2)^2 = t^2 + \dfrac{1}{t^2} - 2$
$\Rightarrow (x^2+y^2)^2 = x^4 + y^4 - 2$
$\Rightarrow 2x^2y^2 = -2$
$\Rightarrow x^2y^2 = -1$
$\Rightarrow y^2 = -\dfrac{1}{x^2} \Rightarrow 2y\dfrac{dy}{dx} = \dfrac{2}{x^3} \Rightarrow x^3y\dfrac{dy}{dx} = 1$
18 **(b)**
Let $f(x) = |x-1| + |x-5|$
$$\Rightarrow f(x) = \begin{cases} -2x+6, & x < 1 \\ 4 & 1 \le x < 5 \\ 2x-6 & x \ge 5 \end{cases}$$
$$\therefore \frac{d}{dx}(f(x)) = \begin{cases} -2, & x < 1 \\ 0 & 1 < x < 5 \\ 2 & x > 5 \end{cases}$$
Hence, $\left(\frac{d}{dx}(f(x))\right)_{x=3} = 0$
19 **(b)**

$$\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$$
$$\Rightarrow \sin^{-1} x = \frac{\pi}{2} - \sin^{-1} y$$
$$\Rightarrow \sin^{-1} x = \cos^{-1} y$$
$$\Rightarrow y = \sqrt{1 - x^2}$$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}(-2x) = -\frac{x}{y}$$

20 **(c)**

$$f[f(f(x))] = f\left[f\left(\frac{1}{1-x}\right)\right] = f\left(\frac{1}{1-\frac{1}{1-x}}\right)$$

$$\left[\because f(x) = \frac{1}{1-x}\right]$$

$$= f\left(\frac{1-x}{-x}\right) = \frac{1}{1+\left(\frac{1-x}{x}\right)}$$

$$\Rightarrow f[f(f(x))] = x$$

$\therefore$ The derivative of composite function is equal to 1.

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | B | D | B | C | C | A | C | B | B | D |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | D | A | C | B | A | B | B | B | B | C |
| | | | | | | | | | | |

**SESSION : 2025-26**

# DPP
DAILY PRACTICE PROBLEMS

**CLASS : XIIth**
**DATE :**

**SOLUTIONS**

**SUBJECT : MATHS**
**DPP NO. :5**

## Topic :-DIFFERENTITATION

1 **(a)**

Given that, $x = \exp\left\{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right\}$

Taking log on both sides, we get

$$\log x = \tan^{-1}\left(\frac{y-x^2}{x^2}\right)$$

$$\Rightarrow \frac{y-x^2}{x^2} = \tan(\log x)$$

$$\Rightarrow y = x^2 \tan(\log x) + x^2$$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = 2x\tan(\log x) + x^2\frac{\sec^2(\log x)}{x} + 2x$$

$$\Rightarrow \frac{dy}{dx} = 2x\tan(\log x) + x\sec^2(\log x) + 2x$$

$$\Rightarrow \frac{dy}{dx} = 2x[1 + \tan(\log x)] + x\sec^2(\log x)$$

2 **(b)**

Given, $x = \frac{\sin y}{\sin(a+y)}$

$\Rightarrow \frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin\cos(a+y)}{\sin^2(a+y)}$

$$= \frac{\sin(a+y-y)}{\sin^2(a+y)}$$

$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

3 **(b)**

Given, $f(x) = e^x \sin x$

$\Rightarrow \quad f'(x) = e^x\cos x + \sin x e^x$

$\Rightarrow \quad f''(x) = e^x\cos x - e^x \sin x + e^x\sin x + e^x \cos x$

$= 2e^x\cos x$

4 **(b)**

We know that, $\cos A\cos 2A\cos 2^2 A \ldots \cos 2^{n-1}A = \frac{\sin(2^n A)}{2^n \sin A}$

$$\therefore \cos x\cos 2x\cos 4x\cos 8x\cos 16x = \frac{\sin 32x}{32\sin x}$$

$$\Rightarrow f(x) = \frac{1}{32} . \frac{\sin 32x}{\sin x}$$

$$\therefore f'(x) = \frac{1}{32} \times \frac{\sin x(32 \cos 32x) - \sin 32x \cos x}{\sin^2 x}$$

$$\Rightarrow f'\left(\frac{\pi}{4}\right) = \frac{1}{\sin \frac{\pi}{4}} = \sqrt{2}$$

5 **(a)**

Given, $\frac{1+x}{1-y} = \sec a$

$$\Rightarrow y \sec a = \sec a - 1 - x$$

$$\Rightarrow \frac{dy}{dx} \sec a = -1$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sec a} = \frac{-1}{\left(\frac{1+x}{1-y}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-1}{x+1}$$

6 **(c)**

Let $u = a^{\sec x}$ and $v = a^{\tan x}$

$\Rightarrow \quad \frac{du}{dx} = a^{\sec x} \log_e a . \sec x \tan x$

and $\frac{dv}{dx} = a^{\tan x} \log_e a . \sec^2 x$

$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{a^{\sec x} \log_e a \sec x \tan x}{a^{\tan x} \log_e a \sec^2 x}$

$= a^{\sec x - \tan x} \sin x$

7 **(b)**

On differentiating given curves w.r.t.θ respectively, we get

$$\frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\tan\left(\frac{\theta}{2}\right)} . \sec^2 \frac{\theta}{2} . \frac{1}{2}\right)$$

and $\frac{dy}{d\theta} = a \cos\theta$

$\Rightarrow \frac{dx}{d\theta} = \frac{a \cos^2\theta}{\sin\theta}$ and $\frac{dy}{d\theta} = a\cos\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta}{a\cos^2\theta/\sin\theta} = \tan\theta$

8 **(c)**

Since, $\phi(x) = f^{-1}(x) \quad \Rightarrow \quad x = f\{\phi(x)\}$

On differentiating w.r.t. $x$, we get

$$1 = f'\{\phi(x)\}. \phi'(x)$$

$\Rightarrow \quad \phi'(x) = \frac{1}{f'\{\phi(x)\}}$ ... (i)

But $f'\{\phi(x)\} = \frac{1}{1+\{\phi(x)\}^5} \quad \left(\because f'(x) = \frac{1}{1+x^5}\right)$

$\therefore$ From Eq. (i),

$\phi'(x) = \frac{1}{f'\{\phi(x)\}} = 1 + \{\phi(x)\}^5$

9 **(b)**

We have,

$f(x) = 3\, e^{x^2} \Rightarrow f'(x) = 6x\, e^{x^2}$

$\therefore f(0) = 3$ and $f'(0) = 0$

Now,

$$f'(x) - 2\,x\,f(x) + \frac{1}{3}f(0) - f'(0) = 6\,x\,e^{x^2} - 6\,x\,e^{x^2} + \frac{1}{3}(3) - 0 = 1$$

10 **(c)**

On differentiating partially the given equation w.r.t. $x$ and $y$

$$\frac{\partial u}{\partial x} = \frac{(x+y)}{(x^2+\ y^2)} \times \frac{x^2 - y^2 + 2xy}{(x+y)^2}$$

$$\Rightarrow\ x\frac{\partial u}{\partial x} = \frac{x}{(x^2+y^2)} \times \frac{x^2-y^2+2xy}{(x+y)} \qquad \text{...(i)}$$

and $\dfrac{\partial u}{\partial y} = \dfrac{(x+y)}{(x^2+y^2)} \times \dfrac{y^2-x^2+2xy}{(x+y)^2}$

$$\Rightarrow y\frac{\partial u}{\partial y} = \frac{y}{(x^2+y^2)} \times \frac{(y^2-x^2+2xy)}{(x+y)} \qquad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{(x^2+y^2)(x+y)}$$

$$[x(x^2 - y^2 + 2xy) + y(y^2 - x^2 + 2xy)]$$

$$= \frac{1}{(x^2+y^2)(x+y)} \times (x^2+y^2)(x+y)$$

$= 1$

11 **(a)**

We have,

$$F(x) = \frac{1}{x^2}\int_4^x (4t^2 - 2F'(t))\,dt$$

$$\Rightarrow x^2F(x) = \int_4^x (4t^2 - 2F'(t))\,dt$$

Differentiating both sides with respect to $x$, we get

$2x\,F(x) + x^2F'(x) = 4x^2 - 2F'(x)$

Putting $x = 4$, we get

$8F(4) + 16F'(4) = 64 - 2F'(4)$

$\Rightarrow 18F'(4) = 64 \quad [\because F(4) = 0]$

$$\Rightarrow F'(4) = \frac{32}{9}$$

12 **(d)**

Put $x^2 = \cos 2\theta$ in the given equation, we get

$$y = \tan^{-1}\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}$$

$$= \tan^{-1}\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$= \tan^{-1}\tan\left(\frac{\pi}{4} - \theta\right)$

$\Rightarrow\ y = \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x^2$

$\Rightarrow\ \frac{dy}{dx} = 0 - \frac{1}{2}\left(-\frac{(2x)}{\sqrt{1-x^4}}\right) = \frac{x}{\sqrt{1-x^4}}$

13 **(b)**

We have,

$f(x) = |x^2 - 5x + 6|$

$\Rightarrow f(x) = \begin{cases} x^2 - 5x + 6, & \text{if } x \geq 3 \text{ or } x \leq 2 \\ -(x^2 - 5x + 6), & \text{if } 2x < x < 3 \end{cases}$

$\therefore f'(x) = \begin{cases} (2x - 5), & \text{if } x > 3 \text{ or } x < 2 \\ -(2x - 5), & \text{if } 2 < x < 3 \end{cases}$

15 **(c)**

Let $y = x^6 + 6^x$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = 6x^5 + 6^x \log 6$$

16 **(b)**

We have,

$f(x) = \cos^2 x + \cos^2(x + \pi/3) + \sin x \sin(x + \pi/3)$

$\Rightarrow f(x) = \frac{1}{2}\left[1 + \cos\ 2x + 1 + \cos\left(2x + \frac{2\pi}{3}\right) + \cos\frac{\pi}{3} - \cos\left(2x + \frac{\pi}{3}\right)\right]$

$\Rightarrow f(x) = \frac{1}{2}\left[\frac{5}{2} + \cos\ 2x + \cos\left(2x + \frac{2\pi}{3}\right) - \cos\left(2x + \frac{\pi}{3}\right)\right]$

$\Rightarrow f(x) = \frac{1}{2}\left[\frac{5}{2} + 2\cos\left(2x + \frac{\pi}{3}\right)\cos\frac{\pi}{3} - \cos\left(2x + \frac{\pi}{3}\right)\right]$

$\Rightarrow f(x) = \frac{5}{4}$

$\therefore gof(x) = g(5/4) = 3$ for all $x$

$\Rightarrow \frac{d}{dx}\left(gof(x)\right) = 0$ for all $x$

17 **(c)**

We have,

$y = \sin^{-1}x + \cos^{-1}y \Rightarrow y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$

18 **(b)**

Given, $z = y + f(v)$ ...(i)

Where $v = \left(\frac{x}{y}\right)$

On differentiating partially Eq. (i) w.r.t. $x$, we get

$\frac{\partial z}{\partial x} = f'\left(\frac{x}{y}\right).\left(\frac{1}{y}\right)$

$\Rightarrow v\frac{\partial z}{\partial x} = f'\left(\frac{x}{y}\right).\left(\frac{1}{y}\right)\left(\frac{x}{y}\right) = f'\left(\frac{x}{y}\right)\left(\frac{x}{y^2}\right)$ ...(ii)

Now, differentiating partially Eq. (i) w.r.t. $y$, we get

$\frac{\partial z}{\partial y} = 1 + f'\left(\frac{x}{y}\right)\left(\frac{-x}{y^2}\right)$ ...(iii)

On adding Eqs. (ii) and (iii), we get

$v\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = f'\left(\frac{x}{y}\right)\left(\frac{x}{y^2}\right) + 1 + f'\left(\frac{x}{y}\right)\left(\frac{-x}{y^2}\right) = 1$

19 **(d)**

$$y = \tan^{-1}\left(\frac{\sqrt{x} - x}{1 + x^{3/2}}\right)$$
$$= \tan^{-1}\left(\frac{\sqrt{x} - x}{1 + \sqrt{x}.x}\right)$$
$$= \tan^{-1}(\sqrt{x}) - \tan^{-1}(x)$$

On differentiating w.r.t. $x$, we get

$$y' = \frac{1}{1 + x}.\frac{1}{2\sqrt{x}} - \frac{1}{1 + x^2}$$
$$\Rightarrow y'(1) = \frac{1}{2}.\frac{1}{2} - \frac{1}{2} = -\frac{1}{4}$$

20 **(c)**

We have,

$f(1) = 1$ and $f'(1) = 2$

$\therefore \lim_{x\to 1}\frac{\sqrt{f(x)} - 1}{\sqrt{x} - 1}$

$= \lim_{x\to 1}\frac{\frac{f'(x)}{2\sqrt{f(x)}}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to 1}\frac{\sqrt{x}\,f'(x)}{\sqrt{f(x)}} = \frac{f'(1)}{\sqrt{f(1)}} = 2$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | A | B | B | B | A | C | B | C | B | C |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | A | D | B | C | C | B | C | B | D | C |
| | | | | | | | | | | |

# DPP

DAILY PRACTICE PROBLEMS

**SESSION : 2025-26**

**CLASS : XIIth**
**DATE :**

**SOLUTIONS**

**SUBJECT : MATHS**
**DPP NO. :6**

## Topic :-DIFFERENTITATION

1 **(a)**

Let $u = \cos^3 x, v = \sin^3 x$

$\frac{du}{dx} = -3\cos^2 x \sin x, \frac{dv}{dx} = 3\sin^2 x \cos x$

Now, $\frac{du}{dv} = \frac{-3\cos^2 x \sin x}{3\sin^2 x \cos x} = -\cot x$

2 **(b)**

We have,

$y = \log\left(\frac{1+x}{1-x}\right)^{1/4} - \frac{1}{2}\tan^{-1} x$

$\Rightarrow y = \frac{1}{4}\log(1+x) - \frac{1}{4}\log(1-x) - \frac{1}{2}\tan^{-1} x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{4(1+x)} + \frac{1}{4(1-x)} - \frac{1}{2(1+x^2)}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2(1-x^2)} - \frac{1}{2(1+x^2)} = \frac{x^2}{1-x^4}$

3 **(a)**

Given, $y = \left(x + \sqrt{1+x^2}\right)^n$

$\frac{dy}{dx} = n\left[x + \sqrt{1+x^2}\right]^{n-1}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$

$= \frac{n[x+\sqrt{1+x^2}]^n}{\sqrt{1+x^2}}$

$\Rightarrow (1+x^2)\left(\frac{dy}{dx}\right)^2 = n^2 y^2$

$\Rightarrow 2\frac{dy}{dx}\frac{d^2y}{dx^2}(1+x^2) + 2x\left(\frac{dy}{dx}\right)^2 = 2n^2 y\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2}(1+x^2) + x\frac{dy}{dx} = n^2 y$

4 **(d)**

Let $y = \tan^{-1}\left\{\frac{3\sqrt{x} - x^{3/2}}{1-3x}\right\}$

Again let $\sqrt{x} = \tan t$

$\therefore\ y = \tan^{-1}\left\{\frac{3\tan t - \tan^3 t}{1 - 3\tan^2 t}\right\} = \tan^{-1}(\tan 3t)$

$\Rightarrow \ y = 3\tan^{-1}\sqrt{x}$

$\Rightarrow \ \dfrac{dy}{dx} = \dfrac{3}{1+x}.\dfrac{1}{2\sqrt{x}} = \dfrac{3}{2(1+x)\sqrt{x}}$

5 **(d)**

We have, $y = \tan^{-1}\left[\frac{\sin x+\cos x}{\cos x-\sin x}\right]$

$$\Rightarrow y = \tan^{-1}\left[\frac{1+\tan x}{1-\tan x}\right]$$

$$\Rightarrow y = \tan^{-1}\left[\frac{\tan\left(\frac{\pi}{4}\right)+\tan x}{1-\tan\left(\frac{\pi}{4}\right)\tan x}\right]$$

$$\Rightarrow y = \tan^{-1}\tan\left(\frac{\pi}{4}+x\right)$$

$$\Rightarrow y = \left(\frac{\pi}{4}\right)+x$$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = 1$$

6 **(d)**

Given, $x = \cos\theta, \quad y = \sin 5\theta$

$\Rightarrow \ \dfrac{dx}{d\theta} = \ -\sin\theta, \quad \dfrac{dy}{d\theta} = 5\cos 5\theta$

$\therefore \ \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = -\dfrac{5\cos 5\theta}{\sin\theta}$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{d}{d\theta}\left(\dfrac{dy}{dx}\right).\dfrac{d\theta}{dx}$

$= \dfrac{d}{d\theta}\left(\dfrac{-5\cos 5\theta}{\sin\theta}\right)\dfrac{1}{-\sin\theta}$

$= \left(\dfrac{\sin\theta\sin 5\theta.25+5\cos 5\theta\cos\theta}{\sin^2\theta}\right).\dfrac{1}{-\sin\theta}$

$= -\dfrac{25\sin 5\theta}{\sin^2\theta} - \dfrac{5\cos 5\theta\cos\theta}{\sin^3\theta}$

$\therefore (1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx}$

$= (1-\cos^2\theta)\left(\dfrac{-25\sin 5\theta}{\sin^2\theta} - \dfrac{5\cos 5\theta\cos\theta}{\sin^3\theta}\right)$

$$-\cos\theta\left(\tfrac{-5\cos 5\theta}{\sin\theta}\right)$$

$= \ \sin^2\theta\left(\dfrac{-25\sin 5\theta}{\sin^2\theta} - \dfrac{5\cos\theta\cos 5\theta}{\sin^3\theta}\right) + \dfrac{5\cos\theta\cos 5\theta}{\sin\theta}$

$= -25\sin 5\theta - \dfrac{5\cos\theta\cos 5\theta}{\sin\theta} + \dfrac{5\cos\theta\cos 5\theta}{\sin\theta}$

$= -25y$

7 **(d)**

$$\because f(x) = a+bx$$

$$f\{f(x)\} = a+b(a+bx)$$

$$= ab+a+b^2x = a(1+b)+b^2x$$

$$f[f\{f(x)\}] = f\{a(1+b)+b^2x\}$$
$$= a + b\{a(1+b)+b^2x\}$$
$$= a(1+b+b^2)+b^3x$$
$$\therefore f'(x) = a(1+b+b^2+\ldots+b^{r-1})+b^rx$$
$$= a\left(\frac{b^r-1}{b-1}\right)+b^rx$$

8 **(c)**

We have,

$x^y = e^{x-y} \Rightarrow y\log x = (x-y) \Rightarrow y = \dfrac{x}{1+\log x}$

Differentiating w.r.t. $x$, we get $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

9 **(d)**

Let $u = \sin^2 x$ and $v = \cos^2 x$

On differentiating w.r.t. $x$, we get

$$\frac{du}{dx} = 2\sin x\cos x = \sin 2x$$

and $\frac{dv}{dx} = -2\cos x\sin x = -\sin 2x$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\sin 2x}{-\sin 2x} = -1$$

10 **(a)**

We have,

$x^p\, y^q = (x+y)^{p+q}$

$\Rightarrow p\log x + q\log y = (p+q)\log(x+y)$

Diff w.r.t. $x$, we get

$\dfrac{p}{x}+\dfrac{q}{y}\dfrac{dy}{dx} = \dfrac{p+q}{x+y}\left(1+\dfrac{dy}{dx}\right)$

$\Rightarrow \dfrac{dy}{dx}\left(\dfrac{q}{y}-\dfrac{p+q}{x+y}\right) \Rightarrow \dfrac{p+q}{x+y}-\dfrac{p}{x} \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x}$

12 **(a)**

Let $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$

Putting $x = \cos\theta$, we get

$y = \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\cos\dfrac{\theta}{2}+\dfrac{1}{\sqrt{2}}\sin\dfrac{\theta}{2}\right) = \sin^{-1}\left\{\sin\left(\dfrac{\pi}{4}+\dfrac{\theta}{2}\right)\right\}$

$\Rightarrow y = \dfrac{\pi}{4}+\dfrac{1}{2}\theta = \dfrac{\pi}{4}+\dfrac{1}{2}\cos^{-1}x$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{1}{2\sqrt{1-x^2}}$

13 **(c)**

Let $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

Put $x = \cos 2\theta$

$$\therefore \quad y=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$
$$=\tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$$
$$=\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)=\tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\theta\right)\right\}$$
$$=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

On differentiating w.r.t. $x$, we get

$\frac{dy}{dx}=0+\frac{1}{2}.\frac{1}{\sqrt{1-x}}=\frac{1}{2\sqrt{1-x^2}}$

14 **(c)**

$y=\tan^{-1}x+\cot^{-1}x+\sec^{-1}x+\operatorname{cosec}^{-1}x$

$=\ \frac{\pi}{2}+\frac{\pi}{2}=\pi$

$\frac{dy}{dx}\ =0$

15 **(c)**

$$\because y=\left(\frac{ax+b}{cx+d}\right)$$

or $c\,xy+dy=ax+b$

On differentiating both sides w.r.t. $x$, we get

$$c\left\{x\frac{dy}{dx}+y.1\right\}+d\frac{dy}{dx}=a$$

or $x\frac{dy}{dx}+y+\left(\frac{d}{c}\right)\frac{dy}{dx}=\left(\frac{a}{c}\right)$

Again differentiating both sides w.r.t. $x$, we get

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}+\left(\frac{d}{c}\right)\frac{d^2y}{dx^2}=0$$

or $x+\frac{2\frac{dy}{dx}}{\left(\frac{d^2y}{dx^2}\right)}+\frac{d}{c}=0$

Again, on differentiating both sides w.r.t. $x$, we get

$$1+\frac{\left(\frac{d^2y}{dx^2}.2\frac{d^2y}{dx^2}-2\frac{dy}{dx}.\frac{d^3y}{dx^3}\right)}{\left(\frac{d^2y}{dx^2}\right)}+0=0$$
$$\Rightarrow 2\frac{dy}{dx}.\frac{d^3y}{dx^3}=3\left(\frac{d^2y}{dx^2}\right)^2$$

16 **(d)**

Given, $y=(\log_{\cos x}\sin x)(\log_{\sin x}\cos x)+\sin^{-1}\frac{2x}{1+x^2}$

At $x=\frac{\pi}{2}$, $\log_{\sin x}$ cos x is not defined.

Hence, we cannot determined the derivative at $x=\frac{\pi}{2}$.

17 **(b)**

$y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

$= \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right)$

$$= \tan^{-1}\left[\tan\left\{\tan^{-1}\left(\frac{a}{b}\right) - x\right\}\right]$$

$\Rightarrow \quad y = \tan^{-1}\left(\frac{a}{b}\right) - x$

$\therefore \quad \frac{dy}{dx} = 0 - 1 = -1$

18 **(d)**

We have, $x\cos\theta$ , $y = \sin 5\theta$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{5\cos 5\theta}{\sin\theta}$

$\Rightarrow \frac{d^2y}{dx^2} = -5\frac{d}{d\theta}\left(\frac{\cos 5\theta}{\sin\theta}\right)\cdot\frac{d\theta}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-25\sin\theta\sin 5\theta - 5\cos\theta\cos 5\theta}{\sin^3\theta}$

$\therefore (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -25\sin 5\theta = -25y$

19 **(c)**

Let $y = \tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$

Put $x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1} x$

$$\therefore y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta} - \sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{1 - \tan\theta}{1 + \tan\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \theta\right)\right)$$

$$\Rightarrow y = \frac{\pi}{4} - \theta$$

$$\Rightarrow y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = -\frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{1}{2\sqrt{1-x^2}}$$

20 **(b)**

$\because \ f(x) = (x-2)(x-4)(x-6)\ldots(x-2n)$

Taking log on both sides in the given equation, we get

$\log f(x) = \log(x-2) + \log(x-4) + \ldots + \log(x-2n)$

on differentiating w.r.t. $x$, we get

$$\frac{1}{f(x)} f'(x) = \frac{1}{(x-2)} + \frac{1}{(x-4)} + \ldots + \frac{1}{(x-2n)}$$

$$\Rightarrow \quad f'(x) = (x-4)(x-6)\ldots(x-2n)$$
$$+(x-2)(x-6)\ldots(x-2n)$$
$$+\cdots+(x-2)(x-6)\ldots(x-2(n-1))$$

$\therefore \ f'(2) = (-2)(-4)\ldots(2-2n)$

$= (-2)^{n-1}(1.2\ldots\ldots(n-1) = (-2)^{n-1}(n-1)!$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | A | B | A | D | D | D | D | C | D | A |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | B | A | C | C | C | D | B | D | C | B |
| | | | | | | | | | | |

SESSION : 2025-26

**DPP**
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

**SOLUTIONS**

SUBJECT : MATHS
DPP NO. :7

# Topic :-DIFFERENTITATION

1 **(a)**

Given, $x = 2\cos\theta - \cos 2\theta$

and $y = 2\sin\theta - \sin 2\theta$

$\frac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$

and $\frac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$\therefore \quad \frac{dy}{dx} = \frac{2\cos\theta - 2\cos 2\theta}{-2\sin\theta + 2\sin 2\theta}$

$$= \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$$

$= \frac{2\sin\left(\frac{\theta+2\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)}{2\cos\left(\frac{\theta+2\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)} = \tan\frac{3\theta}{2}$

2 **(c)**

Since, $f'(x) > \phi'(x)$

$\Rightarrow 2^{2x-1} 2\log 2 > -2^x \log 2 + 2\log 2$

$\Rightarrow 2^{2x} > -2^x + 2$

$\Rightarrow 2^{2x} + 2^x - 2 > 0$

$\Rightarrow (2^x - 1)(2^x + 2) > 0$

$\Rightarrow 2^x - 1 > 0 \qquad [\because 2^x + 2 > 0 \text{ for all } x]$

$\Rightarrow 2^x > 1$

$\therefore x > 0$

3 **(b)**

We have,

$x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 - y^2 = -x^2y + xy^2$

$\Rightarrow (x-y)(x+y) = -xy(x-y)$

$\Rightarrow x + y = -xy$

$\Rightarrow y = -\frac{x}{1+x} \Rightarrow \frac{dy}{dx} = -\left\{\frac{(1+x)-x}{(1+x)^2}\right\} = -\frac{1}{(1+x)^2}$

4 **(c)**

$\because y = e^{(1/2)\log(1+\tan^2 x)}$

$$\Rightarrow \quad y = (\sec^2 x)^{1/2} = \sec x$$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = \sec x \tan x$$

5 **(a)**

Given,

$$f(x) = \frac{1}{4}\left[\frac{x-1}{1} + \frac{(x-1)^3}{3} + \frac{(x-1)^5}{5} + \frac{(x-1)^7}{7} + \cdots\right]$$

$$\Rightarrow \ f(x) = \frac{1}{4}\left[\frac{1}{2}\log(\frac{1+(x-1)}{1-(x-1)}\right] = \frac{1}{8}\log\left(\frac{x}{2-x}\right)$$

$$\Rightarrow \ f'(x) = \frac{1}{8} \times \frac{1}{\left(\frac{x}{2-x}\right)}\left[\frac{(2-x)1 - x(-1)}{(2-x)^2}\right] = \frac{1}{4x(2-x)}$$

6 **(d)**

$$f(x) = \begin{vmatrix} x^3 & x^2 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow f(x) = x^3(-6p^3 - 4p^2) - x^2(p^3 - 4p) + 3x^2(p^2 + 6p)$$

$$\Rightarrow f(x) = -6p^3x^3 - 4p^2x^3 - x^2p^3 + 4px^2 + 3p^2x^2 + 18px^2$$

On differentiating w.r.t. $x$, we get

$$\frac{d}{dx}f(x) = -18p^3x^2 - 12p^2x^2 - 2xp^3 + 8px + 6p^2x + 36px$$

Again differentiating w.r.t. $x$, we get

$$\frac{d^2}{dx^2}f(x) = -36p^3x - 24p^2x - 2p^3 + 8p + 6p^2 + 36p$$

Again differentiating w.r.t. $x$, we get

$\frac{d^3}{dx^3}f(x) = -36p^3 - 24p^2$ =constant

7 **(d)**

We have,

$f(x) = \arctan\left(\frac{x^x - x^{-x}}{2}\right)$

$\Rightarrow f(x) = \tan^{-1}\left\{\frac{x^{2x} - 1}{2x^x}\right\}$

$\Rightarrow f(x) = -\tan^{-1}\left\{\frac{1 - x^{2x}}{2x^x}\right\}$

$\Rightarrow f(x) = -\cot^{-1}\left\{\frac{2x^x}{1 - x^{2x}}\right\}$

$\Rightarrow f(x) = \frac{-\pi}{2} + \tan^{-1}\left\{\frac{2x^x}{1 - x^{2x}}\right\}$

$\Rightarrow f(x) = \begin{cases} \frac{-\pi}{2} + 2\tan^{-1}(x^x), & \text{if } 0 < x < 1 \\ \frac{-\pi}{2} - \pi + 2\tan^{-1}(x^x), & \text{if } x > 1 \end{cases}$

$$\Rightarrow f(x)=\begin{cases}\dfrac{-\pi}{2}+2\tan^{-1}(x^x), & \text{if } 0<x<1\\ \dfrac{-3\pi}{2}+2\tan^{-1}(x^x), & \text{if } x>1\end{cases}$$

$\Rightarrow f'(x)=\frac{2}{1+x^{2x}}\times x^x(1+\log_e x)$ for all $x>0, x\neq 1$

Clearly, $f'(1)$ does not exist

8 **(a)**

We have,

$f(x)+f(y)+f(x)f(y)=1$ for all $x,y\in R$ ...(i)

Putting $x=y=0$, we get

$2f(0)+\{f(0)\}^2=1$

$\Rightarrow \{f(0)\}^2+2f(0)-1=0$

$$\Rightarrow f(0)=\frac{-2\pm\sqrt{4+4}}{2}$$

$\Rightarrow f(0)=-1\pm\sqrt{2}$

$\Rightarrow f(0)=\sqrt{2}-1$ $[\because f(x)>0$ for all $x]$

Putting $y=x$ in (i), we get

$\{f(x)\}^2+2f(x)-1=0$ for all $x$

$\Rightarrow 2f(x)f'(x)+2f'(x)=0$ for all $x$

$\Rightarrow 2\{f(x)+1\}f'(x)=0$ for all $x$

$\Rightarrow f'(x)=0$ for all $x[\because f(x)>0$ for all $x]$

9 **(b)**

Given, $f(x)=e^x$, $g(x)=\sin^{-1}x$

Since, $h(x)=f[g(x)]=e^{\sin^{-1}x}$

Now, $h'(x)=e^{\sin x}.\frac{1}{\sqrt{1-x^2}}$

$$\Rightarrow h'(x)=h(x).\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{h'(x)}{h(x)}=\frac{1}{\sqrt{1-x^2}}$$

10 **(b)**

$$\because f(x,y)=\frac{\cos(x-4y)}{\cos(x+4y)}$$

$$\therefore f\left(x,\frac{\pi}{2}\right)=\frac{\cos(x-2\pi)}{\cos(x+2\pi)}=\frac{\cos x}{\cos x}=1$$

$$\therefore \frac{\partial f}{\partial x}=0$$

11 **(d)**

Given, $y=\sin^n x\cos nx$

$$\frac{dy}{dx}=n\sin^{\mathrm{n}-1}x\cos x\cos nx-n\sin^{\mathrm{n}}x\sin nx$$

$=n\sin^{\mathrm{n}-1}x[\cos x\cos nx-\sin x\sin nx]$

$=n\sin^{\mathrm{n}-1}x\cos(\mathrm{n}+1)\,x$

12 **(a)**

Let $f(x) = 3e^{2x}$

Now, $f'(x) = 6e^{2x} = 2f(x)$

Therefore, our assumption is true.

$\therefore (2) = 3e^{2\times 2} = 3e^4$

13 **(c)**

$y^2 = P(x) \Rightarrow 2yy' = P'(x)$ ...(i)

$$\Rightarrow (2y)y'' + y'(2y') = P''(x)$$
$$\Rightarrow 2yy'' = P''(x) - 2(y')^2$$
$$\Rightarrow 2y^3y'' = y^2P''(x) - 2(yy')^2$$
$$= y^2P''(x) - 2\frac{\{P'(x)\}^2}{4}$$

[from Eq.(i)]

$$\Rightarrow 2y^3.y'' = P(x)P''(x) - \frac{1}{2}\{P'(x)\}^2$$
$$\therefore \frac{d}{dx}(2y^3.y'')$$
$$= P(x)P'''(x) + P''(x)P'(x) - P'(x)P''(x)]$$
$$= P(x).P'''(x)$$
$$\Rightarrow 2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right) = P(x)P'''(x)$$

14 **(c)**

Given, $x = e^{y+x}$

$$\Rightarrow \log x = (y+x) \Rightarrow \frac{1}{x} = \frac{dy}{dx} + 1$$

$\Rightarrow \frac{dy}{dx} = \frac{1-x}{x}$

15 **(a)**

Let $y = a\sin^3 t$ and $x = a\cos^3 t$, then

On differentiating w.r.t. $t$, we get

$$\frac{dy}{dt} = 3a\sin^2 t\cos t$$

and $\frac{dx}{dt} = 3a\cos^2 t(-\sin t)$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{3a\sin^2 t\cos t}{3a\cos^2 t(-\sin t)} = -\tan t$$

Again differentiating w.r.t. $x$, we get

$$\frac{d^2y}{dx^2} = -\sec^2 t\frac{dt}{dx} = \frac{-\sec^2 t}{3a\cos^2 t(-\sin t)}$$
$$= \frac{1}{3a}\left(\frac{\sec^4 t}{\sin t}\right)$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}} = \frac{1}{3a}\cdot\frac{4}{\frac{1}{\sqrt{2}}} = \frac{4\sqrt{2}}{3a}$$

16 **(c)**

Let $u = \sin x^3$ and $v = \cos x^3$.

On differentiating w.r.t. $x$, we get

$$\frac{du}{dx} = \cos x^3 . 3x^2 \text{ and} \frac{dv}{dx} = -\sin x^3 . 3x^2$$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{3x^2 \cos x^3}{-3x^2 \sin x^3} = -\cot x^3$$

17 **(d)**

$$\frac{\cos x}{1+\sin x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}$$

$$= \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}} \quad = \frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}$$

$$= \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\therefore = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right) = \frac{\pi}{4} - \frac{x}{2}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{2}$$

18 **(a)**

Given,= $\tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$

$$= \tan^{-1}\sqrt{\frac{1-\cos\left(\frac{\pi}{2} - x\right)}{1+\cos\left(\frac{\pi}{2} - x\right)}}$$

$$= \tan^{-1}\left|\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right|$$

$$= \frac{\pi}{4} - \frac{x}{2} \qquad \left[\because x = \frac{\pi}{6}\right]$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}$$

19 **(a)**

Given, $y = \frac{\log \sin x}{\log \cos x}$

$$\Rightarrow \frac{dy}{dx} = \frac{\cot x \log \cos x + \tan x \log \sin x}{(\log \cos x)^2}$$

20 **(d)**

Given, $y = 2^x . 3^{2x-1}$

$$\Rightarrow \frac{dy}{dx} = 2^x . 3^{2x-1} 2 \log 3 + 3^{2x-1} . 2^x \log 2$$

$$= 2^x 3^{2x-1} \log 18 = y \log 18$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx}\log 18$$
$$= y(\log 18)^2$$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | A | C | B | C | A | D | D | A | B | B |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | D | A | C | C | A | C | D | A | A | D |
| | | | | | | | | | | |

# Topic :-DIFFERENTITATION

1 **(b)**

$\because \quad y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$$= \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$$

$\Rightarrow y = \frac{\pi}{2} \quad \Rightarrow \quad \frac{dy}{dx} = 0$

2 **(a)**

Since, $f(x) = e^{g(x)}$

$\Rightarrow \; e^{g(x+1)} = f(x+1) = xf(x) = xe^{g(x)}$

and $\quad g(x+1) = \log x + g(x)$

$\Rightarrow \; g(x+1) - g(x) = \log x \; \ldots$(i)

Replacing $x$ by $x - \frac{1}{2}$, we get

$g\left(x + \frac{1}{2}\right) - g\left(x - \frac{1}{2}\right) = \log(x - \frac{1}{2})$

$= \log(2x - 1) - \log 2$

$\therefore \; g''\left(x + \frac{1}{2}\right) - g''\left(x - \frac{1}{2}\right) = -\frac{4}{(2x-1)^2} \quad \ldots$(ii)

On substituting, $x = 1,2,3, \ldots N$ in Eq. (ii) and adding, we get

$g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right) = -4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N-1)^2}\right\}$

3 **(b)**

We have,

$y = \log_{x^2+4}(7x^2 - 5x + 1) = \frac{\log_e(7x^2 - 5x + 1)}{\log_e(x^2 + 4)} = \frac{\log_e f(x)}{\log_e g(x)}$

$\therefore \frac{dy}{dx} = \frac{\log_e(g(x) \cdot \frac{f'(x)}{f(x)} - \log_e f(x) \cdot \frac{g'(x)}{g(x)}}{(\log_e g(x))^2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\log_e g(x)}\left\{\frac{f'(x)}{f(x)} - y\frac{g'(x)}{g(x)}\right\}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\log_e(x^2 + 4)}\left\{\frac{14x - 5}{7x^2 - 5x + 1} - \frac{2xy}{x^2 + 4}\right\}$

5 **(a)**

We have, $\; y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$, where $x = \tan\theta$

$\Rightarrow y = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{1}{2}\theta = \frac{1}{2}\tan^{-1} x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2(1+x^2)} \Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{2}$

6 **(b)**

Let $y = \sqrt{\sec\sqrt{x}}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\sec\sqrt{x}}}.\sec\sqrt{x}.\tan\sqrt{x}.\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{4\sqrt{x}}(\sec\sqrt{x})^{3/2}.\sin x$$

7 **(c)**

We have,

$Sgn\, x = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1, & \text{for } x < 0 \end{cases}$

$\therefore g(x) = Sgn \sin x = \begin{cases} \sin x, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -\sin x, & \text{for } x < 0 \end{cases}$

$\Rightarrow g'(x) = \begin{cases} \cos x, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -\cos x, & \text{for } x < 0 \end{cases}$

$\Rightarrow g'(1) = \cos 1$

8 **(c)**

$$\because y = x^{\text{In } x}$$

On taking log on both sides, we get

$\text{In } y = (\text{In } x)^2$

On differentiating w.r.t. $x$,we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{2\text{ In } x}{x}$$

$$\Rightarrow \frac{dy}{dx} = y\frac{2\text{ In } x}{x} = \frac{2\left(x^{\text{In } x}\right)\text{In } x}{x}$$

$$\Rightarrow \frac{dy}{dx} = 2x^{\text{In } x - 1}\text{In } x$$

9 **(d)**

Since, $x = e^t \sin t$ and $y = e^t \cos t$

$\Rightarrow \frac{dx}{dt} = e^t \cos t + \sin t\, e^t$

and $\frac{dy}{dt} = -e^t \sin t + e^t \cos t$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{(\cos t - \sin t)}{(\cos t + \sin t)}$

$\frac{d^2y}{dx^2} = \frac{\begin{bmatrix}(\cos t + \sin t)(-\sin t - \cos t) \\ -(\cos t - \sin t)(-\sin t + \cos t)\end{bmatrix}}{(\cos t + \sin t)^2}\frac{dt}{dx}$

$$= \frac{-(\sin t + \cos t)^2 - (\cos t - \sin t)^2}{(\cos t + \sin t)^2}$$

$$\times \frac{1}{e^t(\cos t + \sin t)}$$

$$= -\frac{2}{e^t(cos\, t + sin\, t)^3}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{(x=\pi)} = \frac{-2}{e^\pi(\cos \pi + \sin \pi)^3} = \frac{2}{e^\pi}$$

10 **(b)**

We have,

$f'(x) = \sin(\log x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$

$$\therefore \frac{dy}{dx} = f'\left(\frac{2x+3}{3-2x}\right) \times \frac{d}{dx}\left(\frac{2x+3}{3-2x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \sin\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\} \times \frac{12}{(3-2x)^2} \quad [\because f'(x) = \sin(\log x)]$$

11 **(d)**

Given, $r = \left[2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)\right]^{1/2}$

$$\frac{dr}{d\phi} = \frac{\left[2 - 2\cos\left(2\phi + \frac{\pi}{4}\right)\sin\left(2\phi + \frac{\pi}{4}\right).2\right]}{2\sqrt{2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)}}$$

$$= \frac{\left[1 - \sin\left(4\phi + \frac{\pi}{2}\right)\right]}{\sqrt{2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)}}$$

$$\Rightarrow \left(\frac{dr}{d\phi}\right)_{\phi=\pi/4} = \frac{\left[1 - \sin\left(\pi + \frac{\pi}{2}\right)\right]}{\sqrt{2.\frac{\pi}{4} + \cos^2\left(\frac{\pi}{2} + \frac{\pi}{4}\right)}}$$

$$= \frac{1+1}{\sqrt{\frac{\pi}{2} + \frac{1}{2}}} = 2\sqrt{\frac{2}{1+\pi}}$$

12 **(d)**

$$\because y = 1 + x + x^2 + \ldots \infty$$

$$\therefore y = \frac{1}{1-x} = (1-x)^{-1}$$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = -\frac{1}{(1-x)^2}(-1) = \frac{1}{(1-x)^2}$$

$$\therefore \frac{dy}{dx} - y = \frac{1}{(1-x)^2} - \frac{1}{(1-x)}$$

$$= \frac{1-1+x}{(1-x)^2} = \frac{x}{(1-x)^2}$$

$$\Rightarrow \frac{dy}{dx} - y = xy^2$$
$$\Rightarrow \frac{dy}{dx} = xy^2 + y$$

13 **(a)**

$f'(5) = \lim_{h\to 0} \frac{f(5+h)-f(5)}{h}$

$$= \lim_{h\to 0} \frac{2f(5)f(h) - f(5)}{h}$$
$$= \lim_{h\to 0} 2f(5)[\frac{f(h)-\frac{1}{2}}{h}]$$
$$\Rightarrow 1024\log 2 = 2f(5)f'(0)$$

Again now, $f(2+3) = 2f(2)f(3)$ ... (i)

$$\Rightarrow \frac{1024\log 2}{2f'(0)} = 2\times 8\times f(3)$$

$\Rightarrow f(3) = \frac{32\log 2}{f'(0)}$ ...(ii)

$\therefore f'(3) = \lim_{h\to 0} \log \frac{f(3+h)-f(3)}{h}$

$$= \lim_{h\to 0} \frac{2f(3)f(h) - f(3)}{h}$$
$$= 2f(3)f'(0)$$
$$= 2\times \frac{32\log 2f'(0)}{f'(0)}$$

$=64\log 2$ [from Eq. (ii)]

14 **(d)**

Let $u = e^{x^2}$ and $v = e^{2x-1}$

On differentiating w.r.t. $x$, we get

$\frac{du}{dx} = e^{x^2}.2x$ and $\frac{dv}{dx} = e^{2x-1}(2)$

$\therefore \frac{du}{dv} = \frac{e^{x^2}.2x}{e^{2x-1}.2}$

$$\Rightarrow \frac{du}{dv} = xe^{x^2-2x+1}$$

$\Rightarrow \left(\frac{du}{dv}\right)_{(x=1)} = 1.e^{1-2+1} = 1$

15 **(b)**

Given, $x = log_e t$

$\Rightarrow e^x = t$ and $y+1 = t^2 \Rightarrow$ y $= e^{2x} - 1$

On differentiating w.r.t. $y$, we get

$2e^{2x}\frac{dx}{dy} = 1$

$\Rightarrow \frac{dx}{dy} = \frac{1}{2e^{2x}}$

Again, differentiating w.r.t. $y$, we get

$$\frac{d^2x}{dy^2} = \frac{1}{2}e^{-2x}(-2)\frac{dx}{dy}$$
$$= -e^{-2x}.\frac{1}{2e^{2x}}$$
$$= -\frac{1}{2}\, e^{-4x}$$

16 **(a)**

Given, $y = \cot^{-1}(\cos 2x)^{1/2}$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{1+\cos 2x} \times \frac{1}{2\sqrt{\cos 2x}} \times -2\sin 2x$$
$$= \frac{2\sin x \cos x}{2\cos^2 x\sqrt{\cos 2x}}$$
$$\Rightarrow \frac{dy}{dx} = \frac{\tan x}{\sqrt{\cos 2x}}$$
$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{6}} = \frac{1/\sqrt{3}}{\sqrt{1/2}} = \sqrt{\frac{2}{3}}$$

17 **(c)**

Polynomial $P(x)$, satisfying the given relation can be taken as $x$

$$ie, P(x) = x$$
$$\therefore P'(x) = 1$$
$$\Rightarrow P'(0) = 1$$

18 **(c)**

$y = (\cos x^2)^2$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = 2\cos x^2\,(-\sin x^2)2x = -2x\sin 2x^2$$

19 **(c)**

We have,

$f(x) = 2^{2x-1}$ and $g(x) = -2^x + 2x\log 2$

$\therefore f'(x) > g'(x)$

$\Rightarrow 2 \times 2^{2x-1}\log 2 > -2^x \log 2 + 2\log 2$

$\Rightarrow 2^{2x} > -2^x + 2$

$\Rightarrow 2^{2x} + 2^x - 2 > 0$

$\Rightarrow (2^x - 1)(2^x + 2) > 0$

$\Rightarrow 2^x - 1 > 0$

$\Rightarrow 2^x > 1 \Rightarrow x > 0 \Rightarrow x \in (0, \infty)$

20 **(c)**

Let $I = \frac{d}{dx}\left\{\begin{matrix}\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) \\ -\tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)\end{matrix}\right\}$

Put $x = \tan\theta$ the given equation

$\therefore \quad I = \frac{d}{dx}\{\tan^{-1}(\tan 2\theta) + \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 4\theta)\}$

$= \frac{d}{dx}(\theta) = \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | B | A | B | C | A | B | C | C | D | B |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | D | D | A | D | B | A | C | C | C | C |
| | | | | | | | | | | |

# Topic :-DIFFERENTITATION

1 **(a)**

$$y=\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\times\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}+\sqrt{1+\sin x}}$$

$$=\frac{2(1+\cos x)}{-2\sin x}=-\cot\frac{x}{2}$$

$$\therefore \frac{dy}{dx}=\frac{1}{2}\text{cosec}^2\frac{x}{2}$$

2 **(a)**

Since, $y=x^{\sin x}+\sqrt{x}$

Let $y_1=x^{\sin x}$ and $y_2=\sqrt{x}$

Now, $y_1=x^{\sin x}$ $\Rightarrow$ $\log y_1=\sin x\log x$

On differentiating w.r.t. $x$, we get

$$\frac{1}{y_1}.\frac{dy_1}{dx}=\cos x\log x+\frac{1}{x}\sin x$$

$$\Rightarrow \frac{dy_1}{dx}=x^{\sin x}[\cos x\log x+\frac{1}{x}\sin x]$$

$$\Rightarrow \left(\frac{dy_1}{dx}\right)_{x=\frac{\pi}{2}}=\left(\frac{\pi}{2}\right)^{\sin\frac{\pi}{2}}\left[\cos\frac{\pi}{2}\log\frac{\pi}{2}+\frac{2}{\pi}\sin\frac{\pi}{2}\right]$$

$$=\frac{\pi}{2}\times\frac{2}{\pi}=1$$

Now, $y_2=\sqrt{x}$ $\Rightarrow$ $\frac{dy_2}{dx}=\frac{1}{2\sqrt{x}}$

$$\Rightarrow\left(\frac{dy_2}{dx}\right)_{x=\frac{\pi}{2}}=\frac{1}{2\sqrt{\frac{\pi}{2}}}=\sqrt{\frac{1}{2\pi}}$$

Since, $y=y_1+y_2$

$\therefore$ At $x=\frac{\pi}{2}$, $\frac{dy}{dx}=\frac{dy_1}{dx}+\frac{dy_2}{dx}$ $\Rightarrow$ $\frac{dy}{dx}=1+\frac{1}{\sqrt{2\pi}}$

3 **(a)**

Since, $y=\frac{3at^2}{1+t^2}$ and $x=\frac{3at}{1+t^3}$

On differentiating given curves w.r.t. $t$ respectively

$$\frac{dy}{dt}=\frac{(1+t^3)(6at)-3at^2(3t^2)}{(1+t^3)^2}=\frac{6at-3at^4}{(1+t^3)^2}$$

and $\frac{dx}{dt}=\frac{(1+t^3)(3a)-3at(3t^2)}{(1+t^3)^2}=\frac{3a-6at^3}{(1+t^3)^2}$

$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3at(2-t^3)}{3a(1-2t^3)}=\frac{t(2-t^3)}{(1-2t^3)}$

4 **(d)**

Given, $y=\frac{\log x}{\log a}+\frac{\log a}{\log x}+1+1$

$\Rightarrow \frac{dy}{dx}=\frac{1}{x\log a}-\frac{\log a}{x(\log x)^2}$

25 **(c)**

We have,

$8f(x)+6f\left(\frac{1}{x}\right)=x+5$ ...(i)

Replacing $x$ by $\frac{1}{x}$, we get

$6f(x)+8f\left(\frac{1}{x}\right)=\frac{1}{x}+5$ ....(ii)

Eliminating $f\left(\frac{1}{x}\right)$ from these two equations, we get

$f(x)=\frac{1}{28}\left(8x-\frac{6}{x}+10\right)$

$\therefore y=x^2f(x)=\frac{1}{28}(8x^3-6x+10x^2)$

$\Rightarrow \frac{dy}{dx}=\frac{1}{28}(24x^2-6+20x)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=-1}=\frac{1}{28}(24-6-20)=-\frac{1}{14}$

6 **(d)**

Since, $y=\sqrt{\frac{1-x}{1+x}}$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx}=\frac{\sqrt{1+x}\times\frac{(-1)}{2\sqrt{1-x}}-\sqrt{1-x}\times\frac{1}{2\sqrt{1+x}}}{(\sqrt{1+x})^2}$$

$$\Rightarrow \frac{dy}{dx}=-\frac{1}{(1+x)\sqrt{1-x^2}}\times\frac{1-x}{1-x}$$

$$\Rightarrow (1-x^2)\frac{dy}{dx}+y=0$$

7 **(a)**

$x^y=e^{2(x-y)}$

$\therefore y\ \log x=2(x-y)$

$\Rightarrow y\,(\log x+2)=2x$

$y=\frac{2x}{\log x+2}$

$\frac{dy}{dx}=\frac{(\log x+2)(2)-2x.\frac{1}{x}}{(\log x+2)^2}$

$$= \frac{2\log x + 4 - 2}{(\log x + 2)^2} = \frac{2(\log x + 1)}{(\log x + 2)^2}$$

8 **(a)**

$x = a(1 + \cos\theta), \quad y = a(\theta + \sin\theta)$

$\frac{dx}{d\theta} = -a\sin\theta, \frac{dy}{d\theta} = a(1 + \cos\theta)$

$\therefore \frac{dy}{dx} = \frac{1+\cos\theta}{-\sin\theta} = \frac{2\cos^2\frac{\theta}{2}}{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$\Rightarrow \frac{dy}{dx} = -\cot\frac{\theta}{2}$

$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}\left(\frac{dy}{dx}\right).\frac{d\theta}{dx}$

$= \frac{d}{d\theta}\left(-\cot\frac{\theta}{2}\right).\frac{1}{-a\sin\theta}$

$= \frac{1}{2}\text{cosec}^2\frac{\theta}{2}.\frac{1}{-a\sin\theta}$

$\therefore \left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{2}} = \frac{1}{2}.2.\frac{1}{-a} = -\frac{1}{a}$

9 **(a)**

Given, $y^2 = ax^2 + bx + c \Rightarrow 2y\frac{dy}{dx} = 2ax + b$

$\Rightarrow 2\left(\frac{dy}{dx}\right)^2 + 2y\left(\frac{d^2y}{dx^2}\right) = 2a$

$\Rightarrow y\frac{d^2y}{dx^2} = a - \left(\frac{dy}{dx}\right)^2$

$\Rightarrow y\frac{d^2y}{dx^2} = a - \left(\frac{2ax+b}{2y}\right)^2$

$\Rightarrow y\frac{d^2y}{dx^2} = \frac{4ay^2 - (2ax+b)^2}{4y^2}$

$\Rightarrow 4y^3\frac{d^2y}{dx^2} = 4a(ax^2 + bx + c) - (4a^2x^2 + 4abx + b^2)$

$\Rightarrow 4y^3\frac{d^2y}{dx^2} = 4ac - b^2$

$\Rightarrow y^3\frac{d^2y}{dx^2} = \frac{4ac-b^2}{4}$ =constant

10 **(d)**

Given, $y = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \dots = \frac{1}{1-\frac{1}{x}}$

$\Rightarrow \quad y = \frac{x}{x-1}$(GP series) ...(i)

$\frac{dy}{dx} = \frac{1(x-1)-x.1}{(x-1)^2} = -\frac{1}{(x-1)^2}$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{y^2}{x^2}$ [from Eq.(i)]

11 **(c)**

Given, $f(x,y) = 2(x-y)^2 - x^4 - y^4$

On differentiating partially w.r.t. $x$, twicely

$f_x = 4(x-y) - 4x^3$

$\Rightarrow f_{xx} = 4 - 12x^2$

$\Rightarrow (f_{xx})_{(0,0)} = 4 - 0 = 4$

Similarly, $f_{yy} = 4 - 12y^2$

$\Rightarrow (f_{yy})_{(0,0)} = 4 - 0 = 4$

and $f_{xy} = -4 + 0$

$\Rightarrow (f_{xy})_{(0,0)} = -4$

$\therefore \left|f_{xx}\, f_{yy} - f_{xy}^2\right|_{(0,0)} = |4(4) - (-4)^2| = 0$

12 **(c)**

$$y = \tan^{-1}\frac{1}{1+x+x^2} + \tan^{-1}\frac{1}{x^2+3x+3} +$$

...+upto $n$ terms

$$= \tan^{-1}\frac{(x+1)-x}{1+x(x+1)} + \tan^{-1}\frac{(x+2)-(x+1)}{1+(x+1)(x+2)} +$$

...+ upto $n$ terms

$$= [\tan^{-1}(x+1) - \tan^{-1}x + \tan^{-1}(x+2) - \tan^{-1}(x+1) + \cdots + \tan^{-1}(x+n) - \tan^{-1}\{x+(n-1)\}]$$

$$= \tan^{-1}(x+n) - \tan^{-1}x$$

$$\therefore\ y'(x) = \frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$$

$$\Rightarrow\ y'(0) = \frac{1}{1+n^2} - 1 = \frac{n^2}{1+n^2}$$

13 **(b)**

Let $f(x) = x^2$

On differentiating w.r.t. $x$, we get

$f'(x) = 2x$

Given that, $f'(a+b) = f'(a) + f'(b)$

$$\Rightarrow 2(a+b) = 2a + 2b$$

$$\Rightarrow 2a + 2b = 2a + 2b$$

14 **(c)**

Let $y = (x+1)^n$

$\therefore\ \frac{dy}{dx} = n(x+1)^{n-1}$

$\frac{d^2y}{dx^2} = n(n-1)(x+1)^{n-2}$

Similarly, $\frac{d^2y}{dx^n} = n(n-1)(n-2)\ldots 3.2.1 = n!$

15 **(d)**

Given, $y = 2^x.3^{2x-1}$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = 2^x.3^{2x-1}\log 3(2) + 2^x.3^{2x-1}\log 2$$

$$\Rightarrow \frac{dy}{dx} = 2^x.3^{2x-1}[2\log 3 + \log 2]$$

$$\Rightarrow \frac{dy}{dx} = y \log 18$$

16 **(a)**

Given, $y = x^2 + \frac{1}{y} \quad \Rightarrow \quad y^2 = x^2 y + 1$

$\Rightarrow 2y \frac{dy}{dx} = y.2x + x^2 \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{2xy}{2y-x^2}$

17 **(d)**

$$y = \frac{\frac{(e^x + e^{-x})}{2}}{\frac{(e^x - e^{-x})}{2}} = \frac{\cosh x}{\sinh x}$$

$\Rightarrow y = \coth x \Rightarrow \frac{dy}{dx} = -\text{cosech}^2\, x$

18 **(c)**

$$f(f(x)) = f(|x-2)$$
$$= ||x-2|-2|$$
$$= x - 4 \quad (\because x > 20)$$
$$\Rightarrow g(x) = x - 4$$
$$\therefore g'(x) = 1$$

19 **(b)**

$$5f(x) + 3f\left(\frac{1}{x}\right) = x + 2 \text{ ... (i)}$$

On replacing $x$ by $\frac{1}{x}$, we get

$$5f\left(\frac{1}{x}\right) + 3f(x) = \frac{1}{x} + 2 \text{ ... (ii)}$$

On multiplying Eq. (i) by 5 and Eq. (ii) by 3 and then on subtracting, we get

$$\therefore 16f(x) = 5x - \frac{3}{x} + 4$$

$$\Rightarrow xf(x) = \frac{5x^2 - 3 + 4x}{16} = y$$

$$\therefore \frac{dy}{dx} = \frac{10x + 4}{16}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{10 + 4}{16} = \frac{7}{8}$$

20 **(b)**

$f(\log x) = \log\log(x)$

$\Rightarrow \frac{d}{dx}\{f(\log x)\} = \frac{1}{x}.\frac{1}{\log x} = (x \log x)^{-1}$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | A | A | A | D | C | D | A | A | A | D |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | C | C | B | C | D | A | D | C | B | B |
| | | | | | | | | | | |

SESSION : 2025-26

# DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

**SOLUTIONS**

SUBJECT : MATHS
DPP NO. :10

## Topic :-DIFFERENTITATION

2 **(a)**

Given, $f(x)=|x|^3=\begin{cases}0, & x=0\\ x^3, & x>0\\ -x^3, & x<0\end{cases}$

Now, $Rf'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$

$$=\lim_{h\to 0}\frac{h^3-0}{h}=0$$

and $Lf'(0)=\lim_{h\to 0}\frac{f(-h)-f(0)}{-h}$

$$=\lim_{h\to 0}\frac{-h^3-0}{-h}=0$$

$$\because Rf'(0)=Lf'(0)=0$$

$$\therefore f'(0)=0$$

3 **(c)**

We have, $y=\log|x|=\begin{cases}\log x, & x>0\\ \log(-x), & x<0\end{cases}$

$$\therefore \frac{dy}{dx}=\begin{cases}\frac{1}{x}, & x>0\\ \frac{1}{-x}(-1)=\frac{1}{x}, & x<0\end{cases}$$

$\Rightarrow \frac{dy}{dx}=\frac{1}{x}, x\neq 0$

5 **(b)**

We have, $f(x)=x+2$

$\therefore f'(x)=1$ for all $x\Rightarrow f'(x)=1$ for all $x$

6 **(c)**

We have,

$f(x)-\log_x(\log_e x)$

$\Rightarrow f(x)=\frac{\log_e(\log_e x)}{\log_e x}$

$\Rightarrow f'(x)=\frac{\log_e x\times\frac{1}{x\log_e x}-\frac{1}{x}\log_e(\log_e x)}{(\log_e x)^2}$

$\Rightarrow f'(e)=\frac{1}{e}-\frac{1}{e}\times\log(1)=\frac{1}{e}$

7 **(b)**

Given, $\sin(x+y)+\cos(x+y)=\log(x+y)$

On differentiating w.r.t. $x$, we get

$$\cos(x+y)\left(1+\frac{dy}{dx}\right)-\sin(x+y)\left(1+\frac{dy}{dx}\right)$$

$$=\frac{1}{(x+y)}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow 1+\frac{dy}{dx}=0 \quad \Rightarrow \quad \frac{d^2y}{dx^2}=0$$

8 **(b)**

$$10^{-x\tan x}\frac{d}{dx}(10^{x\tan x})$$
$$=10^{-x\tan x}10^{x\tan x}\log 10(\tan x+x\sec^2 x)$$
$$=\log 10(\tan x+x\sec^2 x)$$

9 **(c)**

Given, $y=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}\dots$

$$\Rightarrow \quad y = e^{-x} \quad \Rightarrow \quad \frac{dy}{dx}=-e^{-x}$$

$$\therefore \frac{d^2y}{dx^2}=e^{-x}=y$$

10 **(b)**

Let $y_1=\sec^{-1}\frac{1}{2x^2-1}$ and $y_2=\sqrt{1-x^2}$

$\Rightarrow \frac{dy_1}{dx}=\frac{-2}{\sqrt{1-x^2}}$ and $\frac{dy_2}{dx}=\frac{-x}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy_1}{dy_2}=\frac{2}{x} \quad \Rightarrow \quad \left(\frac{dy_1}{dy_2}\right)_{x=1/2}=4$

11 **(d)**

We have,

$$y=\cos 2x\cos 3x=\frac{1}{2}[\cos 5x+\cos x]$$

$$\therefore y_n=\frac{1}{2}\left\{\frac{d^n}{dx^n}(\cos 5x)+\frac{d^n}{dx^n}(\cos x)\right\}$$

$$\Rightarrow y_n=\frac{1}{2}\left\{5^n\cos\left(\frac{n\pi}{2}+5x\right)+\cos\left(\frac{n\pi}{2}+x\right)\right\}$$

12 **(c)**

Given, $f(x)=\log_{x^2}(\log_e x)=\frac{1}{2}\log_x(\log_e x)$

$$\Rightarrow (x)=\frac{1}{2}\frac{\log_e\log_e x}{\log_e x}$$

$$\Rightarrow f'(x)=\frac{1}{2}\frac{\log_e x\left(\frac{1}{x\log_e x}\right)-\log_e\log_e x\times\frac{1}{x}}{(\log_e x)^2}$$

$\Rightarrow\ f'(x) = \frac{1}{2}\frac{\frac{1}{x} - \frac{1}{x}\log_e \log_e x}{(\log_e x)^2}$

At $x = e, f'(e) = \frac{1}{2}\frac{\frac{1}{e} - \frac{1}{e}\log_e 1}{(1)^2}$

$\Rightarrow\ f'(e) = \frac{1}{2e}$

13 **(d)**

Given, $f(x) = \sin x, g(x) = x^2$

and $h(x) = \log_e x$

Also , $F(x) = (hogof)(x)$

$$\therefore\ (hogof)(x) = (hog)(\sin x)$$

$$\Rightarrow \quad = h(\sin x^2)$$

$$\Rightarrow F(x) = 2\log\sin x$$

On differentiating, we get

$$F'(x) = 2\cot x$$

Again differentiating, we get

$$F''(x) = -2\,\text{cosec}^2 x$$

14 **(c)**

Given, $\quad x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

and

$y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

$\Rightarrow = \tan^{-1} t,$

and $\ y = \tan^{-1} t$

$\therefore\ \ y = x \Rightarrow \frac{dy}{dx} = 1$

15 **(c)**

$$\because y = \tan^{-1}\left(\frac{\log\left(\frac{e}{x^2}\right)}{\log ex^2}\right) + \tan^{-1}\left(\frac{3 + 2\log x}{1 - 6\log x}\right)$$

$$= \tan^{-1}\left(\frac{1 - \log x^2}{1 + \log x^2}\right) + \tan^{-1}\left(\frac{3 + 2\log x}{1 - 6\log x}\right)$$

$$= \tan^{-1}(1) - \tan^{-1}(2\log x) + \tan^{-1}(3) + \tan^{-1}(2\log x)$$

$$\therefore y = \tan^{-1}(1) + \tan^{-1}(3)$$

$$\Rightarrow \frac{dy}{dx} = 0\ \Rightarrow\ \frac{d^2y}{dx^2} = 0$$

16 **(b)**

Let $y = \sin^2\cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}$

Put $x = \cos\theta\ \Rightarrow \theta = \cos^{-1} x$

$$\Rightarrow\ y = \sin^2\cot^{-1}\left\{\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right\}$$

$$\Rightarrow y = \sin^2 \cot^{-1}\left\{\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\right\}$$

$$= \sin^2 \cot^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$\Rightarrow y = \sin^3\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

On differentiating w.r.t. θ, we get

$$\frac{dy}{d\theta} = 2\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) - \left(-\frac{1}{2}\right)$$

$$\Rightarrow \frac{dy}{d\theta} = -\frac{\sin(\pi - \theta)}{2} = -\frac{\sin\theta}{2} = -\frac{1}{2}\sqrt{1 - x^2}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{d\theta}.\frac{d\theta}{dx} = \frac{-1}{2}\sqrt{1 - x^2}\frac{d}{dx}(\cos^{-1} x)$$

$$= -\frac{\sqrt{1 - x^2}}{2}\left(\frac{-1}{\sqrt{1 - x^2}}\right) = \frac{1}{2}$$

17 **(c)**

In the neighbourhood of $x = \sqrt[5]{\frac{\pi}{2}}$, we have

$[x] = 1$

Therefore, in the neighbourhood of $x = \sqrt[5]{\frac{\pi}{2}}$, we have

$f(x) = \sin\left\{\frac{\pi}{2}[x] - x^5\right\} = \sin\left(\frac{\pi}{2} - x^5\right) = \cos x^5$

$\Rightarrow f'(x) = -5x^4 \sin x^5$

$\Rightarrow f'\left(\sqrt[5]{\frac{\pi}{2}}\right) = -5\left(\frac{\pi}{2}\right)^{4/5} \sin\frac{\pi}{2} = -5\left(\frac{\pi}{2}\right)^{4/5}$

18 **(b)**

Since, $h'(x) = 2f(x)f'(x) + 2g(x)g'(x)$

Now, $f'(x) = g(x)$ and $f''(x) = -f(x)$

$\Rightarrow f''(x) = g'(x)$ and $f''(x) = -f(x)$

$$\Rightarrow -f(x) = g'(x)$$

Thus, $f'(x) = g(x)$ and $g'(x) = -f(x)$

$$\therefore h'(x) = -2g(x)\,g'(x) + 2g(x)g'(x)$$

$= 0, \forall x$

$\Rightarrow h(x)$ =constant for all $x$

But $h(5) = 11$

Hence, $h(x) = 11$ for all $x$

19 **(d)**

$$f(x) = \begin{vmatrix} x^3 & x^4 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow \frac{d}{dx} f(x) = \begin{vmatrix} 3x^2 & 4x^3 & 6x \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow \frac{d^2}{dx^2} f(x) = \begin{vmatrix} 6x & 12x^2 & 6 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow \frac{d^3}{dx^3} f(x) = \begin{vmatrix} 6 & 24x & 0 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow \frac{d^4}{dx^4} f(x) = \begin{vmatrix} 0 & 24 & 0 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$= -24 \begin{vmatrix} 1 & 4 \\ p & p^3 \end{vmatrix} = -24(p^3 - 4p)$$

Hence, $\frac{d^4}{dx^4} f(x)$ is a constant.

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | A | A | C | B | B | C | B | B | C | B |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | D | C | D | C | C | B | C | B | D | B |
| | | | | | | | | | | |